OKLAHOMA ACADEMIC STANDARDS

MATHEMATICS

OKLAHOMA STATE DEPARTMENT OF EDUCATION

CHAMPION EXCELLENCE

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Introduction

The Oklahoma Academic Standards for Mathematics 2016 is the result of the contributions of hundreds of mathematics teachers, mathematics educators, and mathematicians from across the state of Oklahoma. This document reflects a balanced synthesis of the work of all members of the Oklahoma Academic Standards for Mathematics Writing Committee and feedback from teachers, mathematicians, external reviews, and numerous education stakeholders including business, industry and commerce, parent groups, career tech, higher education, and external reviewers.

The Oklahoma Academic Standards for Mathematics 2016 specify what students should know and be able to do as learners of mathematics at the end of each grade level or course. Students are held responsible for learning standards listed at earlier grade levels as well as their current grade level. Throughout this document, the standards are written to allow time for study of additional material at every grade level. The order of the standards at any grade level is not meant to imply a sequence of topics and should be considered flexible for the organization of any course. The document provides standards for PK-7, Pre-Algebra, Algebra I, Geometry, and Algebra II with Algebra I as the pre-requisite for both Geometry and Algebra II.

Development of the Oklahoma Academic Standards for Mathematics

The Oklahoma Academic Standards for Mathematics writing team drew on the work of the National Council of Teachers of Mathematics (NCTM) standards documents; the National Research Council's report Adding It Up, the Oklahoma Priority Academic Standards (PASS), and other states' standards documents and curriculum framework guides (e.g., Minnesota, Virginia, and Massachusetts). Please see the reference list at the end of this document for a more complete list of all resources consulted.

Vision and Guiding Principles

These standards envision all students in Oklahoma will become mathematically proficient and literate through a strong mathematics program that emphasizes and engages them in problem solving, communicating, reasoning and proof, making connections, and using representations. Mathematically proficient and literate students can confidently and effectively use mathematics concepts, computation skills, and numbers to problem-solve, reason, and analyze information. Developing mathematical proficiency and literacy for Oklahoma students depends in large part on a clear, comprehensive, coherent, and developmentally appropriate set of standards to guide curricular decisions. The understanding and implementation of these standards throughout PK-12 mathematics experience for students is based on the following guiding principles:

Guiding Principle 1: Excellence in mathematics education requires equity—high expectations and strong support for all students.
All students must have opportunities to study—and support to learn—mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students.

Guiding Principle 2: Mathematical ideas should be explored in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.
Students need to understand mathematics deeply and use it effectively. To achieve mathematical understanding, students should be actively engaged in doing meaningful mathematics, discussing mathematical ideas, and applying mathematics in interesting, thought provoking situations. Student understanding is

further developed through ongoing reflection about cognitively demanding and tasks relevant to their lives.

Tasks should challenge and engage students in mathematics in multiple ways. Short- and long-term investigations that connect procedures and skills with conceptual understanding are integral components of an effective mathematics program. Activities should build upon curiosity and prior knowledge, and enable students to solve progressively deeper, broader, and more sophisticated problems. Mathematical tasks reflecting significant mathematics should generate active classroom talk, promote the development of conjectures, and lead to an understanding of the necessity for mathematical reasoning.

Guiding Principle 3: An effective mathematics program focuses on problem solving.
Mathematical problem solving is the hallmark of an effective mathematics program. Skill in mathematical problem solving requires practice with a variety of mathematical problems as well as a firm grasp of mathematical techniques and their underlying principles. Students who possess a deeper knowledge of mathematics can then use mathematics in a flexible way to attack various problems and devise different ways of solving any particular problem. Mathematical problem solving calls for reflective thinking, persistence, and learning from the ideas of others. Success in solving mathematical problems helps to create an abiding interest in mathematics.

Guiding Principle 4: Technology is essential in teaching and learning mathematics.
Technology enhances the mathematics curriculum in many ways. Technology enables students to communicate ideas within the classroom or to search for needed information. It can be especially helpful in assisting students with special needs in regular and special classrooms, at home, and in the community. Technology changes what mathematics is to be learned and when and how it is learned. Tools such as measuring instruments, manipulatives (such as base ten blocks and fraction pieces), scientific and graphing calculators, and computers with appropriate software, if properly used, contribute to a rich learning environment for developing and applying mathematical concepts. Appropriate use of calculators is essential; calculators should not be used as a replacement for basic understanding and skills. Although the use of a graphing calculator can help middle and secondary students to visualize properties of functions and their graphs, graphing calculators should be used to enhance their understanding and skills rather than replace them.

Standards Overview

The Oklahoma Academic Standards for Mathematics are developed around four main content strands, Algebraic Reasoning and Algebra, Number and Operations, Geometry and Measurement, and Data and Probability organize the content standards throughout PK-7 and Pre-Algebra. The standards for Algebra I, Algebra II, and Geometry are fundamentally organized around these strands as well. The process standards are defined as the Mathematical Actions and Processes and are comprised of the skills and abilities students should develop and be engaged in throughout their PK-12 mathematics education. Among these are the ability to problem solve, communicate, and reason about mathematics which will help students be ready for the mathematics expectations of college and the skills desired by many employers. While the process and content standards work in concert to create clear, concise, and rigorous mathematics standards and expectations for Oklahoma students with the aim of helping them be college and career ready, it is not intended that each mathematical action and process will be utilized or developed with each standard. Certainly some standards and objectives can be achieved more readily with particular mathematics actions and processes. For example, an objective that involves explaining a particular concept may be best accomplished by also engaging students in communicating mathematically. Whereas, standards and objectives that focus in the early grades on fluency with operations will align well with the mathematical action and process focused on procedural fluency.

