



DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS

FOR

Mathematics

The Dynamic Learning Maps Essential Elements are copyrighted by the University of Kansas Center for Research. They are based substantially on an earlier version that was copyrighted by Edvantia. The Essential Elements may be reprinted, with appropriate citation, in part or entirety by anyone. However, no text in the document may be modified. Comments, explanations, or other additional materials may be interspersed as long as they are clearly indicated that they are not part of the Essential Elements and are not the opinions of DLM. Appropriate citations follows.

Dynamic Learning Maps Consortium (2013). *Dynamic Learning Maps Essential Elements for English Language Arts*. Lawrence, KS: University of Kansas.

and

Dynamic Learning Maps Consortium (2013). *Dynamic Learning Maps Essential Elements for Mathematics*. Lawrence, KS: University of Kansas.

Background on the Dynamic Learning Maps Essential Elements

The Dynamic Learning Maps Essential Elements are specific statements of knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities. The initial draft of the Dynamic Learning Maps Essential Elements (then called the Common Core Essential Elements) was released in the spring of 2012.

The initial version of the Dynamic Learning Maps Essential Elements (DLM EEs) was developed by a group of educators and content specialists from the 12 member states of the Dynamic Learning Maps Alternate Assessment Consortium (DLM) in the spring of 2011. Led by Edvantia, Inc., a sub-contractor of DLM, representatives from each state education agency and the educators and content specialists they selected developed the original draft of DLM EEs. Experts in mathematics and English language arts, as well as individuals with expertise in instruction for students with significant cognitive disabilities reviewed the draft documents. Edvantia then compiled the information into the version released in the spring of 2012.

Concurrent with the development of the DLM EEs, the DLM consortium was actively engaged in building learning maps in mathematics and English language arts. The DLM learning maps are highly connected representations of how academic skills are acquired, as reflected in research literature. In the case of the DLM project, the Common Core State Standards helped to specify academic targets, while the surrounding map content clarified how students could reach the specified standard. Learning maps of this size had not been previously developed, and as a

result, alignment between the DLM EEs and the learning maps was not possible until the fall of 2012, when an initial draft of the learning maps was available for review.

Alignment of the DLM EEs to the DLM Learning Maps

Teams of content experts worked together to revise the initial version of the DLM EEs and the learning maps to ensure appropriate alignment of these two critical elements of the project. Alignment involved horizontal alignment of the DLM EEs with the Common Core State Standards and vertical alignment of the DLM EEs with meaningful progressions in the learning maps. The alignment process began when researchers Caroline Mark and Kelli Thomas compared the learning maps with the initial version of the DLM EEs to determine how the map and the DLM EEs should be adjusted to improve their alignment. The teams of content experts most closely involved with this alignment work included:

Mathematics

Kelli Thomas, Ph.D. (co-lead)
Angela Broaddus, Ph.D. (co-lead)
Perneet Sood
Kristin Joannou
Bryan Candea Kromm

English Language Arts

Caroline Mark, Ph.D. (lead)
Jonathan Schuster, Ph.D.
Russell Swinburne Romine, Ph.D.
Suzanne Peterson

These teams worked in consultation with Sue Bechard, Ph.D. and Karen Erickson, Ph.D., who offered guidance based on their experience in alternate assessments of students with significant cognitive disabilities.

The Alignment Process

The process of aligning the learning map and the DLM EEs began by identifying nodes in the maps that represented the essential elements in mathematics and English language arts. This process revealed areas in the maps where additional nodes were needed to account for incremental growth reflected from an essential element in one grade to the next. Also identified

were areas in which an essential element was out of place developmentally, according to research, with other essential elements. For example, adjustments were made when an essential element related to a higher-grade map node appeared earlier on the map than an essential element related to a map node from a lower grade (e.g., a fifth-grade skill preceded a third-grade skill). Finally, the alignment process revealed DLM EEs that were actually written as instructional tasks rather than learning outcomes.

This initial review step provided the roadmap for subsequent revision of both the learning maps and the DLM EEs. The next step in the DLM project was to develop the claims document, which served as the basis for the evidence-centered design of the DLM project and helped to further refine both the modeling of academic learning in the maps and the final revisions to the DLM EEs.

