The Partnership for Assessment of Readiness for College and Careers

Webinar: Model Content Frameworks for Mathematics

November 21, 2011

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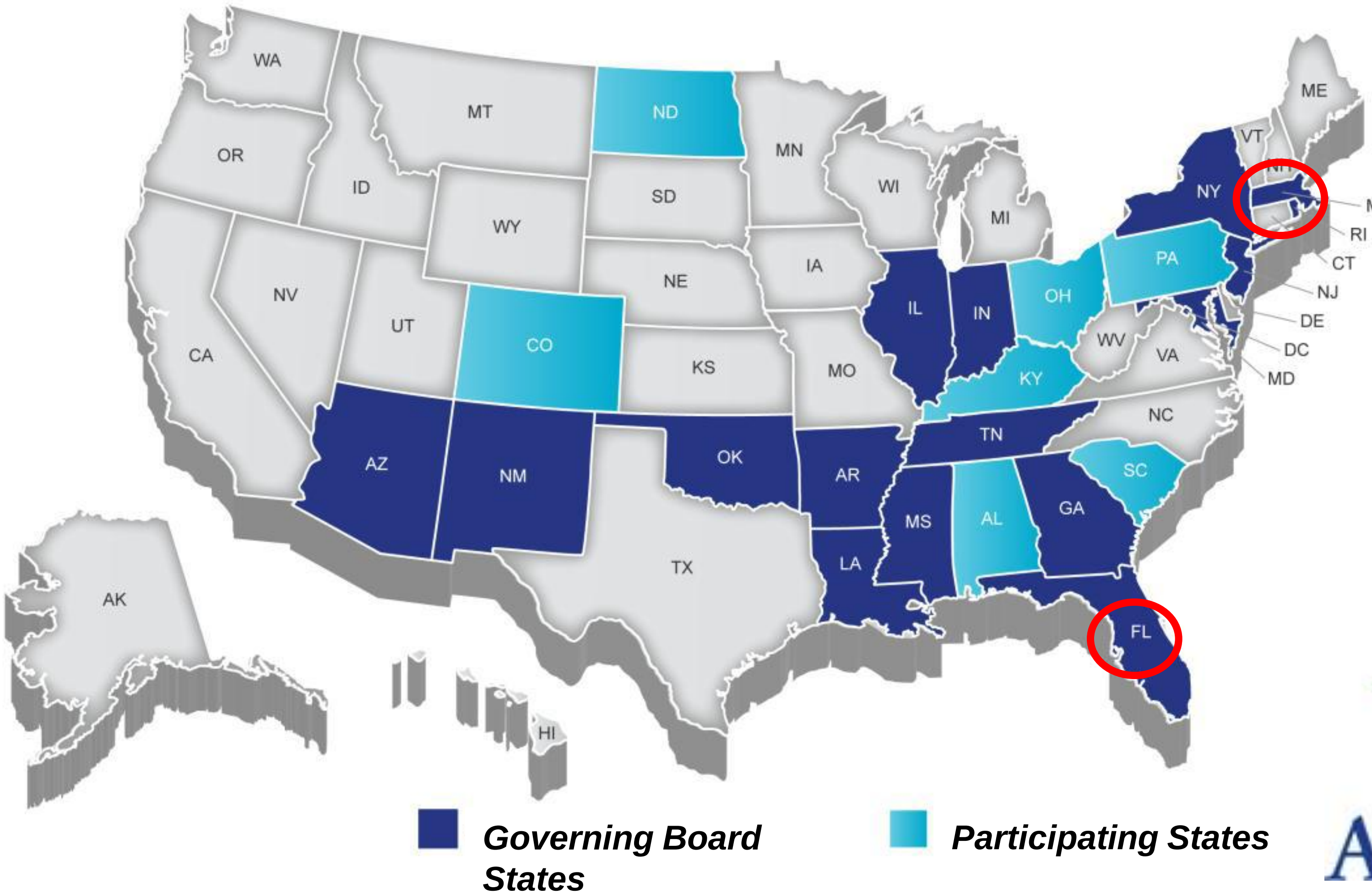
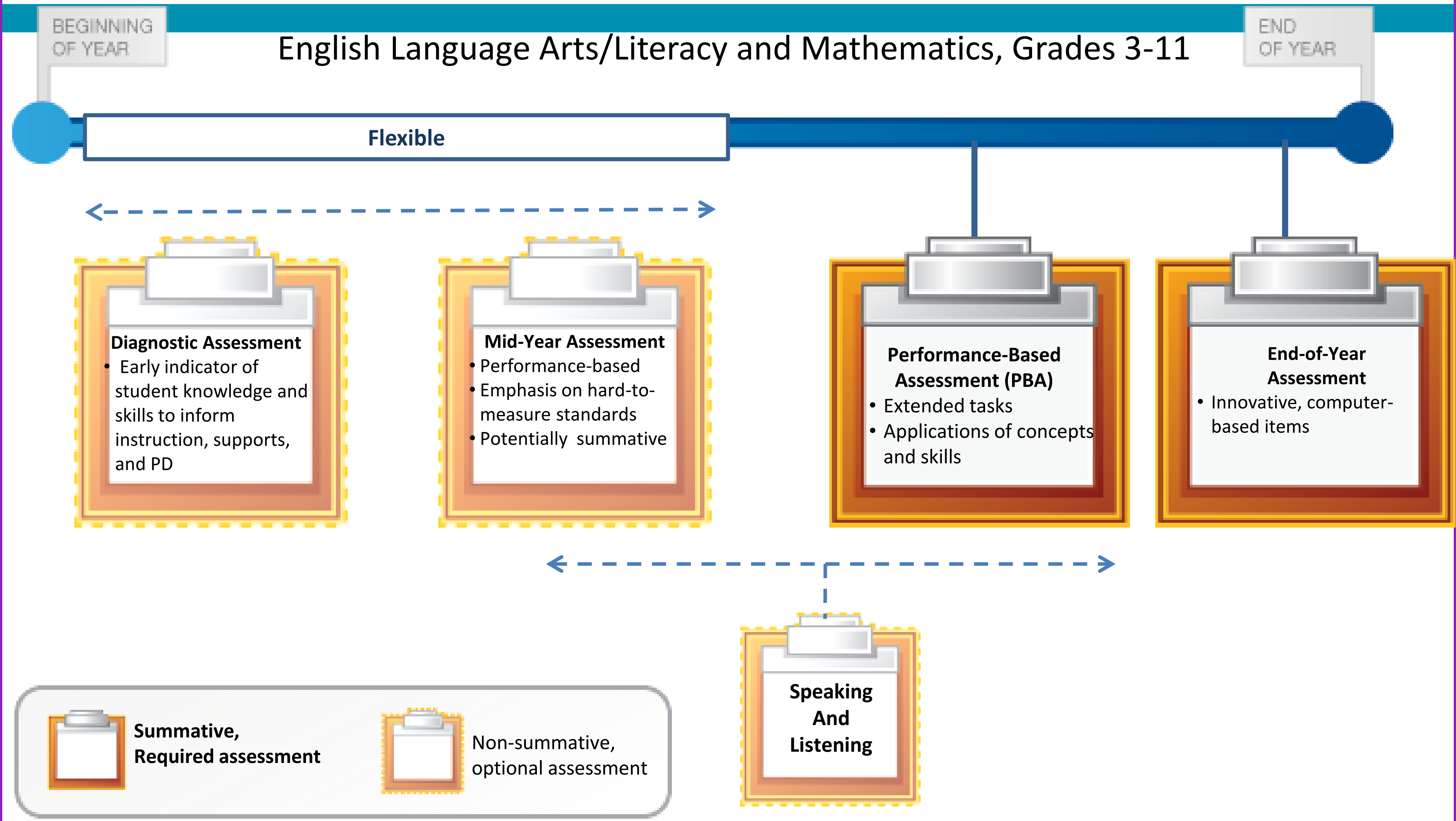
Document title: PARCC MCF for Mathematics Webinar_112111.ppt