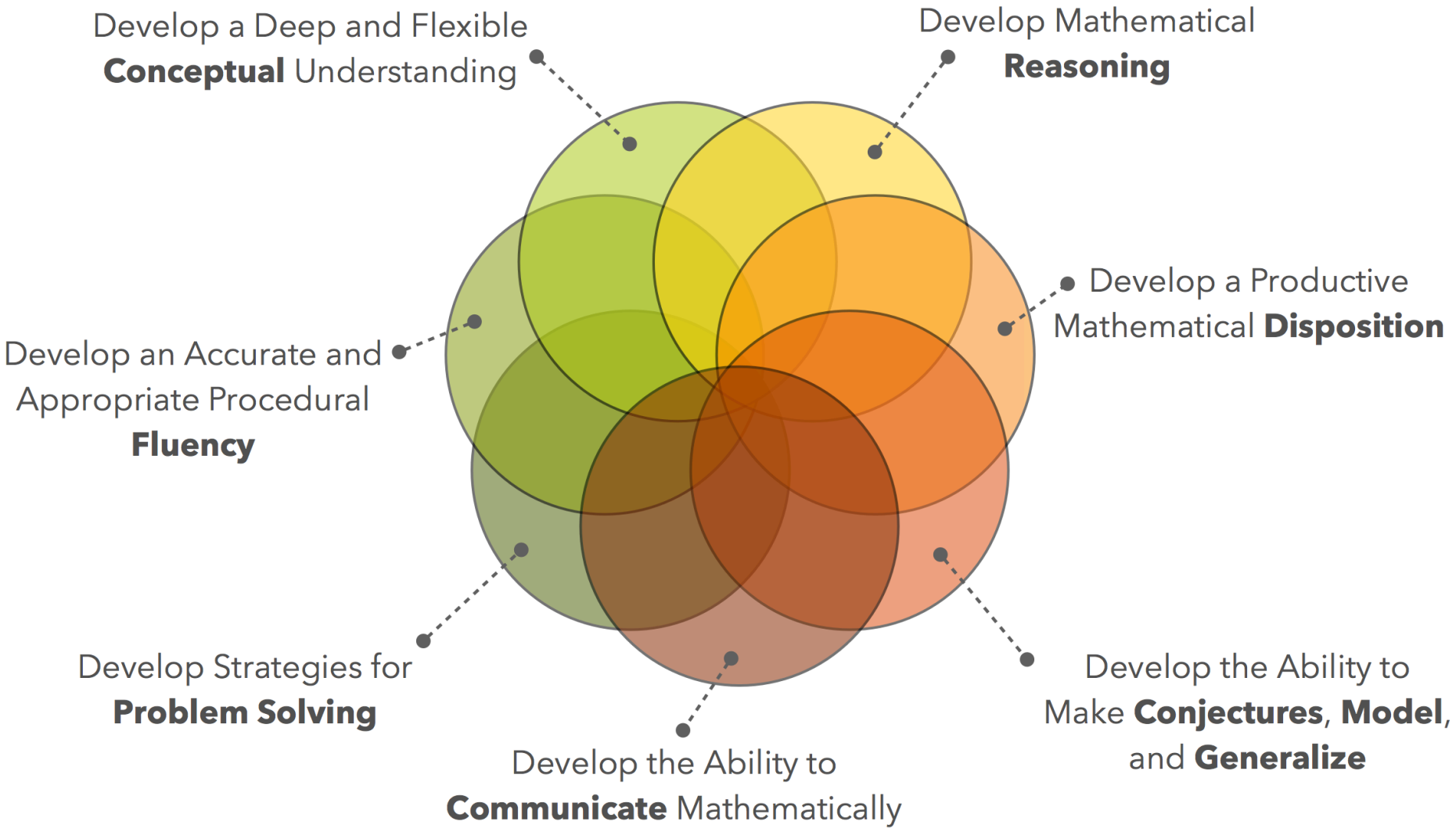
Number and Operations Strand: A focus on number and operations is the cornerstone of a strong mathematics program. Developing students' fluency with number and operations throughout their PK-12 mathematics experience requires a balance and connection between conceptual understanding and computational proficiency and efficiency. This strand provides focus on the importance of students' understanding of numbers, ways of representing numbers, relationships among numbers, relationships among number systems, and meanings of operations and how they relate to one another. An emphasis is placed on the development of estimation so students can determine the reasonableness of solutions and answers. Further, it requires that students should be able to compute efficiently and proficiently.

Algebraic Reasoning and Algebra Strand: All students should be able to reason algebraically and learn algebra. This strand provides focus for the PK-7 and Pre-Algebra standards around the notion that algebra is more than moving symbols around. It is about understanding patterns, relations and functions, representing and analyzing mathematical situations and structures using algebraic symbols, using mathematical models to represent and understand quantitative relationships, and analyzing change in various contexts. Understanding change is fundamental to algebraic reasoning and the concept of function with depth. This understanding is critical for success in college-level mathematics. It is also fundamental to understanding many real-world problems and situations students will face in their future careers.

Geometry and Measurement Strand: All students should gain experience using a variety of visual and coordinate representations to analyze and solve problems and learn how to use appropriate units and tools for measuring. This strand provides focus for the PK-7 and Geometry standards around the notion that geometry and measurement help students understand and represent ideas and solve problems they will encounter in their daily lives. A focus on geometry should enable students to analyze characteristics of two- and three-dimensional objects, develop arguments based on geometric relationships, describe spatial relationships using coordinate geometry and other representational systems, apply transformations and symmetry to analyze mathematical situations, and utilize visualization, spatial reasoning and geometric modeling to solve problems. A focus on measurement should enable students to understand measureable attributes of objects and the units, systems, and processes of measurement, and apply appropriate techniques, tools, and formulas to determine measurements.

Data and Probability Strand: An increased emphasis on understanding data should span all grade levels. Making sense of data and probability has become a part of our daily lives, supporting the importance of this strand throughout a students' PK-12 mathematics experience. A focus on data and probability should enable all students to formulate questions that can be addressed with data, and to collect, organize, and display relevant data to answer them. Students should select and use appropriate statistical methods to analyze data, develop and evaluate inferences and predictions that are based on data, and understand and apply basic concepts of probability. The study of data is also an opportunity to apply the basic skills of computing with numbers and being an educated consumer of information presented in the news and media while the study of probability provides application and use of fractions in daily life.

