DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS

FOR

Mathematics

Background on the Dynamic Learning Maps Essential Elements

The Dynamic Learning Maps Essential Elements are specific statements of knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities. The initial draft of the Dynamic Learning Maps Essential Elements (then called the Common Core Essential Elements) was released in the spring of 2012.

The initial version of the Dynamic Learning Maps Essential Elements (DLM EEs) was developed by a group of educators and content specialists from the 12 member states of the Dynamic Learning Maps Alternate Assessment Consortium (DLM) in the spring of 2011. Led by Edvantia, Inc., a sub-contractor of DLM, representatives from each state education agency and the educators and content specialists they selected developed the original draft of DLM EEs. Experts in mathematics and English language arts, as well as individuals with expertise in instruction for students with significant cognitive disabilities reviewed the draft documents. Edvantia then compiled the information into the version released in the spring of 2012.

Concurrent with the development of the DLM EEs, the DLM consortium was actively engaged in building learning maps in mathematics and English language arts. The DLM learning maps are highly connected representations of how academic skills are acquired, as reflected in research literature. In the case of the DLM project, the Common Core State Standards helped to specify academic targets, while the surrounding map content clarified how students could reach the specified standard. Learning maps of this size had not been previously developed, and as a result, alignment between the DLM EEs and the learning maps was not possible until the fall of 2012, when an initial draft of the learning maps was available for review.

Alignment of the DLM EEs to the DLM Learning Maps

Teams of content experts worked together to revise the initial version of the DLM EEs and the learning maps to ensure appropriate alignment of these two critical elements of the project. Alignment involved horizontal alignment of the DLM EEs with the Common Core State Standards and vertical alignment of the DLM EEs with meaningful progressions in the learning maps. The alignment process began when researchers Caroline Mark and Kelli Thomas compared the learning maps with the initial version of the DLM EEs to determine how the map and the DLM EEs should be adjusted to improve their alignment. The teams of content experts most closely involved with this alignment work included:

Mathematics	**English Language Arts**
Kelli Thomas, Ph.D. (co-lead)	Caroline Mark, Ph.D. (lead)
Angela Broaddus, Ph.D. (co-lead)	Jonathan Schuster, Ph.D.
Perneet Sood	Russell Swinburne Romine, Ph.D.
Kristin Joannou	Suzanne Peterson
Bryan Candea Kromm	

These teams worked in consultation with Sue Bechard, Ph.D. and Karen Erickson, Ph.D., who offered guidance based on their experience in alternate assessments of students with significant cognitive disabilities.

The Alignment Process

The process of aligning the learning map and the DLM EEs began by identifying nodes in the maps that represented the essential elements in mathematics and English language arts. This process revealed areas in the maps where additional nodes were needed to account for incremental growth reflected from an essential element in one grade to the next. Also identified were areas in which an essential element was out of place developmentally, according to research, with other essential elements. For example, adjustments were made when an essential element related to a higher-grade map node appeared earlier on the map than an essential element related to a map node from a lower grade (e.g., a fifth-grade skill preceded a third-grade skill). Finally, the alignment process revealed DLM EEs that were actually written as instructional tasks rather than learning outcomes.

This initial review step provided the roadmap for subsequent revision of both the learning maps and the DLM EEs. The next step in the DLM project was to develop the claims document, which served as the basis for the evidence-centered design of the DLM project and helped to further refine both the modeling of academic learning in the maps and the final revisions to the DLM EEs.

Claims and Conceptual Areas

The DLM system uses a variant of evidence-centered design (ECD) as the framework for developing the DLM Alternate Assessment System. While ECD is multifaceted, it starts with a set of claims regarding important knowledge in the domains of interest (mathematics and English language arts), as well as an understanding of how that knowledge is acquired. Two sets of claims have been developed for DLM that identify the major domains of interest within mathematics and English language arts for students with significant cognitive disabilities. These claims are broad statements about expected student learning that serve to focus the scope of the assessment. Because the learning map identifies particular paths to the acquisition of academic skills, the claims also help to organize the structures in the learning map for this population of students. Specifically, conceptual areas within the map further define the knowledge and skills required to meet the broad claims identified by DLM.

The claims are also significant because they provide another means through which to evaluate alignment between the DLM EEs and the learning map nodes, and serve as the foundation for evaluating the validity of inferences made from test scores. DLM EEs related to a particular claim and conceptual area must clearly link to one another, and the learning map must reflect how that knowledge is acquired. Developing the claims and conceptual areas for DLM provided a critical framework for organizing nodes on the learning maps and, accordingly, the DLM EEs that align with each node.