Partnership for Assessment of Readiness for College and Careers (PARCC)



The PARCC Assessment Design



Purpose and Audience of the Model Content Frameworks

Purpose

- Support implementation of the Common Core State Standards
- Inform development of item specifications and blueprints for the PARCC assessments in grades 3–8 and high school.

Audience

- Primary audience is state and local curriculum directors
- Frameworks will also be accessible for teachers and building administrators to use as a resource

Development Process

- State-led process that included math content experts in PARCC member states and members of the Common Core State Standards writing team
  - Mathematics Rapid Response Feedback Group (content experts from a subset of PARCC states): Tommy Coy (AR), Gladys Cruz (NY), Robin Hill (KY), Rachel Jachino (IL), Mary Knuck (AZ), Barbara Libby (MA), Jim Mirabelli (IN), Brian Roget (OH), Carolyn Sessions (LA), John Svendsen (NY), Donna Watts (MD), Dee Ann Wilson (FL), Sandi Woodall (GA)

- Three rounds of feedback; nearly 1,000 individual comments were submitted from K–12 educators, principals, superintendents, higher education faculty, school board members, parents, and students during August public review

Approach of the Model Content Frameworks for Mathematics

- PARCC Model Content Frameworks provide a deep analysis of the CCSS, leading to more guidance on how focus, coherence, content and practices all work together.
- They **focus on framing the critical advances in the standards:**
  - Focus and coherence
  - Content knowledge, conceptual understanding, and expertise
  - Content and mathematical practices
- Model Content Frameworks for grades 3-8, Algebra I, Geometry, Algebra II, Mathematics I, Mathematics II, Mathematics III

Themes from Public Feedback to Draft Model Content Frameworks for Mathematics

Feedback: Include grades K-2
Revision: K-2 frameworks will be released late 2012

Feedback: Need for more concise language. Shorten the document and simplify some of the technical terminology
Revision: Introduction shortened and redundancies removed. Use of concise language; terminology revised with audience in mind.

Feedback: Revisit priorities and explain rationale behind the prioritization
Revision: Priorities revised to include a range of emphases and explanation of how standards relate to each other

Themes from Public Feedback to Draft Model Content Frameworks for Mathematics

Feedback: Include more specifics in the High School Mathematics Model Content Frameworks
Revision: More detail added about possible courses and suggested areas of emphasis

Feedback: Consider reformatting and adding more visual elements and provide quick reference points for educators
Revision: Revisions made with readability and navigation in mind, including addition of Table of Contents

Feedback: Add more examples and non-examples at each grade level to assist curriculum directors and educators in implementation
Revision: Sample tasks were not included in frameworks but will be available (expected Summer 2012) as a result of prototyping work being conducting by PARCC.

Key Elements of the Model Content Frameworks

- Examples of key advances from the previous grade
- Fluency expectations or examples of culminating standards
- Examples of major within-grade dependencies
- Examples of opportunities for connections among standards, clusters or domains
- Examples of opportunities for in-depth focus
- Examples of opportunities for connecting mathematical content and mathematical practices
- Content emphases by cluster

Grade 3 Example

Examples of Key Advances from Grade 2 to Grade 3

- Students in grade 3 begin to enlarge their concept of number by developing an understanding of fractions as numbers. This work will continue in grades 3–6, preparing the way for work with the complete rational number system in grades 6 and 7.
- Students in grades K–2 worked on number; place value; and addition and subtraction concepts, skills and problem solving. Beginning in grade 3, students will learn concepts, skills and problem solving for multiplication and division. This work will continue in grades 3, 4 and 5, preparing the way for work with ratios and proportions in grades 6 and 7.

Fluency Expectations or Examples of Culminating Standards

3.OA.7 Students fluently multiply and divide within 100. By the end of grade 3, they know all products of two one-digit numbers from memory.

3.NBT.2 Students fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (Although 3.OA.7 and 3.NBT.2 are both fluency standards, these two standards do not represent equal investments of time in grade 3. Note that students in grade 2 were already adding and subtracting within 1000, just not fluently. That makes 3.NBT.2 a relatively small and incremental expectation. By contrast, multiplication and division are new in grade 3, and meeting the multiplication and division fluency standard 3.OA.7 with understanding is a major portion of students' work in grade 3.)

Grade 3 Example

Examples of Major Within-Grade Dependencies

- Students must begin work with multiplication and division (3.OA) at or near the very start of the year to allow time for understanding and fluency to develop. Note that area models for products are an important part of this process (3.MD.7). Hence, work on concepts of area (3.MD.5–6) should likely begin at or near the start of the year as well.