Claims and Conceptual Areas

The DLM system uses a variant of evidence-centered design (ECD) as the framework for developing the DLM Alternate Assessment System. While ECD is multifaceted, it starts with a set of claims regarding important knowledge in the domains of interest (mathematics and English language arts), as well as an understanding of how that knowledge is acquired. Two sets of claims have been developed for DLM that identify the major domains of interest within mathematics and English language arts for students with significant cognitive disabilities. These claims are broad statements about expected student learning that serve to focus the scope of the assessment. Because the learning map identifies particular paths to the acquisition of academic skills, the claims also help to organize the structures in the learning map for this population of students. Specifically, conceptual areas within the map further define the knowledge and skills required to meet the broad claims identified by DLM.

The claims are also significant because they provide another means through which to evaluate alignment between the DLM EEs and the learning map nodes, and serve as the foundation for evaluating the validity of inferences made from test scores. DLM EEs related to a particular claim and conceptual area must clearly link to one another, and the learning map must reflect how that knowledge is acquired. Developing the claims and conceptual areas for DLM provided a critical framework for organizing nodes on the learning maps and, accordingly, the DLM EEs that align with each node.

The table below reveals the relationships among the claims, conceptual areas, and DLM EEs in mathematics. The DLM EEs are represented with codes that reflect the domains in mathematics. For example, the first letter or digit represents the grade of record, the next code reflects the domain, followed by the number that aligns with the Common Core State Standard grade level expectation. As such, K.CC.1 is the code for the DLM EE that aligns with kindergarten (K), counting and cardinality (CC), standard 1. Keys to the codes can be found under the table.

Clearly articulated claims and conceptual areas for DLM served as an important evidence-centered framework within which this version of the DLM EEs was developed. With the claims and conceptual areas in place, the relationship between DLM EEs within a claim and conceptual area or across grade levels is easier to track and strengthen. The learning maps, as well as the claims and conceptual areas, had not yet been developed when the original versions of the DLM EEs were created. As such, the relationship of DLM EEs within and across grade levels was more difficult to evaluate at that time.

Table 1. Dynamic Learning Maps Claims and Conceptual Areas for Students with Significant Cognitive Disabilities in Mathematics

<p>Claim 1</p>	<p>Number Sense: Students demonstrate increasingly complex understanding of number sense.</p> <p>Conceptual Areas in the Dynamic Learning Map:</p> <p>MC 1.1 Understand number structures (counting, place value, fraction) <i>Essential Elements Included:</i> K.CC.1, 4, 5; 1.NBT.1a-b; 2.NBT.2a-b,3; 3.NBT.1,2,3; 3.NF.1-3; 4.NF.1-2,3; 5.NF.1,2; 6.RP.1; 7.RP.1-3; 7.NS.2.c-d; 8.NS.2.a</p> <p>MC 1.2 Compare, compose, and decompose numbers and sets <i>Essential Elements Included:</i> K.CC.6; 1.NBT.2, 3, 4, 6; 2.NBT.1, 4, 5b; 4.NBT.2, 3; 5.NBT.1, 2, 3, 4; 6.NS.1, 5-8; 7.NS.3; 8.NS.2.b; 8.EE.3-4;</p> <p>MC 1.3 Calculate accurately and efficiently using simple arithmetic operations <i>Essential Elements Included:</i> 2.NBT.5.a, 6-7; 3.OA.4; 4.NBT.4, 5.NBT.5, 6-7; 6.NS.2, 3; 7.NS.1, 2.a, 2.b; 8.NS.1; 8.EE.1; N-CN.2.a, 2.b, 2.c; N-RN.1; S-CP.1-5; S-IC.1-2</p>
<p>Claim 2</p>	<p>Geometry: Students demonstrate increasingly complex spatial reasoning and understanding of geometric principles.</p> <p>Conceptual Areas in the Dynamic Learning Map:</p> <p>MC 2.1 Understand and use geometric properties of two- and three-dimensional shapes <i>Essential Elements Included:</i> K.MD.1-3; K.G.2-3; 1.G.1, 2; 2.G.1; 3.G.1; 4.G.1, 2; 4.MD.5, 6; 5.G.1-4; 5.MD.3; 7.G.1, 2, 3, 5; 8.G.1, 2, 4, 5; G-CO.1, 4-5, 6-8; G-GMD.1-3, 4</p> <p>MC 2.2 Solve problems involving area, perimeter, and volume <i>Essential Elements Included:</i> 1.G.3; 3.G.2; 4.G.3; 4.MD.3; 5.MD.4-5; 6.G.1, 2; 7.G.4, 6; 8.G.9; G-GMD.1-3; G-GPE.7</p>
<p>Claim 3</p>	<p>Measurement Data and Analysis: Students demonstrate increasingly complex understanding of measurement, data, and analytic procedures.</p> <p>Conceptual Areas in the Dynamic Learning Map:</p> <p>MC 3.1 Understand and use measurement principles and units of measure <i>Essential Elements Included:</i> 1.MD.1-2, 3.a, 3.b, 3.c, 3.d; 2.MD.1, 3-4, 5, 6, 7, 8; 3.MD.1, 2, 4; 4.MD.1, 2.a, 2.b, 2.c, 2.d; 5.MD.1.a, 1.b, 1.c; N-Q.1-3</p> <p>MC 3.2 Represent and interpret data displays <i>Essential Elements Included:</i> 1.MD.4; 2.MD.9-10; 3.MD.3; 4.MD.4.a, 4.b; 5.MD.2; 6.SP.1-2, 5; 7.SP.1-2, 3, 5-7; 8.SP.4; S-ID. 1-2, 3, 4</p>
<p>Claim 4</p>	<p>Algebraic and functional reasoning: Students solve increasingly complex mathematical problems, making productive use of algebra and functions.</p> <p>Conceptual Areas in the Dynamic Learning Map:</p> <p>MC 4.1. Use operations and models to solve problems <i>Essential Elements Included:</i> K.OA.1, 1.a, 1.b, 2, 5.a, 5.b; 2.OA.3, 4; 3.OA.1-2, 8; 4.OA.1-</p>