Mathematical Actions and Processes



Mathematical Actions and Processes

The Mathematical Actions and Processes simultaneously reflect the holistic nature of mathematics as a discipline in which patterns and relationships among quantities, numbers, and space are studied (National Academies of Sciences, 2014) and as a form of literacy such that all students are supported in accessing and understanding mathematics for life, for the workplace, for the scientific and technical community, and as a part of cultural heritage (NCTM, 2000). The seven Mathematical Actions and Processes leverage both the NCTM Process Standards and the Five Mathematical Proficiencies (NRC, 2001) to capture the mathematical experience of Oklahoma students as they pursue mathematical literacy.

Throughout their Pk-12 education experience, mathematically literate students will:

Develop a Deep and Flexible Conceptual Understanding
Demonstrate a deep and flexible conceptual understanding of mathematical concepts, operations, and relations while making mathematical and real-world connections. Students will develop an understanding of how and when to apply and use the mathematics they know to solve problems.

Develop Accurate and Appropriate Procedural Fluency
Learn efficient procedures and algorithms for computations and repeated processes based on a strong sense of numbers. Develop fluency in addition, subtraction, multiplication, and division of numbers and expressions. Students will generate a sophisticated understanding of the development and application of algorithms and procedures.

Develop Strategies for Problem Solving
Analyze the parts of complex mathematical tasks and identify entry points to begin the search for a solution. Students will select from a variety of problem solving strategies and use corresponding multiple representations (verbal, physical, symbolic, pictorial, graphical, tabular) when appropriate. They will pursue solutions to various tasks from real-world situations and applications that are often interdisciplinary in nature. They will find methods to verify their answers in context and will always question the reasonableness of solutions.

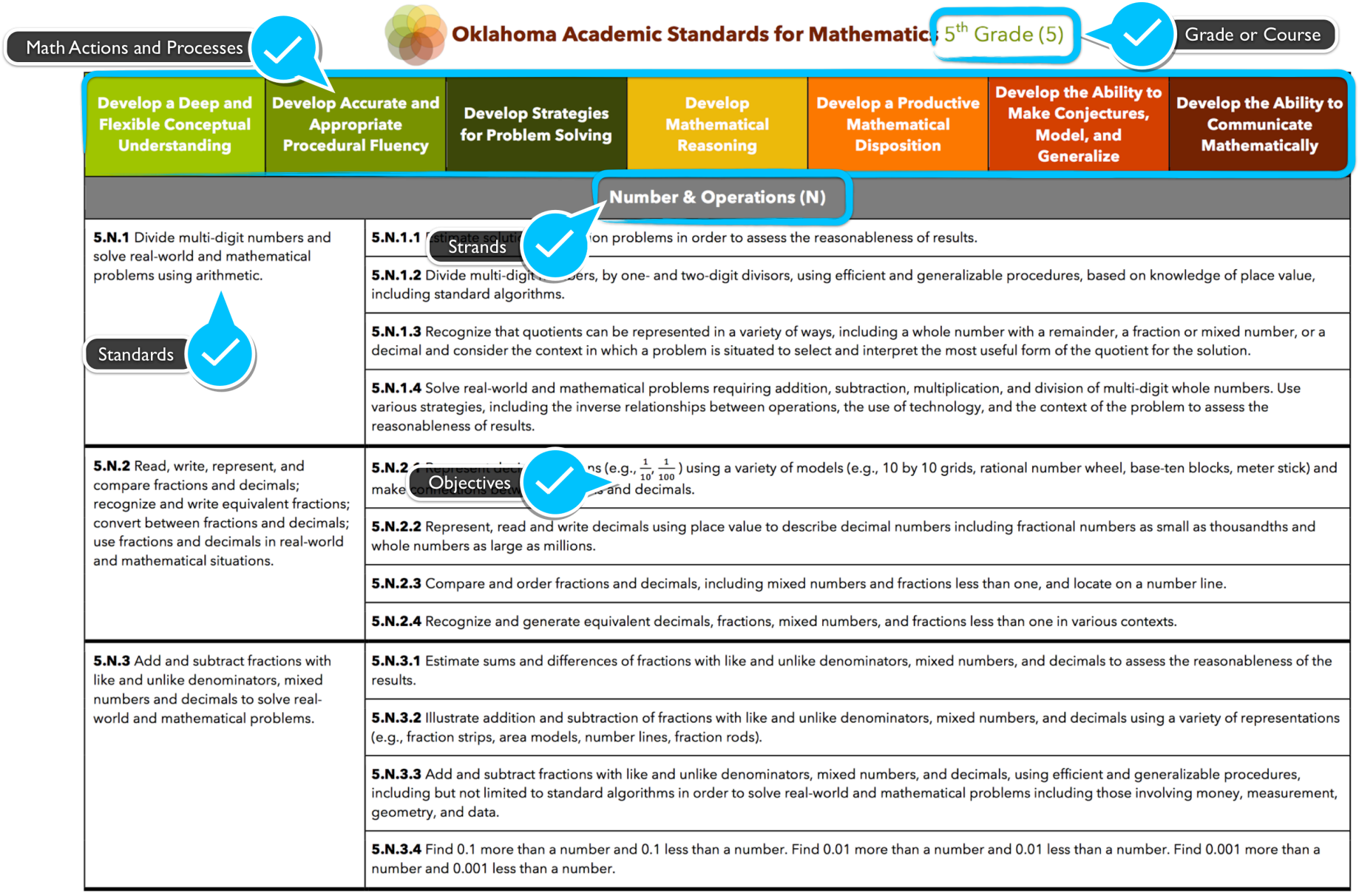
Develop Mathematical Reasoning
Explore and communicate a variety of reasoning strategies to think through problems. Students will apply their logic to critique the thinking and strategies of others to develop and evaluate mathematical arguments, including making arguments and counterarguments and making connections to other contexts.

