Priority Academic Student Skills

OVERVIEW

High School

The *Priority Academic Student Skills (PASS)* in mathematics for high school establishes a framework for a curriculum that reflects the needs of all students. Such a curriculum recognizes that they will spend their adult lives in a society increasingly dominated by technology and quantitative methods.

A broadened view of mathematics will include the traditional topics of algebra and geometry but must also include the mathematical processes of problem-solving, communication, reasoning, connections, and representation. Although they are stated separately for emphasis, these process standards should be integrated throughout the high school core curriculum.

A school's curriculum in mathematics should be organized to permit all students to progress as far into the mathematics proposed here as their achievement with the objectives allows. Schools should use this material to create a curriculum most beneficial to their students. Those students planning to continue their mathematics education should study additional advanced mathematics topics such as trigonometry and calculus.

The curriculum is intended to provide a common body of mathematical ideas accessible to all students. It is recognized that students entering high school differ in many ways, including mathematical achievement, but it is believed these differences are best addressed by extensions of the proposed content rather than by deletions.

The increasing role of technology in instruction will alter the teaching and learning of mathematics. Calculators and computers should be integrated throughout the curriculum so that students will concentrate on the problem-solving process as well as the calculations associated with problems.

PROCESS STANDARDS

High School

The National Council of Teachers of Mathematics (NCTM) has identified five process standards: Problem Solving, Reasoning and Proof, Communication, Connections, and Representation. Active involvement by students using these processes is likely to broaden mathematical understandings and lead to increasingly sophisticated abilities required to meet mathematical challenges in meaningful ways.

Process Standard 1: Problem Solving

1. Apply a wide variety of problem-solving strategies (identify a pattern, use equivalent representations) to solve problems from within and outside mathematics.

2. Identify the problem from a described situation, determine the necessary data and apply appropriate problem-solving strategies.

Process Standard 2: Communication

1. Use mathematical language and symbols to read and write mathematics and to converse with others.

2. Demonstrate mathematical ideas orally and in writing.

3. Analyze mathematical definitions and discover generalizations through investigations.

Process Standard 3: Reasoning

1. Use various types of logical reasoning in mathematical contexts and real-world situations.

2. Prepare and evaluate suppositions and arguments.

3. Verify conclusions, identify counterexamples, test conjectures, and justify solutions to mathematical problems.

4. Justify mathematical statements through proofs.

Process Standard 4: Connections

1. Link mathematical ideas to the real world (e.g., statistics helps qualify the confidence we can have when drawing conclusions based on a sample).

2. Apply mathematical problem-solving skills to other disciplines.

3. Use mathematics to solve problems encountered in daily life.

4. Relate one area of mathematics to another and to the integrated whole (e.g., connect equivalent representations to corresponding problem situations or mathematical concepts).

Process Standard 5: Representation

1. Use algebraic, graphic, and numeric representations to model and interpret mathematical and real world situations.

2. Use a variety of mathematical representations as tools for organizing, recording, and communicating mathematical ideas (e.g., mathematical models, tables, graphs, spreadsheets).

3. Develop a variety of mathematical representations that can be used flexibly and appropriately.

MATHEMATICS CONTENT STANDARDS

Algebra I
(Updated August 2006)

The following skills are required of all students completing Algebra I. **Major Concepts** should be taught in depth using a variety of methods and applications (concrete to the abstract). **Maintenance Concepts** have been taught previously and are a necessary foundation for this course. The major concepts are considered minimal exit skills and districts are strongly encouraged to exceed these skills when building an Algebra I curriculum. Visual and physical models, calculators, and other technologies are recommended when appropriate and can enhance both instruction and assessment.