The table below reveals the relationships among the claims, conceptual areas, and DLM EEs in mathematics. The DLM EEs are represented with codes that reflect the domains in mathematics. For example, the first letter or digit represents the grade of record, the next code reflects the domain, followed by the number that aligns with the Common Core State Standard grade level expectation. As such, K.CC.1 is the code for the DLM EE that aligns with kindergarten (K), counting and cardinality (CC), standard 1. Keys to the codes can be found under the table.

Clearly articulated claims and conceptual areas for DLM served as an important evidence-centered framework within which this version of the DLM EEs was developed. With the claims and conceptual areas in place, the relationship between DLM EEs within a claim and conceptual area or across grade levels is easier to track and strengthen. The learning maps, as well as the claims and conceptual areas, had not yet been developed when the original versions of the DLM EEs were created. As such, the relationship of DLM EEs within and across grade levels was more difficult to evaluate at that time.

Table 1. Dynamic Learning Maps Claims and Conceptual Areas for Students with Significant Cognitive Disabilities in Mathematics

Claim 1	**Number Sense: Students demonstrate increasingly complex understanding of number sense.** Conceptual Areas in the Dynamic Learning Map: **MC 1.1 Understand number structures (counting, place value, fraction)** *Essential Elements Included: K.CC.1, 4 ,5; 1.NBT.1a-b; 2.NBT.2a-b,3; 3.NBT.1,2,3; 3.NF.1-3; 4.NF.1-2,3; 5.NF.1,2; 6.RP.1; 7.RP.1-3; 7.NS.2.c-d; 8.NS.2.a* **MC 1.2 Compare, compose, and decompose numbers and sets** *Essential Elements Included: K.CC.6; 1.NBT.2, 3, 4, 6; 2.NBT.1, 4, 5b; 4.NBT.2, 3; 5.NBT.1, 2, 3, 4; 6.NS.1, 5-8; 7.NS.3; 8.NS.2.b; 8.EE.3-4;* **MC 1.3 Calculate accurately and efficiently using simple arithmetic operations** *Essential Elements Included: 2.NBT.5.a, 6-7; 3.OA.4; 4.NBT.4, 5.NBT.5, 6-7; 6.NS.2, 3; 7.NS.1, 2.a, 2.b; 8.NS.1; 8.EE.1; N-CN.2.a, 2.b, 2.c; N-RN.1; S-CP.1-5; S-IC.1-2*
Claim 2	**Geometry: Students demonstrate increasingly complex spatial reasoning and understanding of geometric principles.** Conceptual Areas in the Dynamic Learning Map: **MC 2.1 Understand and use geometric properties of two- and three-dimensional shapes** *Essential Elements Included: K.MD.1-3; K.G.2-3; 1.G.1, 2; 2.G.1; 3.G.1; 4.G.1, 2; 4.MD.5, 6; 5.G.1-4; 5.MD.3; 7.G.1, 2, 3, 5; 8.G.1, 2, 4, 5; G-CO.1, 4-5, 6-8; G-GMD.1-3, 4* **MC 2.2 Solve problems involving area, perimeter, and volume** *Essential Elements Included: 1.G.3; 3.G.2; 4.G.3; 4.MD.3; 5.MD.4-5; 6.G.1, 2; 7.G.4, 6; 8.G.9; G-GMD.1-3; G-GPE.7*
Claim 3	**Measurement Data and Analysis: Students demonstrate increasingly complex understanding of measurement, data, and analytic procedures.** Conceptual Areas in the Dynamic Learning Map: **MC 3.1 Understand and use measurement principles and units of measure** *Essential Elements Included: 1.MD.1-2, 3.a, 3.b, 3.c, 3.d; 2.MD.1, 3-4, 5, 6, 7, 8; 3.MD.1, 2, 4; 4.MD.1, 2.a, 2.b, 2.c, 2.d; 5.MD.1.a, 1.b, 1.c; N-Q.1-3* **MC 3.2 Represent and interpret data displays** *Essential Elements Included: 1.MD.4; 2.MD.9-10; 3.MD.3; 4.MD.4.a, 4.b; 5.MD.2; 6.SP.1-2, 5; 7.SP.1-2, 3, 5-7; 8.SP.4; S-ID. 1-2, 3, 4*
Claim 4	**Algebraic and functional reasoning: Students solve increasingly complex mathematical problems, making productive use of algebra and functions.** Conceptual Areas in the Dynamic Learning Map: **MC 4.1. Use operations and models to solve problems** *Essential Elements Included: K.OA.1, 1.a, 1.b, 2, 5.a, 5.b; 2.OA.3, 4; 3.OA.1-2, 8; 4.OA.1-*