Develop a Productive Mathematical Disposition
Hold the belief that mathematics is sensible, useful and worthwhile. Students will develop the habit of looking for and making use of patterns and mathematical structures. They will persevere and become resilient, effective problem solvers.

Develop the Ability to Make Conjectures, Model, and Generalize
Make predictions and conjectures and draw conclusions throughout the problem solving process based on patterns and the repeated structures in mathematics. Students will create, identify, and extend patterns as a strategy for solving and making sense of problems.

Develop the Ability to Communicate Mathematically
Students will discuss, write, read, interpret and translate ideas and concepts mathematically. As they progress, students' ability to communicate mathematically will include their increased use of mathematical language and terms and analysis of mathematical definitions.

Reading the Oklahoma Academic Standards for Mathematics



5.N.1 Divide multi-digit numbers and solve real-world and mathematical problems using arithmetic.	**5.N.1.1** [illegible] on problems in order to assess the reasonableness of results.
	5.N.1.2 Divide multi-digit [illegible]bers, by one- and two-digit divisors, using efficient and generalizable procedures, based on knowledge of place value, including standard algorithms.
	5.N.1.3 Recognize that quotients can be represented in a variety of ways, including a whole number with a remainder, a fraction or mixed number, or a decimal and consider the context in which a problem is situated to select and interpret the most useful form of the quotient for the solution.
	5.N.1.4 Solve real-world and mathematical problems requiring addition, subtraction, multiplication, and division of multi-digit whole numbers. Use various strategies, including the inverse relationships between operations, the use of technology, and the context of the problem to assess the reasonableness of results.
5.N.2 Read, write, represent, and compare fractions and decimals; recognize and write equivalent fractions; convert between fractions and decimals; use fractions and decimals in real-world and mathematical situations.	**5.N.2.1** [illegible]ns (e.g., $\frac{1}{10}$, $\frac{1}{100}$) using a variety of models (e.g., 10 by 10 grids, rational number wheel, base-ten blocks, meter stick) and make [illegible] and decimals.
	5.N.2.2 Represent, read and write decimals using place value to describe decimal numbers including fractional numbers as small as thousandths and whole numbers as large as millions.
	5.N.2.3 Compare and order fractions and decimals, including mixed numbers and fractions less than one, and locate on a number line.
	5.N.2.4 Recognize and generate equivalent decimals, fractions, mixed numbers, and fractions less than one in various contexts.
5.N.3 Add and subtract fractions with like and unlike denominators, mixed numbers and decimals to solve real-world and mathematical problems.	**5.N.3.1** Estimate sums and differences of fractions with like and unlike denominators, mixed numbers, and decimals to assess the reasonableness of the results.
	5.N.3.2 Illustrate addition and subtraction of fractions with like and unlike denominators, mixed numbers, and decimals using a variety of representations (e.g., fraction strips, area models, number lines, fraction rods).
	5.N.3.3 Add and subtract fractions with like and unlike denominators, mixed numbers, and decimals, using efficient and generalizable procedures, including but not limited to standard algorithms in order to solve real-world and mathematical problems including those involving money, measurement, geometry, and data.
	5.N.3.4 Find 0.1 more than a number and 0.1 less than a number. Find 0.01 more than a number and 0.01 less than a number. Find 0.001 more than a number and 0.001 less than a number.

Develop a Deep and Flexible Conceptual Understanding	Develop Accurate and Appropriate Procedural Fluency	Develop Strategies for Problem Solving	Develop Mathematical Reasoning	Develop a Productive Mathematical Disposition	Develop the Ability to Make Conjectures, Model, and Generalize	Develop the Ability to Communicate Mathematically

Number & Operations (N)	
PK.N.1 Know number names and count in sequence.	**PK.N.1.1** Count aloud forward in sequence by 1s to 20.
	PK.N.1.2 Recognize and name written numerals 0-10.
	PK.N.1.3 Recognize that zero represents the count of no objects.
PK.N.2 Count to tell the number of objects.	**PK.N.2.1** Identify the number of objects, up to 10, in a row or column.
	PK.N.2.2 Use one-to-one correspondence in counting objects and matching groups of objects.
	PK.N.2.3 Understand the last numeral spoken, when counting aloud, tells how many total objects are in a set.
	PK.N.2.4 Count up to 5 items in a scattered configuration; not in a row or column.
PK.N.3 Compare sets using number.	**PK.N.3.1** Compare two sets of 1-5 objects using comparative language such as same, more, or fewer.
Algebraic Reasoning & Algebra (A)	
PK.A.1 Recognize, duplicate, and extend patterns.	**PK.A.1.1** Sort and group up to 5 objects into a set based upon characteristics such as color, size, and shape and explain verbally what the objects have in common.
	PK.A.1.2 Recognize, duplicate, and extend repeating patterns involving manipulatives, sound, movement, and other contexts.
Geometry & Measurement (GM)	
PK.GM.1 Identify common shapes.	**PK.GM.1.1** Identify circles, squares, rectangles, and triangles by pointing to the shape when given the name.
PK.GM.2 Describe and compare measureable attributes.	**PK.GM.2.1** Identify measurable attributes of objects. Describe them as little, big, long, short, tall, heavy, light, or other age appropriate vocabulary.
	PK.GM.2.2 Directly compare two objects with a common measurable attribute using words such as longer/shorter; heavier/lighter; or taller/shorter.
	PK.GM.2.3 Sort objects into sets by one or more attributes.

Data & Probability (D)	
PK.D.1 Collect and organize categorical data.	**PK.D.1.1** Collect and organize information about objects and events in the environment.
	PK.D.1.2 Use categorical data to create real-object graphs.