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with partitioning shapes (3.G.2) relates to visual fraction models (3.NF).
- Scaled picture graphs and scaled bar graphs (3.MD.3) can be a visually appealing context for solving multiplication and division problems.

Grade 3 Example

Examples of Opportunities for In-Depth Focus

3.OA.3 Word problems involving equal groups, arrays and measurement quantities can be used to build students' understanding of and skill with multiplication and division, as well as to allow students to demonstrate their understanding of and skill with these operations.

3.OA.7 Finding single-digit products and related quotients is a required fluency for grade 3. Reaching fluency will take much of the year for many students. These skills and the understandings that support them are crucial; students will rely on them for years to come as they learn to multiply and divide with multidigit whole numbers and to add, subtract, multiply and divide with fractions. After multiplication and division situations have been established, reasoning about patterns in products (e.g., products involving factors of 5 or 9) can help students remember particular products and quotients. Practice — and if necessary, extra support — should continue all year for those who need it to attain fluency.

3.NF.2 Developing an understanding of fractions as numbers is essential for future work with the number system. It is critical that students at this grade are able to place fractions on a number line diagram and understand them as a related component of their ever-expanding number system.

3.MD.2 Continuous measurement quantities such as liquid volume, mass and so on are an important context for fraction arithmetic (cf. 4.NF.4c, 5.NF.7c, 5.NF.3). In grade 3, students begin to get a feel for continuous measurement quantities and solve whole-number problems involving such quantities.

3.MD.7 Area is a major concept within measurement, and area models must function as a support for multiplicative reasoning in grade 3 and beyond.

Grade 3 Example

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- Students learn and use strategies for finding products and quotients that are based on the properties of operations; for example, to find 4×7, they may recognize that $7 = 5 + 2$ and compute $4 \times 5 + 4 \times 2$. This is an example of seeing and making use of structure (MP.7). Such reasoning processes amount to brief arguments that students may construct and critique (MP.3).

- Students will analyze a number of situation types for multiplication and division, including arrays and measurement contexts. Extending their understanding of multiplication and division to these situations requires that they make sense of problems and persevere in solving them (MP.1), look for and make use of structure (MP.7) as they model these situations with mathematical forms (MP.4), and attend to precision (MP.6) as they distinguish different kinds of situations over time (MP.8).

Grade 3 Example

Content Emphases by Cluster[8]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Grade 3 Example

Key: Major Clusters; Supporting Clusters; Additional Clusters

Operations and Algebraic Thinking	
Major	Represent and solve problems involving multiplication and division.
Major	Understand properties of multiplication and the relationship between multiplication and division.
Major	Multiply and divide within 100.
Major	Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Number and Operations in Base Ten	
Additional	Use place value understanding and properties of operations to perform multi-digit arithmetic.
Number and Operations — Fractions	
Major	Develop understanding of fractions as numbers.
Measurement and Data	
Major	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
Supporting	Represent and interpret data.
Major	Geometric measurement: understand concepts of area and relate area to multiplication and addition.
Additional	Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
Geometry	
Supporting	Reason with shapes and their attributes.

Grade 3 Example

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Represent and interpret data: Students multiply and divide to solve problems using information presented in scaled bar graphs (3.MD.3). Pictographs and scaled bar graphs are a visually appealing context for one- and two-step word problems.

- Reason with shapes and their attributes: Work toward meeting 3.G.2 should be positioned in support of area measurement and understanding of fractions.

Structure of the Model Content Frameworks for High School Mathematics

- General analysis:
  - Examples of opportunities for connections among standards, clusters, domains, or conceptual categories
  - Examples of opportunities for connecting mathematical content and mathematical practices
  - Instructional emphases by cluster
- Course-specific analysis:
  - Examples of key advances from previous grades or courses
  - Fluency recommendations
  - Discussion of mathematical practices in relation to course content
  - Examples of major within-course dependencies
  - Examples of opportunities for in-depth focus

High School Example: Examples of Content Standards that Apply to Two or More High School Courses

- PARCC will develop assessments aligned with two sequences

The standards:

A-REI.4: Solve quadratic equations in one variable.

a. **Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.**

b. **Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.**

A-APR: Understand the relationship between zeros and factors of polynomials.

Algebra I (and Mathematics II)

Students develop an array of techniques for solving quadratic equations, and they reason about the connections among them, which are based on the idea that different forms of an expression or equation facilitate seeing different features (A-SSE.3).