	2, 3, 4; 6.EE.1-2, 3, 5-7; 7.EE.1, 4; 8.EE.7; A-CED.1, 2-4; A-SSE.1, 3 **MC 4.2 Understand patterns and functional thinking** *Essential Elements Included: 3.OA.9; 4.OA.5; 5.OA.3; 7.EE.2; 8.EE.5-6; 8.F.1-3, 4, 5; A-REI.10-12; A-SSE.4; F-BF.1, 2; F-IF.1-3, 4-6; F-LE.1*

A-CED = creating equations; A-SSE = seeing structure in equations BF = building functions; CC = counting & cardinality; EE = expressions & equations; F-BF = basic fractions; F-IF = interpreting functions; G = geometry; G-GMD = geometric measurement & dimension; G-GPE = general properties & equations; MD = measurement & data; NBT = numbers & operations in base ten; N-CN = complex number system; NF = numbers & operations - fractions; N-RN = real number system; NS = number systems; N-Q = number & quantity; OA = operations & algebraic thinking; RP = ratios & proportional relationships; S-IC- statistics & probability - making inferences/justifying conclusions; S-ID = statistics & probability - interpreting categorical & quantitative data; SP = statistics & probability

Resulting Changes to the DLM Essential Elements

The development of the entire DLM Alternate Assessment System guided a final round of revisions to the DLM EEs, which can be organized into four broad categories: alignment across grade levels, language specificity, common core alignment, and defining learning expectations (rather than instructional tasks). The first type of revision was required to align the DLM EEs across grade levels, both vertically and horizontally. The maps, and the research supporting them, were critical in determining the appropriate progression of skills and understandings from grade to grade. This alignment across grade levels was important within and across standards, strands, and domains. For example, in determining when it was appropriate to introduce concepts in mathematics regarding the relative position of objects, we had to consider the grade level at which prepositions that describe relative position were introduced in English language arts. Examining the research-based skill development outlined in the learning map aided in these kinds of determinations.

The articulation of the claims and conceptual areas reinforced the need for specific language in the DLM EEs to describe learning within an area. Because teams assigned to grade bands developed the first round of DLM EEs, the language choices from one grade to the next were not consistent. Even when closely related skills, concepts, or understandings were targeted, the same terms were not always selected to describe the intended learning outcome. The teams of content experts who worked on this revised version of the DLM EEs were very intentional in selecting a common set of terms to reflect the claims and conceptual areas and applied them consistently across the entire set of DLM EEs.

Another important change in this version of the DLM EEs involved alignment to the Common Core State Standards (CCSS). Given that the DLM EEs are intended to clarify the bridge to the CCSS expectations for students with the most significant cognitive disabilities, it is critical that alignment be as close as possible without compromising learning and development over time. While there was never a one-to-one correspondence between the CCSS and the DLM EEs, the revisions have made the alignment between the two more precise than it was in the first version.

Finally, revisions to the DLM EEs involved shifting the focus of a small number of DLM EEs that were written in the form of instructional tasks rather than learning expectations, and adding "With guidance and support" to the beginning of a few of the DLM EEs in the primary grades in English language arts to reflect the expectations articulated in the CCSS.

Members of the DLM consortium reviewed each of the changes to the original version of the DLM EEs. Four states provided substantive feedback on the revisions, and this document incorporates the changes those teams suggested.

Access to Instruction and Assessment

The DLM EEs specify learning targets for students with significant cognitive disabilities; however, they do not describe all of the ways that students can engage in instruction or demonstrate understanding through an assessment. Appropriate modes of communication, both for presentation or response, are not stated in the DLM EEs unless a specific mode is an expectation. Where no limitation has been stated, no limitation should be inferred. Students' opportunities to learn and to demonstrate learning during assessment should be maximized by providing whatever communication, assistive technologies, augmentative and alternative communication (AAC) devices, or other access tools that are necessary and routinely used by the student during instruction.