Develop a Deep and Flexible Conceptual Understanding	Develop Accurate and Appropriate Procedural Fluency	Develop Strategies for Problem Solving	Develop Mathematical Reasoning	Develop a Productive Mathematical Disposition	Develop the Ability to Make Conjectures, Model, and Generalize	Develop the Ability to Communicate Mathematically

Number & Operations (N)

K.N.1 Understand the relationship between quantities and whole numbers.	**K.N.1.1** Count aloud forward in sequence to 100 by 1's and 10's.
	K.N.1.2 Recognize that a number can be used to represent how many objects are in a set up to 10.
	K.N.1.3 Use ordinal numbers to represent the position of an object in a sequence up to 10.
	K.N.1.4 Recognize without counting (subitize) the quantity of a small group of objects in organized and random arrangements up to 10. **Clarification statement**: Subitizing is defined as instantly recognizing the quantity of a set without having to count. "Subitizing" is not a vocabulary word and is not meant for student discussion at this age.
	K.N.1.5 Count forward, with and without objects, from any given number up to 10.
	K.N.1.6 Read, write, discuss, and represent whole numbers from 0 to at least 10. Representations may include numerals, pictures, real objects and picture graphs, spoken words, and manipulatives.
	K.N.1.7 Find a number that is 1 more or 1 less than a given number up to 10.
	K.N.1.8 Using the words more than, less than or equal to compare and order whole numbers, with and without objects, from 0 to 10.
K.N.2 Develop conceptual fluency with addition and subtraction (up to 10) using objects and pictures.	**K.N.2.1** Compose and decompose numbers up to 10 with objects and pictures.
K.N.3 Understand the relationship between whole numbers and fractions through fair share.	**K.N.3.1** Distribute equally a set of objects into at least two smaller equal sets.
K.N.4 Identify coins by name.	**K.N.4.1** Identify pennies, nickels, dimes, and quarters by name.

Algebraic Reasoning & Algebra (A)	
K.A.1 Duplicate patterns in a variety of contexts.	**K.A.1.1** Sort and group up to 10 objects into a set based upon characteristics such as color, size, and shape. Explain verbally what the objects have in common.
	K.A.1.2 Recognize, duplicate, complete, and extend repeating, shrinking and growing patterns involving shape, color, size, objects, sounds, movement, and other contexts.
Geometry & Measurement (GM)	
K.GM.1 Recognize and sort basic two-dimensional shapes and use them to represent real-world objects.	**K.GM.1.1** Recognize squares, circles, triangles, and rectangles.
	K.GM.1.2 Sort two-dimensional objects using characteristics such as shape, size, color, and thickness.
	K.GM.1.3 Identify attributes of two-dimensional shapes using informal and formal geometric language interchangeably.
	K.GM.1.4 Use smaller shapes to form a larger shape when there is an outline to follow.
	K.GM.1.5 Compose free-form shapes with blocks.
	K.GM.1.6 Use basic shapes and spatial reasoning to represent objects in the real world.
K.GM.2 Compare and order objects according to location and measurable attributes.	**K.GM.2.1** Use words to compare objects according to length, size, weight, position, and location.
	K.GM.2.2 Order up to 6 objects using measurable attributes, such as length and weight.
	K.GM.2.3 Sort objects into sets by more than one attribute.
	K.GM.2.4 Compare the number of objects needed to fill two different containers.
K.GM.3 Tell time as it relates to daily life.	**K.GM.3.1** Develop an awareness of simple time concepts using words such as yesterday, today, tomorrow, morning, afternoon, and night within his/her daily life.
Data & Probability (D)	
K.D.1 Collect, organize, and interpret categorical data.	**K.D.1.1** Collect and sort information about objects and events in the environment.
	K.D.1.2 Use categorical data to create real-object and picture graphs.
	K.D.1.3 Draw conclusions from real-object and picture graphs.

Develop a Deep and Flexible Conceptual Understanding	Develop Accurate and Appropriate Procedural Fluency	Develop Strategies for Problem Solving	Develop Mathematical Reasoning	Develop a Productive Mathematical Disposition	Develop the Ability to Make Conjectures, Model, and Generalize	Develop the Ability to Communicate Mathematically

Number & Operations (N)	
1.N.1 Count, compare, and represent whole numbers up to 100, with an emphasis on groups of tens and ones.	**1.N.1.1** Recognize numbers to 20 without counting (subitize) the quantity of structured arrangements. **Clarification statement**: Subitizing is defined as instantly recognizing the quantity of a set without having to count. "Subitizing" is not a vocabulary word and is not meant for student discussion at this age.
	1.N.1.2 Use concrete representations to describe whole numbers between 10 and 100 in terms of tens and ones.
	1.N.1.3 Read, write, discuss, and represent whole numbers up to 100. Representations may include numerals, addition and subtraction, pictures, tally marks, number lines and manipulatives, such as bundles of sticks and base 10 blocks.
	1.N.1.4 Count forward, with and without objects, from any given number up to 100 by 1s, 2s, 5s and 10s.
	1.N.1.5 Find a number that is 10 more or 10 less than a given number up to 100.
	1.N.1.6 Compare and order whole numbers from 0 to 100.
	1.N.1.7 Use knowledge of number relationships to locate the position of a given whole number on an open number line up to 20.
	1.N.1.8 Use objects to represent and use words to describe the relative size of numbers, such as more than, less than, and equal to.
1.N.2 Solve addition and subtraction problems up to 10 in real-world and mathematical contexts.	**1.N.2.1** Represent and solve real-world and mathematical problems using addition and subtraction up to ten.
	1.N.2.2 Determine if equations involving addition and subtraction are true.
	1.N.2.3 Demonstrate fluency with basic addition facts and related subtraction facts up to 10.
1.N.3 Develop foundational ideas for fractions.	**1.N.3.1** Partition a regular polygon using physical models and recognize when those parts are equal.
	1.N.3.2 Partition (fair share) sets of objects into equal groupings.