	<p>2, 3, 4; 6.EE.1-2, 3, 5-7; 7.EE.1, 4; 8.EE.7; A-CED.1, 2-4; A-SSE.1, 3</p> <p>MC 4.2 Understand patterns and functional thinking</p> <p><i>Essential Elements Included: 3.OA.9; 4.OA.5; 5.OA.3; 7.EE.2; 8.EE.5-6; 8.F.1-3, 4, 5; A-REI.10-12; A-SSE.4; F-BF.1, 2; F-IF.1-3, 4-6; F-LE.1</i></p>
--	---

A-CED = creating equations; A-SSE = seeing structure in equations BF = building functions; CC = counting & cardinality; EE = expressions & equations; F-BF = basic fractions; F-IF = interpreting functions; G = geometry; G-GMD = geometric measurement & dimension; G-GPE = general properties & equations; MD = measurement & data; NBT = numbers & operations in base ten; N-CN = complex number system; NF = numbers & operations - fractions; N-RN = real number system; NS = number systems; N-Q = number & quantity; OA = operations & algebraic thinking; RP = ratios & proportional relationships; S-IC- statistics & probability - making inferences/justifying conclusions; S-ID = statistics & probability - interpreting categorical & quantitative data; SP = statistics & probability

Resulting Changes to the DLM Essential Elements

The development of the entire DLM Alternate Assessment System guided a final round of revisions to the DLM EEs, which can be organized into four broad categories: alignment across grade levels, language specificity, common core alignment, and defining learning expectations (rather than instructional tasks). The first type of revision was required to align the DLM EEs across grade levels, both vertically and horizontally. The maps, and the research supporting them, were critical in determining the appropriate progression of skills and understandings from grade to grade. This alignment across grade levels was important within and across standards, strands, and domains. For example, in determining when it was appropriate to introduce concepts in mathematics regarding the relative position of objects, we had to consider the grade level at which prepositions that describe relative position were introduced in English language arts. Examining the research-based skill development outlined in the learning map aided in these kinds of determinations.

The articulation of the claims and conceptual areas reinforced the need for specific language in the DLM EEs to describe learning within an area. Because teams assigned to grade bands developed the first round of DLM EEs, the language choices from one grade to the next were not consistent. Even when closely related skills, concepts, or understandings were

targeted, the same terms were not always selected to describe the intended learning outcome. The teams of content experts who worked on this revised version of the DLM EEs were very intentional in selecting a common set of terms to reflect the claims and conceptual areas and applied them consistently across the entire set of DLM EEs.