Students with significant cognitive disabilities include a broad range of students with diverse disabilities and communication needs. For some students with significant cognitive disabilities, a range of assistive technologies is required to access content and demonstrate achievement. For other students, AAC devices or accommodations for hearing and visual impairments will be needed. During instruction, teams should meet individual student needs using whatever technologies and accommodations are required. Examples of some of the ways that students may use technology while learning and demonstrating learning are topics for professional development, and include:

- communication devices that compensate for a student's physical inability to produce independent speech.
- alternate access devices that compensate for a student's physical inability to point to responses, turn pages in a book, or use a pencil or keyboard to answer questions or produce writing.

Guidance and Support

The authors of the CCSS use the words "prompting and support" at the earliest grade levels to indicate when students are not expected to achieve standards completely independently. Generally, "prompting" refers to "the action of saying something to persuade, encourage, or remind someone to do or say something" (McKean, 2005). However, in special education, prompting is often used to mean a system of structured cues to elicit desired behaviors that otherwise would not occur. In order to clearly communicate that teacher assistance is permitted during instruction of the DLM EEs and is not limited to structured prompting procedures, the decision was made by the stakeholder group to use the more general term *guidance* throughout the DLM EEs.

Guidance and support during instruction should be interpreted as teacher encouragement, general assistance, and informative feedback to support the student in learning. Some examples of the kinds of teacher behaviors that would be considered guidance and support include verbal supports, such as

- getting the student started (e.g., "Tell me what to do first."),
- providing a hint in the right direction without revealing the answer (e.g., Student wants to write *dog* but is unsure how, so the teacher might say, "See if you can write the first letter in the word, /d/og [phonetically pronounced]."),
- using structured technologies such as task-specific word banks, or
- providing structured cues such as those found in prompting procedures (e.g., least-to-most prompts, simultaneous prompting, and graduated guidance).

Guidance and support as described above applies to instruction and is also linked to demonstrating learning relative to DLM EEs, where guidance and support is specifically called out within the standards.

Conclusion

Developing the research-based model of knowledge and skill development represented in the DLM Learning Maps supported the articulation of assessment claims for mathematics and English language arts. This articulation subsequently allowed for a careful revision of the DLM EEs to reflect both horizontal alignment with the CCSS and vertical alignment across the grades, with the goal of moving students toward more sophisticated understandings in both domains. Though the contributions made by Edvantia and our state partners in developing the initial set of DLM EEs were a critical first step, additional revisions to the DLM EEs were required to ensure consistency across all elements of the Dynamic Learning Maps Alternate Assessment System.

APPENDIX

Development of the Dynamic Learning Maps Essential Elements has been a collaborative effort among practitioners, researchers, and our state representatives. Listed below are the reviews and the individuals involved with each round of improvements to the Dynamic Learning Maps Essential Elements. Thank you to all of our contributors.

Review of Draft Two of Dynamic Learning Maps Essential Elements

A special thanks to all of the experts nominated by their state to review draft two of the Dynamic Learning Maps Essential Elements. We are grateful for your time and efforts to improve these standards for students with significant cognitive disabilities. Your comments have been incorporated into this draft. The states with teams who reviewed draft two include:

Illinois	Oklahoma
Iowa	Utah
Kansas	Virginia
Michigan	West Virginia
Missouri	Wisconsin

Development of the Original Dynamic Learning Maps Common Core Essential Elements

A special thanks to Edvantia and the team of representatives from Dynamic Learning Maps consortium states who developed the original Common Core Essential Elements upon which the revised Dynamic Learning Maps Essential Elements are based. The team from Edvantia who led the original effort included:

Jan Sheinker, Sheinker Educational Services, Inc.
Beth Judy, Director, Assessment, Alignment, and Accountability Services
Nathan Davis, Information Technology Specialist
Kristen Deitrick, Corporate Communications Specialist

Linda Jones, Executive Assistant

Representatives from Dynamic Learning Maps consortium states included:

IOWA
SEA Representatives: Tom Deeter, Emily Thatcher
Stakeholders: Peggy Akins, Judy Hamer, Kathleen Kvamme-Promes, Donna Shaw

KANSAS
SEA Representatives: Debbie Matthews, Kris Shaw
Stakeholders: Debby Byrne, Holly Draper, Dawn Gresham, Linda Hickey

MICHIGAN
SEA Representatives: Joanne Wilkelman, Adam Wyse
Stakeholders: Debra Susan Asano, Thomai Gersh, Marcia O'Brian, Terri Portice