1.N.4 Identify coins and their values.	**1.N.4.1** Identifying pennies, nickels, dimes, and quarters by name and value.
	1.N.4.2 Write a number with the cent symbol to describe the value of a coin.
	1.N.4.3 Determine the value of a collection of pennies, nickels, or dimes up to one dollar counting by ones, fives, or tens.
Algebraic Reasoning & Algebra (A)	
1.A.1 Identify patterns found in real-world and mathematical situations.	**1.A.1.1** Identify, create, complete, and extend repeating, growing, and shrinking patterns with quantity, numbers, or shapes in a variety of real-world and mathematical contexts.
Geometry & Measurement (GM)	
1.GM.1 Recognize, compose, and decompose two- and three-dimensional shapes.	**1.GM.1.1** Identify trapezoids and hexagons by pointing to the shape when given the name.
	1.GM.1.2 Compose and decompose larger shapes using smaller two-dimensional shapes.
	1.GM.1.3 Compose structures with three-dimensional shapes.
	1.GM.1.4 Recognize three-dimensional shapes such as cubes, cones, cylinders, and spheres.
1.GM.2 Select and use nonstandard and standard units to describe length and volume/capacity.	**1.GM.2.1** Use nonstandard and standard measuring tools to measure the length of objects to reinforce the continuous nature of linear measurement.
	1.GM.2.2 Illustrate that the length of an object is the number of same-size units of length that, when laid end-to-end with no gaps or overlaps, reach from one end of the object to the other.
	1.GM.2.3 Measure the same object/distance with units of two different lengths and describe how and why the measurements differ.
	1.GM.2.4 Describe a length to the nearest whole unit using a number and a unit.
	1.GM.2.5 Use standard and nonstandard tools to identify volume/capacity. Compare and sort containers that hold more, less, or the same amount.
1.GM.3 Tell time to the half and full hour.	**1.GM.3.1** Tell time to the hour and half-hour (analog and digital).
Data & Probability (D)	
1.D.1 Collect, organize, and interpret categorical and numerical data.	**1.D.1.1** Collect, sort, and organize data in up to three categories using representations (e.g., tally marks, tables, Venn diagrams).
	1.D.1.2 Use data to create picture and bar-type graphs to demonstrate one-to-one correspondence.
	1.D.1.3 Draw conclusions from picture and bar-type graphs.

Develop a Deep and Flexible Conceptual Understanding	Develop Accurate and Appropriate Procedural Fluency	Develop Strategies for Problem Solving	Develop Mathematical Reasoning	Develop a Productive Mathematical Disposition	Develop the Ability to Make Conjectures, Model, and Generalize	Develop the Ability to Communicate Mathematically

Number & Operations (N)	
2.N.1 Compare and represent whole numbers up to 1,000 with an emphasis on place value and equality.	**2.N.1.1** Read, write, discuss, and represent whole numbers up to 1,000. Representations may include numerals, words, pictures, tally marks, number lines and manipulatives.
	2.N.1.2 Use knowledge of number relationships to locate the position of a given whole number on an open number line up to 100.
	2.N.1.3 Use place value to describe whole numbers between 10 and 1,000 in terms of hundreds, tens and ones. Know that 100 is 10 tens, and 1,000 is 10 hundreds.
	2.N.1.4 Find 10 more or 10 less than a given three-digit number. Find 100 more or 100 less than a given three-digit number.
	2.N.1.5 Recognize when to round numbers to the nearest 10 and 100.
	2.N.1.6 Use place value to compare and order whole numbers up to 1,000 using comparative language, numbers, and symbols (e.g., 425 > 276, 73 < 107, page 351 comes after page 350, 753 is between 700 and 800).
2.N.2 Add and subtract one- and two-digit numbers in real-world and mathematical problems.	**2.N.2.1** Use the relationship between addition and subtraction to generate basic facts up to 20.
	2.N.2.2 Demonstrate fluency with basic addition facts and related subtraction facts up to 20.
	2.N.2.3 Estimate sums and differences up to 100.
	2.N.2.4 Use strategies and algorithms based on knowledge of place value and equality to add and subtract two-digit numbers.
	2.N.2.5 Solve real-world and mathematical addition and subtraction problems involving whole numbers up to 2 digits.
	2.N.2.6 Use concrete models and structured arrangements, such as repeated addition, arrays and ten frames to develop understanding of multiplication.
2.N.3 Explore the foundational ideas of fractions.	**2.N.3.1** Identify the parts of a set and area that represent fractions for halves, thirds, and fourths.
	2.N.3.2 Construct equal-sized portions through fair sharing including length, set, and area models for halves, thirds, and fourths.
2.N.4 Determine the value of a set of coins.	**2.N.4.1** Determine the value of a collection(s) of coins up to one dollar using the cent symbol.
	2.N.4.2 Use a combination of coins to represent a given amount of money up to one dollar.

Algebraic Reasoning & Algebra (A)	
2.A.1 Describe the relationship found in patterns to solve real-world and mathematical problems.	**2.A.1.1** Represent, create, describe, complete, and extend growing and shrinking patterns with quantity and numbers in a variety of real-world and mathematical contexts.
	2.A.1.2 Represent and describe repeating patterns involving shapes in a variety of contexts.
2.A.2 Use number sentences involving unknowns to represent and solve real-world and mathematical problems.	**2.A.2.1** Use objects and number lines to represent number sentences.
	2.A.2.2 Generate real-world situations to represent number sentences and vice versa.
	2.A.2.3 Apply commutative and identity properties and number sense to find values for unknowns that make number sentences involving addition and subtraction true or false.
Geometry & Measurement (GM)	
2.GM.1 Analyze attributes of two-dimensional figures and develop generalizations about their properties.	**2.GM.1.1** Recognize trapezoids and hexagons.
	2.GM.1.2 Describe, compare, and classify two-dimensional figures according to their geometric attributes.
	2.GM.1.3 Compose two-dimensional shapes using triangles, squares, hexagons, trapezoids, and rhombi.
	2.GM.1.4 Recognize right angles and classify angles as smaller or larger than a right angle.
2.GM.2 Understand length as a measurable attribute and explore capacity.	**2.GM.2.1** Explain the relationship between the size of the unit of measurement and the number of units needed to measure the length of an object.
	2.GM.2.2 Explain the relationship between length and the numbers on a ruler by using a ruler to measure lengths to the nearest whole unit.
	2.GM.2.3 Explore how varying shapes and styles of containers can have the same capacity.
2.GM.3 Tell time to the quarter hour.	**2.GM.3.1** Read and write time to the quarter-hour on an analog and digital clock. Distinguish between a.m. and p.m.
Data & Probability (D)	
2.D.1 Collect, organize, and interpret data.	**2.D.1.1** Explain that the length of a bar in a bar graph or the number of objects in a picture graph represents the number of data points for a given category.
	2.D.1.2 Organize a collection of data with up to four categories using pictographs and bar graphs with intervals of 1s, 2s, 5s or 10s.
	2.D.1.3 Write and solve one-step word problems involving addition or subtraction using data represented within pictographs and bar graphs with intervals of one.
	2.D.1.4 Draw conclusions and make predictions from information in a graph.