MAJOR CONCEPTS	MAINTENANCE CONCEPTS

Number Sense and Algebraic Operations -
   Polynomials, Exponents, Expressions
	Number Sense & Algebraic Reasoning-
	   Equations, Inequalities, Exponents,
	   Rational Numbers

Relations and Functions -
   Linear Functions & Slope
   Formulas
	Geometry
	   Volume, Surface Area, Ratio,
	   Proportion, Formulas

Data Analysis, Statistics and Probability-
   Tables, Graphs, Charts, Scatter Plots
	Data Analysis and Statistics -
	   Graphical Representations,
	Measures of Central Tendency

Standard 1: **Number Sense and Algebraic Operations - The student will use expressions and equations to model number relationships.**

   1.   Equations and Formulas

      a.   Translate word phrases and sentences into expressions and equations and vice versa.

      b.   Solve literal equations involving several variables for one variable in terms of the others.

      c.   Use the formulas from measurable attributes of geometric models (perimeter, circumference, area and volume), science, and statistics to solve problems within an algebraic context.

      d.   Solve two-step and three-step problems using concepts such as rules of exponents, rate, distance, ratio and proportion, and percent.

   2.   Expressions

      a.   Simplify and evaluate linear, absolute value, rational and radical expressions.

      b.   Simplify polynomials by adding, subtracting or multiplying.

      c.   Factor polynomial expressions.

Note: Asterisks (*) have been used to identify standards and objectives that must be assessed by the local school district. All other skills may be assessed by the Oklahoma School Testing Program (OSTP).

Standard 2: **Relations and Functions - The student will use relations and functions to model number relationships.**

1.   Relations and Functions

   a.   Distinguish between linear and nonlinear data.

   b.   Distinguish between relations and functions.

   c.   Identify dependent and independent variables, domain and range.

   d.   Evaluate a function using tables, equations or graphs.

2.   Linear Equations and Graphs

   a.   Solve linear equations by graphing or using properties of equality.

   b.   Recognize the parent graph of the functions $y = k$, $y = x$, $y = |x|$, and predict the effects of transformations on the parent graph.

   c.   Slope

      I.   Calculate the slope of a line using a graph, an equation, two points or a set of data points.

      II.   Use the slope to differentiate between lines that are parallel, perpendicular, horizontal, or vertical.

      III.   Interpret the slope and intercepts within the context of everyday life (e.g., telephone charges based on base rate [y-intercept] plus rate per minute [slope]).

   d.   Develop the equation of a line and graph linear relationships given the following: slope and y-intercept, slope and one point on the line, two points on the line, x-intercept and y-intercept, a set of data points.

   e.   Match equations to a graph, table, or situation and vice versa.

3.   Linear Inequalities and Graphs

   a.   Solve linear inequalities by graphing or using properties of inequalities.

   b.   Match inequalities (with 1 or 2 variables) to a graph, table, or situation and vice versa.

4.   Solve a system of linear equations by graphing, substitution or elimination.

*   5.   Nonlinear Functions

   a.   Match exponential and quadratic functions to a table, graph or situation and vice versa.

   b.   Solve quadratic equations by graphing, factoring, or using the quadratic formula.

Standard 3:   Data Analysis, Probability and Statistics - The student will use data analysis, probability and statistics to formulate and justify predictions from a set of data.

1.   Data Analysis

   a. Translate from one representation of data to another and understand that the data can be represented using a variety of tables, graphs, or symbols and that different modes of representation often convey different messages.

   b. Make valid inferences, predictions, and/or arguments based on data from graphs, tables, and charts.

   c. Solve two-step and three-step problems using concepts such as probability and measures of central tendency.

2.   Collect data involving two variables and display on a scatter plot; interpret results using a linear model/equation and identify whether the model/equation is a line best fit for the data.

MATHEMATICS CONTENT STANDARDS

Geometry
(Updated February 2007)

The following skills are required of all students completing Geometry. **Major Concepts** should be taught in depth using a variety of methods and applications (concrete to the abstract). **Maintenance Concepts** have been taught previously and are a necessary foundation for this course. The major concepts are considered minimal exit skills and districts are strongly encouraged to exceed these skills when building a Geometry curriculum. Visual and physical models, calculators, and other technologies are recommended when appropriate and can enhance both instruction and assessment.