They learn the method of completing the square, interpret it geometrically and use it to derive the quadratic formula, the method that is both general and efficient.

Of the techniques for solving quadratic equations, factoring is the most versatile for solving polynomial equations. Factoring depends on the "zero product property" of the real numbers: If a product is 0, at least one of the factors must be 0. In this course, the technique of solving polynomial equations via factoring is applied to quadratic equations. But the principle is perfectly general. Thus, the connection between the linear factors of a polynomial and the zeros of the corresponding polynomial function is a theme that is central to the coherence of high school mathematics.

And even in Algebra I, students can solve higher degree equations if they are given a head start on factorizations. For example, they can solve

$$(x-6)(x^2-5x+6)=0$$

Note: Complex solutions are not emphasized in this course.

Geometry (and Mathematics II)

While these standards will not generally be emphasized in a Geometry course, completing the square arises again in equations of circles (G-GPE.1).

Also, the factorization of a polynomial can reveal structural properties in geometric contexts. An example of this interplay comes from the problem of maximizing the area of a rectangle given a fixed perimeter. One approach to showing that a square does the job is to show how an $a \times b$ rectangle can be dissected to fit inside a square of side-length $\frac{a+b}{2}$. Abstracting from numerical examples, students are led to the algebraic identity

$$\left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 = ab$$

This can be established by a variety of methods, including the factorization of the left-hand side as a difference of squares. This identity shows how far off a given rectangle is from the corresponding square, and it shows precisely when the "error" $\frac{a-b}{2}$ is 0. The identity can even be used to establish the arithmetic-geometric mean inequality: for non-negative real numbers a and b, $\frac{a+b}{2} \geq \sqrt{ab}$, with equality if and only if $a = b$.

Algebra II (and Mathematics III)

Students complete this standard by including in their repertoire the set of complex numbers, and they continue to use the all these techniques when quadratic factors arise in more general contexts of solving polynomial equations graphing polynomial functions.

Factoring remains an important technique more broadly. This course introduces the operation of division with remainder for polynomials in one variable. An analysis of division has several applications that are core to advanced algebra. The most important application is to the factor and remainder theorems (A-APR.2). These theorems are the advancement of the study of solving quadratic equations as they apply to polynomials more generally. The factor theorem deepens the connection between factors of polynomials and solutions to equations that is stated in Algebra I. One of its many applications is that it can be used to show that a polynomial of degree n has at most n roots, and this implies the important result that a polynomial function of degree n is completely determined by $n + 1$ points on its graph.

High School Example: Algebra I

Examples of Key Advances from Grades K–8

- Having already extended arithmetic from whole numbers to fractions (grades 4–6) and from fractions to rational numbers (grade 7), students in grade 8 encountered particular irrational numbers such as $\sqrt{5}$ or π. In Algebra I, students will begin to understand the real number *system*. For more on the extension of number systems, see page 58 of the standards.

- Students in grade 8 worked with integer exponents. In Algebra I, students will extend the properties of exponents to positive real numbers raised to rational powers (N-RN.1, 2).

- Students in middle grades worked with measurement units, including units obtained by multiplying and dividing quantities. In Algebra I, students apply these skills in a more sophisticated fashion to solve problems in which reasoning about units adds insight (N-Q).

- Themes beginning in middle school algebra continue and deepen during high school. As early as grades 6 and 7, students began to use the properties of operations to generate equivalent expressions (6.EE.3, 7.EE.1). By grade 7, they began to recognize that rewriting expressions in different forms could be useful in problem solving (7.EE.2). In Algebra I, these aspects of algebra carry forward as students continue to use properties of operations to rewrite expressions, gaining fluency and engaging in what has been called "mindful manipulation."[24]

- Students in grade 8 extended their prior understanding of proportional relationships to begin working with functions, with an emphasis on linear functions. In Algebra I, students will master linear and quadratic functions. Students encounter other kinds of functions to ensure that general principles are perceived in generality, as well as to enrich the range of quantitative relationships considered in problems.

High School Example: Algebra I

Discussion of Mathematical Practices in Relation to Course Content

Two overarching practices relevant to Algebra I are:

- **Make sense of problems and persevere in solving them (MP.1).**
- **Model with mathematics (MP.4).**

Indeed, other mathematical practices in Algebra I might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together.