MISSOURI
SEA Representatives: Lynn Everett, Jane VanDeZande
Stakeholders: Melia Franklin, Lou Ann Hoover, Debbie Jameson, Kate Sadler

NEW JERSEY
SEA Representatives: Melanie O'Dea
Stakeholders: Brenda Berrios, Neal Webster, Tina Yurcho

NORTH CAROLINA
SEA Representatives: Claire Greer, Sarah Reives
Stakeholders: Emma Hatfield-Sidden, Judy Jennings, Jennifer Michalenok,

OKLAHOMA
SEA Representatives: Jennifer Burnes, Amy Dougherty
Stakeholders: Pam Cox, Dianna Daubenspeck, Sondra LeGrande, Christie Stephenson

UTAH
SEA Representatives: Wendy Carver, Kurt Farnsworth
Stakeholders: James Bray, Janice Hill, Linda Stallviere, Ryan Webb

VIRGINIA
SEA Representative: John Eisenberg
Stakeholders: Maria Beck, Daniel Blegun, Al Klugh, Cheryl Ann Prevatte

WASHINGTON

SEA Representatives: Judy Kraft, Janice Tornow
Stakeholders: Annalisa Brewster, Kim Cook, Jeffrey Dunn, Kimberly Perisho

WEST VIRGINIA
SEA Representatives: Beth Cipoletti, Melissa Gholson
Stakeholders: Robert Bartlett, Gerald Hartley, Angel Harris, Angela See

WISCONSIN
SEA Representatives: Emilie Amundson, Kristen Burton
Stakeholders: Lori Hillyer, Tamara Maxwell, Connie Persike, Sara Vold

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR HIGH SCHOOL

High School Mathematics Domain: Number and Quantity—The Real Number System

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Extend the properties of exponents to rational exponents.	
N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	**EE.N-RN.1.** Determine the value of a quantity that is squared or cubed.
N-RN.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	Not applicable.
CLUSTER: Use properties of rational and irrational numbers.	
N-RN.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	Not applicable.

High School Mathematics Domain: Number and Quantity—Quantities★

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Reason quantitatively, and use units to solve problems.	
N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	**EE.N-Q.1–3.** Express quantities to the appropriate precision of measurement.
N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.	
N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	

High School Mathematics Domain: Number and Quantity—The Complex Number System

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Perform arithmetic operations with complex numbers.	
N-CN.1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	Not applicable.
N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	**EE.N-CN.2.a.** Use the commutative, associative, and distributive properties to add, subtract, and multiply whole numbers.
	EE.N-CN.2.b. Solve real-world problems involving addition and subtraction of decimals, using models when needed.
	EE.N-CN.2.c. Solve real-world problems involving multiplication of decimals and whole numbers, using models when needed.
N-CN.3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	Not applicable.
CLUSTER: Represent complex numbers and their operations on the complex plane.	
N-CN.4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	Not applicable.
N-CN.5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}i)^3 = 8$ *because* $(-1 + \sqrt{3}i)$ *has modulus 2 and argument 120°.*	Not applicable.
N-CN.6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use complex numbers in polynomial identities and equations.	
N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.	Not applicable.
N-CN.8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.	Not applicable.
N-CN.9. (+) Know the Fundamental Theorum of Algebra; show that it is true for quadratic polynomials	Not applicable.

High School Mathematics Domain: Number and Quantity – Vector and Matrix Quantities

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Represent and model with vector quantities.	
N-VM.1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$, v).	Not applicable.
N-VM.2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	Not applicable.
N-VM.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.	Not applicable.
CLUSTER: Perform operations on vectors.	
N-VM.4. (+) Add and subtract vectors.	Not applicable.
N-VM.4.a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	
N-VM.4.b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.	
N-VM.4.c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	
N-VM.5. (+) Multiply a vector by a scalar.	Not applicable.
N-VM.5.a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.	
N-VM.5.b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\lVert c\boldsymbol{v}\rVert = \lvert c\rvert v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\lvert c\rvert v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).	

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Perform operations on matrices, and use matrices in applications.	
N-VM.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	Not applicable.
N-VM.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.	Not applicable.
N-VM.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.	Not applicable.
N-VM.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	Not applicable.
N-VM.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	Not applicable.
N-VM.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	Not applicable.
N-VM.12. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	Not applicable.