Another important change in this version of the DLM EEs involved alignment to the Common Core State Standards (CCSS). Given that the DLM EEs are intended to clarify the bridge to the CCSS expectations for students with the most significant cognitive disabilities, it is critical that alignment be as close as possible without compromising learning and development over time. While there was never a one-to-one correspondence between the CCSS and the DLM EEs, the revisions have made the alignment between the two more precise than it was in the first version.

Finally, revisions to the DLM EEs involved shifting the focus of a small number of DLM EEs that were written in the form of instructional tasks rather than learning expectations, and adding “With guidance and support” to the beginning of a few of the DLM EEs in the primary grades in English language arts to reflect the expectations articulated in the CCSS.

Members of the DLM consortium reviewed each of the changes to the original version of the DLM EEs. Four states provided substantive feedback on the revisions, and this document incorporates the changes those teams suggested.

Access to Instruction and Assessment

The DLM EEs specify learning targets for students with significant cognitive disabilities; however, they do not describe all of the ways that students can engage in instruction or demonstrate understanding through an assessment. Appropriate modes of communication, both

for presentation or response, are not stated in the DLM EEs unless a specific mode is an expectation. Where no limitation has been stated, no limitation should be inferred. Students' opportunities to learn and to demonstrate learning during assessment should be maximized by providing whatever communication, assistive technologies, augmentative and alternative communication (AAC) devices, or other access tools that are necessary and routinely used by the student during instruction.

Students with significant cognitive disabilities include a broad range of students with diverse disabilities and communication needs. For some students with significant cognitive disabilities, a range of assistive technologies is required to access content and demonstrate achievement. For other students, AAC devices or accommodations for hearing and visual impairments will be needed. During instruction, teams should meet individual student needs using whatever technologies and accommodations are required. Examples of some of the ways that students may use technology while learning and demonstrating learning are topics for professional development, and include:

- communication devices that compensate for a student's physical inability to produce independent speech.
- alternate access devices that compensate for a student's physical inability to point to responses, turn pages in a book, or use a pencil or keyboard to answer questions or produce writing.

Guidance and Support

The authors of the CCSS use the words "prompting and support" at the earliest grade levels to indicate when students are not expected to achieve standards completely independently. Generally, "prompting" refers to "the action of saying something to persuade,

encourage, or remind someone to do or say something” (McKean, 2005). However, in special education, prompting is often used to mean a system of structured cues to elicit desired behaviors that otherwise would not occur. In order to clearly communicate that teacher assistance is permitted during instruction of the DLM EEs and is not limited to structured prompting procedures, the decision was made by the stakeholder group to use the more general term *guidance* throughout the DLM EEs.

Guidance and support during instruction should be interpreted as teacher encouragement, general assistance, and informative feedback to support the student in learning. Some examples of the kinds of teacher behaviors that would be considered guidance and support include verbal supports, such as

- getting the student started (e.g., “Tell me what to do first.”),
- providing a hint in the right direction without revealing the answer (e.g., Student wants to write *dog* but is unsure how, so the teacher might say, “See if you can write the first letter in the word, /d/og [phonetically pronounced].”),
- using structured technologies such as task-specific word banks, or
- providing structured cues such as those found in prompting procedures (e.g., least-to-most prompts, simultaneous prompting, and graduated guidance).

Guidance and support as described above applies to instruction and is also linked to demonstrating learning relative to DLM EEs, where guidance and support is specifically called out within the standards.

Conclusion

Developing the research-based model of knowledge and skill development represented in the DLM Learning Maps supported the articulation of assessment claims for mathematics and English language arts. This articulation subsequently allowed for a careful revision of the DLM EEs to reflect both horizontal alignment with the CCSS and vertical alignment across the grades, with the goal of moving students toward more sophisticated understandings in both domains. Though the contributions made by Edvantia and our state partners in developing the initial set of DLM EEs were a critical first step, additional revisions to the DLM EEs were required to ensure consistency across all elements of the Dynamic Learning Maps Alternate Assessment System.

APPENDIX

Development of the Dynamic Learning Maps Essential Elements has been a collaborative effort among practitioners, researchers, and our state representatives. Listed below are the reviews and the individuals involved with each round of improvements to the Dynamic Learning Maps Essential Elements. Thank you to all of our contributors.