- **Reason abstractly and quantitatively (MP.2).** This practice standard refers to one of the hallmarks of algebraic reasoning, the process of decontextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems (A-CED, F-BF) and then transforming the models via algebraic calculations (A-SSE, A-APR, F-IF) to reveal properties of the problems.

- **Use appropriate tools strategically (MP.5).** Spreadsheets, a function modeling language, graphing tools and many other technologies can be used strategically to gain understanding of the ideas expressed by individual content standards and to model with mathematics.

- **Attend to precision (MP.6).** In algebra, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely (A-CED, A-REI) helps students understand the idea in new ways.

High School Example: Algebra I

Fluency Recommendations

A/G Algebra I students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

A-APR.1 Fluency in adding, subtracting and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

A-SSE.1b Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square and other mindful algebraic calculations.

High School Example: Mathematics I

Examples of Key Advances from Grades K–8

- Students build on previous work with solving linear equations and systems of linear equations in two ways: (a) They extend to more formal solution methods, including attending to the structure of linear expressions, and (b) they solve linear inequalities.
- Students formalize their understanding of the definition of a function, particularly their understanding of linear functions, emphasizing the structure of linear expressions. Students also begin to work on exponential functions, comparing them to linear functions.
- Work with congruence and similarity motions that was begun in grades 6–8 progresses. Students also consider sufficient conditions for congruence of triangles.
- Work with the bivariate data and scatter plots in grades 6–8 is extended to working with lines of best fit.

Discussion of Mathematical Practices in Relation to Course Content

- Modeling with mathematics (MP.4) should be a particular focus as students see the purpose and meaning for working with linear and exponential equations and functions.
- Using appropriate tools strategically (MP.5) is also important as students explore those models in a variety of ways, including with technology. For example, students might be given a set of data points and experiment with graphing a line that fits the data.
- As Mathematics I continues to develop a foundation for more formal reasoning, students should engage in the practice of constructing viable arguments and critiquing the reasoning of others (MP.3).

High School Example: Mathematics I

Fluency Recommendations

A/G High school students should become fluent in solving characteristic problems involving the analytic geometry of lines, such as finding the equation of a line given a point and a slope. This fluency can support students in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

G High school students should become fluent in using geometric transformation to represent the relationships among geometric objects. This fluency provides a powerful tool for visualizing relationships, as well as a foundation for exploring ideas both within geometry (e.g., symmetry) and outside of geometry (e.g., transformations of graphs).

S Students should be able to create a visual representation of a data set that is useful in understanding possible relationships among variables.

Intended Uses of the Model Content Frameworks

- Assist in transitioning to the CCSS
  - Help inform curriculum, instruction, and assessment
  - Educator engagement and awareness
- Assist in evaluating resources; provide awareness and quality of other resources
- Provide awareness on the balance of tasks
- Help educators think more deeply about the standards, especially foundational structures
- Grade level analyses

Guidance for Teachers

- Key advances in the standards between grade levels
- Fluency expectations and major within-grade dependencies
- Connections among standards, clusters, or domains
- Opportunities for In-Depth Focus
- Opportunities for Connecting Mathematical Content and Mathematical Practices

Guidance for Building Administrators

- Using the content frameworks to guide discussions around implementation of the standards
- Use the frameworks to support increased focus and coherence in instructional programs
- Using the frameworks to gain a deeper understanding of the role of the practices in cooperation with the content, to support student learning

Guidance for Curriculum Developers

- Using the content frameworks with the standards to sketch out potential model instructional unit plans
- Use the frameworks to support increased focus and coherence in instructional programs
- Recognizing the shifts in the standards from grade to grade and using these shifts as grade level curricula are developed and as materials are purchased to align with the curricula

Releasing the Content Frameworks

- Frameworks are available on www.parcconline.org/parcc-content-frameworks
- Frameworks intended to be dynamic and responsive to evidence and on-going input
  - In Summer 2012, once educators have used the frameworks, PARCC will collect feedback and refine frameworks as necessary
- PARCC will develop additional model instructional tools and supports that build on the frameworks
  - Model Instructional Units
  - Educator Leader Cadres
  - Item specifications
  - Item and task prototypes

QUESTIONS?

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