MAJOR CONCEPTS	MAINTENANCE CONCEPTS
Logical Reasoning	Ratios, Proportions
Properties	Perimeter, Area, Surface Area, Volume
Coordinate Geometry	Equations
Triangles	Formulas

Standard 1: **Logical Reasoning - The student will use deductive and inductive reasoning to solve problems.**

1. Identify and use logical reasoning skills (inductive and deductive) to make and test conjectures, formulate counter examples, and follow logical arguments.

2. State, use, and examine the validity of the converse, inverse, and contrapositive of "if-then" statements.

* 3. Compare the properties of Euclidean geometry to non-Euclidean geometries (for example, elliptical geometry, as shown on the surface of a globe, does not uphold the parallel postulate).

Standard 2: **Properties of 2-Dimensional Figures - The student will use the properties and formulas of geometric figures to solve problems.**

* 1. Use geometric tools (for example, protractor, compass, straight edge) to construct a variety of figures.

2. Line and Angle Relationships

   a. Use the angle relationships formed by parallel lines cut by a transversal to solve problems.

   b. Use the angle relationships formed by two lines cut by a transversal to determine if the two lines are parallel and verify, using algebraic and deductive proofs.

   c. Use relationships between pairs of angles (for example, adjacent, complementary, vertical) to solve problems.

Note: Asterisks (*) have been used to identify standards and objectives that must be assessed by the local school district. All other skills may be assessed by the Oklahoma School Testing Program (OSTP).

3. Polygons and Other Plane Figures

    a. Identify, describe, and analyze polygons (for example, convex, concave, regular, pentagonal, hexagonal, n-gonal).

    b. Apply the interior and exterior angle sum of convex polygons to solve problems, and verify using algebraic and deductive proofs.

    c. Develop and apply the properties of quadrilaterals to solve problems (for example, rectangles, parallelograms, rhombi, trapezoids, kites).

    d. Use properties of 2-dimensional figures and side length, perimeter or circumference, and area to determine unknown values and correctly identify the appropriate unit of measure of each.

4. Similarity

    a. Determine and verify the relationships of similarity of triangles, using algebraic and deductive proofs.

    b. Use ratios of similar 2-dimensional figures to determine unknown values, such as angles, side lengths, perimeter or circumference, and area.

5. Congruence

    a. Determine and verify the relationships of congruency of triangles, using algebraic and deductive proofs.

    b. Use the relationships of congruency of 2-dimensional figures to determine unknown values, such as angles, side lengths, perimeter or circumference, and area.

6. Circles

    a. Find angle measures and arc measures related to circles.

    b. Find angle measures and segment lengths using the relationships among radii, chords, secants, and tangents of a circle.

Standard 3: Triangles and Trigonometric Ratios - The student will use the properties of right triangles and trigonometric ratios to solve problems.

1. Use the Pythagorean Theorem and its converse to find missing side lengths and to determine acute, right, and obtuse triangles, and verify using algebraic and deductive proofs.

2. Apply the 45-45-90 and 30-60-90 right triangle relationships to solve problems, and verify using algebraic and deductive proofs.

3. Express the trigonometric functions as ratios and use sine, cosine, and tangent ratios to solve real-world problems.

* 4. Use the trigonometric ratios to find the area of a triangle.

Standard 4:   Properties of 3-Dimensional Figures - The student will use the properties and formulas of geometric figures to solve problems.

1.   Polyhedra and Other Solids

    a.   Identify, describe, and analyze polyhedra (for example, regular, decahedral).

    b.   Use properties of 3-dimensional figures; side lengths, perimeter or circumference, and area of a face; and volume, lateral area, and surface area to determine unknown values and correctly identify the appropriate unit of measure of each.

2.   Similarity: Use ratios of similar 3-dimensional figures to determine unknown values, such as angles, side lengths, perimeter or circumference of a face, area of a face, and volume.

3.   Create a model of a 3-dimensional figure from a 2-dimensional drawing and make a 2-dimensional representation of a 3-dimensional object (for example, nets, blueprints, perspective drawings).