Review of Draft Two of Dynamic Learning Maps Essential Elements

A special thanks to all of the experts nominated by their state to review draft two of the Dynamic Learning Maps Essential Elements. We are grateful for your time and efforts to improve these standards for students with significant cognitive disabilities. Your comments have been incorporated into this draft. The states with teams who reviewed draft two include:

Illinois	Oklahoma
Iowa	Utah
Kansas	Virginia
Michigan	West Virginia
Missouri	Wisconsin

Development of the Original Dynamic Learning Maps Common Core Essential Elements

A special thanks to Edvantia and the team of representatives from Dynamic Learning Maps consortium states who developed the original Common Core Essential Elements upon which the revised Dynamic Learning Maps Essential Elements are based. The team from Edvantia who led the original effort included:

Jan Sheinker, Sheinker Educational Services, Inc.
Beth Judy, Director, Assessment, Alignment, and Accountability Services
Nathan Davis, Information Technology Specialist
Kristen Deitrick, Corporate Communications Specialist

Linda Jones, Executive Assistant

Representatives from Dynamic Learning Maps consortium states included:

IOWA

SEA Representatives: Tom Deeter, Emily Thatcher

Stakeholders: Peggy Akins, Judy Hamer, Kathleen Kvamme-Promes, Donna Shaw

KANSAS

SEA Representatives: Debbie Matthews, Kris Shaw

Stakeholders: Debby Byrne, Holly Draper, Dawn Gresham, Linda Hickey

MICHIGAN

SEA Representatives: Joanne Wilkelman, Adam Wyse

Stakeholders: Debra Susan Asano, Thomai Gersh, Marcia O'Brian, Terri Portice

MISSOURI

SEA Representatives: Lynn Everett, Jane VanDeZande

Stakeholders: Melia Franklin, Lou Ann Hoover, Debbie Jameson, Kate Sadler

NEW JERSEY

SEA Representatives: Melanie O'Dea

Stakeholders: Brenda Berrios, Neal Webster, Tina Yurcho

NORTH CAROLINA

SEA Representatives: Claire Greer, Sarah Reives

Stakeholders: Emma Hatfield-Sidden, Judy Jennings, Jennifer Michalenok,

OKLAHOMA

SEA Representatives: Jennifer Burnes, Amy Dougherty

Stakeholders: Pam Cox, Dianna Daubenspeck, Sondra LeGrande, Christie Stephenson

UTAH

SEA Representatives: Wendy Carver, Kurt Farnsworth

Stakeholders: James Bray, Janice Hill, Linda Stallviere, Ryan Webb

VIRGINIA

SEA Representative: John Eisenberg

Stakeholders: Maria Beck, Daniel Blegun, Al Klugh, Cheryl Ann Prevatte

WASHINGTON

SEA Representatives: Judy Kraft, Janice Tornow

Stakeholders: Annalisa Brewster, Kim Cook, Jeffrey Dunn, Kimberly Perisho

WEST VIRGINIA

SEA Representatives: Beth Cipoletti, Melissa Gholson

Stakeholders: Robert Bartlett, Gerald Hartley, Angel Harris, Angela See

WISCONSIN

SEA Representatives: Emilie Amundson, Kristen Burton

Stakeholders: Lori Hillyer, Tamara Maxwell, Connie Persike, Sara Vold

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR FOURTH GRADE

Fourth Grade Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use the four operations with whole numbers to solve problems.	
4.OA.1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	EE.4.OA.1-2. Demonstrate the connection between repeated addition and multiplication.
4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	
4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	EE.4.OA.3. Solve one-step real-world problems using addition or subtraction within 100.
CLUSTER: Gain familiarity with factors and multiples.	
4.OA.4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	EE.4.OA.4. Show one way to arrive at a product.
CLUSTER: Generate and analyze patterns.	
4.OA.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. <i>For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</i>	EE.4.OA.5. Use repeating patterns to make predictions.