Develop a Deep and Flexible Conceptual Understanding	Develop Accurate and Appropriate Procedural Fluency	Develop Strategies for Problem Solving	Develop Mathematical Reasoning	Develop a Productive Mathematical Disposition	Develop the Ability to Make Conjectures, Model, and Generalize	Develop the Ability to Communicate Mathematically

Number & Operations (N)

3.N.1 Compare and represent whole numbers up to 10,000 with an emphasis on place value and equality.	**3.N.1.1** Read, write, discuss, and represent whole numbers up to 10,000. Representations may include numerals, expressions with operations, words, pictures, number lines, and manipulatives.
	3.N.1.2 Use place value to describe whole numbers between 1,000 and 10,000 in terms of ten thousands, thousands, hundreds, tens and ones, including expanded form.
	3.N.1.3 Find 1,000 more or 1,000 less than a given four- or five-digit number. Find 100 more or 100 less than a given four- or five-digit number.
	3.N.1.4 Use place value to compare and order whole numbers up to 10,000, using comparative language, numbers, and symbols.
3.N.2 Add and subtract multi-digit whole numbers; multiply with factors up to 10; represent multiplication and division in various ways; Solve real-world and mathematical problems through the representation of related operations.	**3.N.2.1** Represent multiplication facts by using a variety of approaches, such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line and skip counting.
	3.N.2.2 Demonstrate fluency of multiplication facts with factors up to 10.
	3.N.2.3 Use strategies and algorithms based on knowledge of place value and equality to fluently add and subtract multi-digit numbers.
	3.N.2.4 Recognize when to round numbers and apply understanding to round numbers to the nearest ten thousand, thousand, hundred, and ten and use compatible numbers to estimate sums and differences.
	3.N.2.5 Use addition and subtraction to solve real-world and mathematical problems involving whole numbers. Use various strategies, including the relationship between addition and subtraction, the use of technology, and the context of the problem to assess the reasonableness of results.
	3.N.2.6 Represent division facts by using a variety of approaches, such as repeated subtraction, equal sharing and forming equal groups.
	3.N.2.7 Recognize the relationship between multiplication and division to represent and solve real-world problems.
	3.N.2.8 Use strategies and algorithms based on knowledge of place value, equality and properties of addition and multiplication to multiply a two-digit number by a one-digit number.

3.N.3 Understand meanings and uses of fractions in real-world and mathematical situations.	**3.N.3.1** Read and write fractions with words and symbols.
	3.N.3.2 Construct fractions using length, set, and area models.
	3.N.3.3 Recognize unit fractions and use them to compose and decompose fractions related to the same whole. Use the numerator to describe the number of parts and the denominator to describe the number of partitions.
	3.N.3.4 Use models and number lines to order and compare fractions that are related to the same whole.
3.N.4 Determine the value of a set of coins or bills.	**3.N.4.1** Use addition to determine the value of a collection of coins up to one dollar using the cent symbol and a collection of bills up to twenty dollars.
	3.N.4.2 Select the fewest number of coins for a given amount of money up to one dollar.
Algebraic Reasoning & Algebra (A)	
3.A.1 Describe and create representations of numerical and geometric patterns.	**3.A.1.1** Create, describe, and extend patterns involving addition, subtraction, or multiplication to solve problems in a variety of contexts.
	3.A.1.2 Describe the rule (single operation) for a pattern from an input/output table or function machine involving addition, subtraction, or multiplication.
	3.A.1.3 Explore and develop visual representations of growing geometric patterns and construct the next steps.
3.A.2 Use number sentences involving multiplication and unknowns to represent and solve real-world and mathematical problems.	**3.A.2.1** Find unknowns represented by symbols in arithmetic problems by solving one-step open sentences (equations) and other problems involving addition, subtraction, and multiplication. Generate real-world situations to represent number sentences.
	3.A.2.2 Recognize, represent and apply the number properties (commutative, identity, and associative properties of addition and multiplication) using models and manipulatives to solve problems.
Geometry & Measurement (GM)	
3.GM.1 Use geometric attributes to describe and create shapes in various contexts.	**3.GM.1.1** Sort three-dimensional shapes based on attributes.
	3.GM.1.2 Build a three-dimensional figure using unit cubes when picture/shape is shown.
	3.GM.1.3 Classify angles as acute, right, obtuse, and straight.
3.GM.2 Understand measurable attributes of real-world and mathematical objects using various tools.	**3.GM.2.1** Find perimeter of polygon, given whole number lengths of the sides, in real-world and mathematical situations.
	3.GM.2.2 Develop and use formulas to determine the area of rectangles. Justify why length and width are multiplied to find the area of a rectangle by breaking the rectangle into one unit by one unit squares and viewing these as grouped into rows and columns.
	3.GM.2.3 Choose an appropriate measurement instrument and measure the length of objects to the nearest whole centimeter or meter.
	3.GM.2.4 Choose an appropriate measurement instrument and measure the length of objects to the nearest whole yard, whole foot, or half inch.