Standard 5:   Coordinate Geometry - The student will solve problems with geometric figures in the coordinate plane.

1.   Find the distance between two points; the midpoint of a segment; and calculate the slopes of parallel, perpendicular, horizontal, and vertical lines.

2.   Properties of Figures

    a.   Given a set of points determine the type of figure formed based on its properties.

    b.   Use transformations (reflection, rotation, translation) on geometric figures to solve problems within coordinate geometry.

MATHEMATICS CONTENT STANDARDS

Algebra II
(Updated February 2007)

The following skills are required of all students completing Algebra II. **Major Concepts** should be taught in depth using a variety of methods and applications (concrete to the abstract). **Maintenance Concepts** have been taught previously and are a necessary foundation for this course. The major concepts are considered minimal exit skills and districts are strongly encouraged to exceed these skills when building an Algebra II curriculum. Visual and physical models, calculators, and other technologies are recommended when appropriate and can enhance both instruction and assessment.

MAJOR CONCEPTS

MAINTENANCE CONCEPTS

**Number Systems and Algebraic Operations –
    Real and Complex Numbers**
**Functions and Relations -
    Quadratic, Polynomial, Exponential,
    Logarithmic, Rational**
**Data Analysis, Statistics, and Probability
    Relationships, Measures of Central
    Tendency and Variability, Sequences and
    Series**

Polynomials
Exponents
Expressions
Slope
Data Displays

Standard 1: **Number Systems and Algebraic Operations - The student will perform operations with rational, radical, and polynomial expressions, as well as expressions involving complex numbers.**

1. Rational Exponents

    a. Convert expressions from radical notations to rational exponents and vice versa.

    b. Add, subtract, multiply, divide, and simplify radical expressions and expressions containing rational exponents.

2. Polynomial and Rational Expressions

    a. Divide polynomial expressions by lower degree polynomials.

    b. Add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.

3. Complex Numbers

    * a. Recognize that to solve certain problems and equations, number systems need to be extended from real numbers to complex numbers.

Note: Asterisks (*) have been used to identify standards and objectives that must be assessed by the local school district. All other skills may be assessed by the Oklahoma School Testing Program (OSTP).

b. Add, subtract, multiply, divide, and simplify expressions involving complex numbers.

Standard 2: **Relations and Functions - The student will use the relationships among the solution of an equation, zero of a function, x-intercepts of a graph, and factors of a polynomial expression to solve problems involving relations and functions.**

1. Functions and Function Notation

   a. Recognize the parent graphs of polynomial, exponential, radical, quadratic, and logarithmic functions and predict the effects of transformations on the parent graphs, using various methods and tools which may include graphing calculators.

   b. Add, subtract, multiply, and divide functions using function notation.

   c. Combine functions by composition.

   d. Use algebraic, interval, and set notations to specify the domain and range of functions of various types.

   e. Find and graph the inverse of a function, if it exists.

2. Systems of Equations

   a. Model a situation that can be described by a system of equations or inequalities and use the model to answer questions about the situation.

   b. Solve systems of linear equations and inequalities using various methods and tools which may include substitution, elimination, matrices, graphing, and graphing calculators.

   *c. Use either one quadratic equation and one linear equation or two quadratic equations to solve problems.

3. Quadratic Equations and Functions

   a. Solve quadratic equations by graphing, factoring, completing the square and quadratic formula.

   b. Graph a quadratic function and identify the x- and y-intercepts and maximum or minimum value, using various methods and tools which may include a graphing calculator.

   c. Model a situation that can be described by a quadratic function and use the model to answer questions about the situation.

4. Identify, graph, and write the equations of the conic sections (circle, ellipse, parabola, and hyperbola).

5. Exponential and Logarithmic Functions

   a. Graph exponential and logarithmic functions.

   b. Apply the inverse relationship between exponential and logarithmic functions to convert from one form to another.

   c. Model a situation that can be described by an exponential or logarithmic function and use the model to answer questions about the situation.