Fourth Grade Mathematics Domain: Numbers and Operations in Base Ten¹⁵

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Generalize place value understanding for multi-digit whole numbers.	
<p>4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. <i>For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.</i></p>	<p>Not applicable. See EE.5.NBT.1.</p>
<p>4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p>	<p>EE.4.NBT.2. Compare whole numbers to 10 using symbols ($<$, $>$, $=$).</p>
<p>4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place.</p>	<p>EE.4.NBT.3. Round any whole number 0-30 to the nearest ten.</p>
CLUSTER: Use place value understanding and properties of operations to perform multi-digit arithmetic.	
<p>4.NBT.4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p>	<p>EE.4.NBT.4. Add and subtract two-digit whole numbers.</p>
<p>4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>	<p>Not applicable. See EE.4.OA.1.</p>
<p>4.NBT.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>	<p>Not applicable.</p>

¹⁵ Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Fourth Grade Mathematics Domain: Number and Operations—Fractions¹⁶

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Extend understanding of fraction equivalence and ordering.	
<p>4.NF.1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p>	<p>EE.4.NF.1–2. Identify models of one half ($1/2$) and one fourth ($1/4$).</p>
<p>4.NF.2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.</p>	
CLUSTER: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	
<p>4.NF.3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.</p>	<p>EE.4.NF.3. Differentiate between whole and half.</p>
<p>4.NF.3.a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</p>	
<p>4.NF.3.b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. <i>Examples:</i> $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2\ 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$.</p>	
<p>4.NF.3.c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.</p>	
<p>4.NF.3.d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</p>	

¹⁶ Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

CCSS Grade-Level Standards	DLM Essential Elements
<p>4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</p>	<p>Not applicable. See EE.4.OA.1–2 and EE.5.NBT.5.</p>
<p>4.NF.4.a. Understand a fraction a/b as a multiple of $1/b$. <i>For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.</i></p>	
<p>4.NF.4.b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. <i>For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)</i></p>	
<p>4.NF.4.c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. <i>For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</i></p>	
<p>CLUSTER: Understand decimal notation for fractions, and compare decimal fractions.</p>	
<p>4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.¹⁷ <i>For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$.</i></p>	<p>Not applicable. See EE.7.NS.2.c-d.</p>
<p>4.NF.6. Use decimal notation for fractions with denominators 10 or 100. <i>For example, rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.</i></p>	
<p>4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.</p>	

¹⁷ Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Fourth Grade Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	
<p>4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</i></p>	<p>EE.4.MD.1. Identify the smaller measurement unit that comprises a larger unit within a measurement system (inches/foot, centimeter/meter, minutes/hour).</p>
<p>4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p>	<p>EE.4.MD.2.a. Tell time using a digital clock. Tell time to the nearest hour using an analog clock.</p>
	<p>EE.4.MD.2.b. Measure mass or volume using standard tools.</p>
	<p>EE.4.MD.2.c. Use standard measurement to compare lengths of objects.</p>
	<p>EE.4.MD.2.d. Identify coins (penny, nickel, dime, quarter) and their values.</p>
<p>4.MD.3. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. <i>For example, find the width of a rectangular room given the area of the flooring and the length by viewing the area formula as a multiplication equation with an unknown factor.</i></p>	<p>EE.4.MD.3. Determine the area of a square or rectangle by counting units of measure (unit squares).</p>
CLUSTER: Represent and interpret data.	
<p>4.MD.4. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. <i>For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</i></p>	<p>EE.4.MD.4.a. Represent data on a picture or bar graph given a model and a graph to complete.</p>
	<p>EE.4.MD.4.b. Interpret data from a picture or bar graph.</p>

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Geometric measurement: understand concepts of angle and measure angles.	
<p>4.MD.5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:</p>	<p>EE.4.MD.5. Recognize angles in geometric shapes.</p>
<p>4.MD.5.a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $1/360$ of a circle is called a “one-degree angle,” and can be used to measure angles.</p>	
<p>4.MD.5.b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.</p>	
<p>4.MD.6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</p>	<p>EE.4.MD.6. Identify angles as larger and smaller.</p>
<p>4.MD.7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</p>	<p>Not applicable. See EE.4.G.2.a.</p>

Fourth Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	
<p>4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</p>	<p>EE.4.G.1. Recognize parallel lines and intersecting lines.</p>
<p>4.G.2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.</p>	<p>EE.4.G.2. Describe the defining attributes of two-dimensional shapes.</p>
<p>4.G.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures, and draw lines of symmetry.</p>	<p>EE.4.G.3. Recognize that lines of symmetry partition shapes into equal areas.</p>