6. Polynomial Equations and Functions

   a. Solve polynomial equations using various methods and tools which may include factoring and synthetic division.

   b. Sketch the graph of a polynomial function.

   c. Given the graph of a polynomial function, identify the x- and y-intercepts, relative maximums and relative minimums, using various methods and tools which may include a graphing calculator.

   d. Model a situation that can be described by a polynomial function and use the model to answer questions about the situation.

7. Rational Equations and Functions

   a. Solve rational equations.

   b. Sketch the graph of a rational function.

   c. Given the graph of a rational function, identify the x- and y-intercepts, vertical asymptotes, using various methods and tools which may include a graphing calculator.

   d. Model a situation that can be described by a rational function and use the model to answer questions about the situation.

Standard 3: **Data Analysis and Statistics - The student will use data analysis and statistics to formulate and justify predictions from a set of data.**

1. Analysis of Collected Data Involving Two Variables

   a. Interpret data on a scatter plot using a linear, exponential, or quadratic model/equation.

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b. Identify whether the model/equation is a curve of best fit for the data, using various methods and tools which may include a graphing calculator.

* 2.   Measures of Central Tendency and Variability

a. Analyze and synthesize data from a sample using appropriate measures of central tendency (mean, median, mode, weighted average).

b. Analyze and synthesize data from a sample using appropriate measures of variability (range, variance, standard deviation).

c. Use the characteristics of the Gaussian normal distribution (bell-shaped curve) to solve problems.

d. Identify how given outliers affect representations of data.

3.   Identify and use arithmetic and geometric sequences and series to solve problems.

GLOSSARY

addend - in the addition problem $3 + 2 + 6 = 11$, the addends are 3, 2, and 6.

algorithm - step-by-step procedure for solving a problem.

analog time - time displayed on a timepiece having hour and minute hands.

array - (rectangular) an orderly arrangement of objects into a rectangular configuration (e.g., take six tiles and arrange two long and three wide to form a rectangle).

attribute - characteristics (e.g., size, shape, color, weight).

combinations - a selection of objects without regard to order.

complementary angles - two angles whose measure have a sum of 90 degrees.

complex numbers - numbers of the form a + bi, where a and b are real numbers and i equals the square root of -1.

composite numbers - any positive integer exactly divisible by one or more positive integers other than itself and 1.

congruent - geometric figures having exactly the same size and shape.

conic sections - circles, parabolas, ellipses, and hyperbolas which can all be represented by passing a plane through a hollow double cone.

conjecture - a statement believed to be true but not proved.

cosine - in a right triangle, the cosine of an acute angle is the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse.

dependent events - events that influence each other. If one of the events occurs, it changes the probability of the other event.

domain of a relation - the set of all the first elements or x-coordinates of a relation.

exponential function - an exponential function with base b is defined by $y = b^x$, where $b > 0$ and b is not equal to 1.

expression - a mathematical phrase that can include operations, numerals and variables. In algebraic terms: $2m + 3x$; in numeric terms: $2.4 - 1.37$.

Fibonacci sequence - the sequence of numbers, 1, 1, 2, 3, 5, 8, 13, 21, . . . where each number, except the first two, is the sum of the two preceding numbers.

function - a relation in which each element of the domain is paired with exactly one element of the range.

function machine - an input/output box (often made with milk cartons, boxes, or drawn on the board) to show one number entering and a different number exiting. Students guess the rule that produced the second number (e.g., enter 3, exit 5, rule: add 2).

histogram - a bar graph of a frequency distribution.

imaginary number - any complex number, a + bi, for which a = 0 and b does not = 0.

independent events - events that do not influence one another. Each event occurs without changing the probability of the other event.

integers - . . . -2, -1, 0, 1, 2, . . .

intercepts (x & y) - the x (y)-coordinate of the point where a graph intercepts the x (y)- axis.

inverse operations - operations that undo each other (e.g., addition and subtraction are inverse operations; multiplication and division are inverse operations).

irrational numbers - nonterminating, nonrepeating decimals (e.g., square root of 2, pi).

logarithmic functions - logarithmic function with base b is the inverse of the exponential function, and is defined by $x = \log_b y$ (y > 0, b > 0, b not equal to 1).

manipulatives - concrete materials (e.g., buttons, beans, egg and milk cartons, counters, attribute and pattern blocks, interlocking cubes, base-10 blocks, geometric models, geoboards, fractions pieces, rulers, balances, spinners, dot paper) to use in mathematical calculations.

mean - in a set of n numbers, the sum of the numbers divided by n.

median - the middle number in the set, or the mean of the two middle numbers, when the numbers are arranged in order from least to greatest.

mode - a number in a set of data that occurs most often.

multiple - a number that is the product of a given integer and another integer (e.g., 6 and 9 are multiples of 3).

natural numbers - (counting numbers) 1, 2, 3, 4, . . .

nonstandard measurement - a measurement determined by the use of nonstandard units like hands, paper clips, beans, cotton balls, etc.

number sense - involves the understanding of number size (relative magnitude), number representations, number operations, referents for quantities and measurements used in everyday situations, etc.

operation - addition, subtraction, multiplication, division, etc.

order of operations - rules for evaluating an expression: work first within parentheses; then calculate all powers, from left to right; then do multiplications or divisions, from left to right; then do additions and subtractions, from left to right.

ordinal - a number that is used to tell order (e.g., first, fifth).

prime number - an integer greater than one whose only positive factors are 1 and itself (e.g., 2, 3, 5, 7, 11, 13 . . .).

probability - the study and measure of the likelihood of an event happening.

properties of arithmetic - for all real numbers a, b and c:
  commutative property: $a + b = b + a$ and $a \cdot b = b \cdot a$
  associative property: $(a + b) + c = a + (b + c)$ and $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
  distributive property: $a(b + c) = (a \cdot b) + (a \cdot c)$
  identity property: $a + 0 = a$ and $a \cdot 1 = a$
  inverse property: $a + (-a) = 0$ and $a \cdot \frac{1}{a} = 1$

proportion - a statement that ratios are equal.

quadrants - the four regions formed by the axes in a coordinate plane.

quadratic equation - an equation of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers and a is not equal to 0.

quadratic formula - if $ax^2 + bx + c = 0$, where a, b and c are real numbers and a is not equal to

0, then x = $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

range of a relation - the set of all the second elements or y-coordinates of a relation is called the range.

ratio - the comparison of two quantities by division.

rational numbers - quotients of integers (commonly called fractions - includes both positive and negative numbers).

real numbers - the set of all rational and irrational numbers.

recursive patterns - patterns in which each number is found from the previous number by repeating a process (e.g., Fibonacci numbers).

relation - a set of one or more pairs of numbers.

relative magnitude - the size of an object or number compared to other objects and numbers.

scatter plot - a dot or point graph of data.

sequence - a set of numbers arranged in a pattern.

sine - in a right triangle, the sine of an acute angle is the ratio of the length of the leg opposite the angle to the length of the hypotenuse.

slope of a line - the ratio of the change in y to the corresponding change in x. For any

two points (x_1, y_1) and (x_2, y_2), $m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$.

spatial sense - involves building and manipulating mental representations of 2- and 3-dimensional objects and ideas.

standard deviation - measures how much each value in the data differs from the mean of the data.

statistics - the study of data.

stem-and-leaf plot - a frequency distribution made by arranging data in the following way (e.g., student scores on a test were 96, 87, 77, 93, 85, 85, and 75 would be displayed as

      9 | 6, 3
      8 | 7, 5, 5
      7 | 7, 5

supplementary angles - two angles whose measures have a sum of 180 degrees.

supposition - (act of supposing) making a statement or assumption without proof.

tangent - in a right triangle, the tangent is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle.

transformation - motion of a geometric figure (rotation [turn], translation [slide], and reflection [flip]).

whole numbers - 0, 1, 2, 3, 4, . . .