	3.GM.2.5 Using common benchmarks, estimate the lengths (customary and metric) of a variety of objects.
	3.GM.2.6 Use an analog thermometer to determine temperature to the nearest degree in Fahrenheit and Celsius.
	3.GM.2.7 Count cubes systematically to identify number of cubes needed to pack the whole or half of a three-dimensional structure.
	3.GM.2.8 Find the area of two-dimensional figures by counting total number of same size unit squares that fill the shape without gaps or overlaps.
3.GM.3 Solve problems by telling time to the nearest 5 minutes.	**3.GM.3.1** Read and write time to the nearest 5-minute (analog and digital).
	3.GM.3.2 Determine the solutions to problems involving addition and subtraction of time in intervals of 5 minutes, up to one hour, using pictorial models, number line diagrams, or other tools.
Data & Probability (D)	
3.D.1 Summarize, construct, and analyze data.	**3.D.1.1** Summarize and construct a data set with multiple categories using a frequency table, line plot, pictograph, and/or bar graph with scaled intervals.
	3.D.1.2 Solve one- and two-step problems using categorical data represented with a frequency table, pictograph, or bar graph with scaled intervals.

Develop a Deep and Flexible Conceptual Understanding	Develop Accurate and Appropriate Procedural Fluency	Develop Strategies for Problem Solving	Develop Mathematical Reasoning	Develop a Productive Mathematical Disposition	Develop the Ability to Make Conjectures, Model, and Generalize	Develop the Ability to Communicate Mathematically

Number & Operations (N)

4.N.1 Solve real-world and mathematical problems using multiplication and division.	**4.N.1.1** Demonstrate fluency with multiplication and division facts with factors up to 12.
	4.N.1.2 Use an understanding of place value to multiply or divide a number by 10, 100 and 1,000.
	4.N.1.3 Multiply 3-digit by 1-digit or a 2-digit by 2-digit whole numbers, using efficient and generalizable procedures and strategies, based on knowledge of place value, including but not limited to standard algorithms.
	4.N.1.4 Estimate products of 3-digit by 1-digit or 2-digit by 2-digit whole numbers using rounding, benchmarks and place value to assess the reasonableness of results. Explore larger numbers using technology to investigate patterns.
	4.N.1.5 Solve multi-step real-world and mathematical problems requiring the use of addition, subtraction, and multiplication of multi-digit whole numbers. Use various strategies, including the relationship between operations, the use of appropriate technology, and the context of the problem to assess the reasonableness of results.
	4.N.1.6 Use strategies and algorithms based on knowledge of place value, equality and properties of operations to divide 3-digit dividend by 1-digit whole number divisors. (e.g., mental strategies, standard algorithms, partial quotients, repeated subtraction, the commutative, associative, and distributive properties).
	4.N.1.7 Determine the unknown addend or factor in equivalent and non-equivalent expressions. (e.g., 5 + 6 = 4 + □ , 3 x 8 < 3 x □).
4.N.2 Represent and compare fractions and decimals in real-world and mathematical situations; use place value to understand how decimals represent quantities.	**4.N.2.1** Represent and rename equivalent fractions using fraction models (e.g. parts of a set, area models, fraction strips, number lines).
	4.N.2.2 Use benchmark fractions (0, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, 1) to locate additional fractions on a number line. Use models to order and compare whole numbers and fractions less than and greater than one using comparative language and symbols.
	4.N.2.3 Decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations (e.g., $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$).
	4.N.2.4 Use fraction models to add and subtract fractions with like denominators in real-world and mathematical situations.
	4.N.2.5 Represent tenths and hundredths with concrete models, making connections between fractions and decimals.
	4.N.2.6 Represent, read and write decimals up to at least the hundredths place in a variety of contexts including money.

	4.N.2.7 Compare and order decimals and whole numbers using place value, a number line and models such as grids and base 10 blocks.
	4.N.2.8 Compare benchmark fractions ($\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$) and decimals (0.25, 0.50, 0.75) in real-world and mathematical situations.
4.N.3 Determine the value of coins in order to solve monetary transactions.	**4.N.3.1** Given a total cost (whole dollars up to $20 or coins) and amount paid (whole dollars up to $20 or coins), find the change required in a variety of ways. Limited to whole dollars up to $20 or sets of coins.
Algebraic Reasoning & Algebra (A)	
4.A.1 Use multiple representations of patterns to solve real-world and mathematical problems.	**4.A.1.1** Create an input/output chart or table to represent or extend a numerical pattern.
	4.A.1.2 Describe the single operation rule for a pattern from an input/output table or function machine involving any operation of a whole number.
	4.A.1.3 Create growth patterns involving geometric shapes and define the single operation rule of the pattern.
4.A.2 Use multiplication and division with unknowns to create number sentences representing a given problem situation.	**4.A.2.1** Use number sense, properties of multiplication and the relationship between multiplication and division to solve problems and find values for the unknowns represented by letters and symbols that make number sentences true.
	4.A.2.2 Solve for unknowns in problems by solving open sentences (equations) and other problems involving addition, subtraction, multiplication, or division with whole numbers. Use real-world situations to represent number sentences and vice versa.
Geometry & Measurement (GM)	
4.GM.1 Name, describe, classify and construct polygons, and three-dimensional figures.	**4.GM.1.1** Identify points, lines, line segments, rays, angles, endpoints, and parallel and perpendicular lines in various contexts.
	4.GM.1.2 Describe, classify, and sketch quadrilaterals, including squares, rectangles, trapezoids, rhombuses, parallelograms, and kites. Recognize quadrilaterals in various contexts.
	4.GM.1.3 Given two three-dimensional shapes, identify similarities, and differences.
4.GM.2 Understand angle, length, and area as measurable attributes of real-world and mathematical objects. Use various tools to measure angles, length, area, and volume.	**4.GM.2.1** Measure angles in geometric figures and real-world objects with a protractor or angle ruler.
	4.GM.2.2 Find the area of polygons that can be decomposed into rectangles.
	4.GM.2.3 Using a variety of tools and strategies, develop the concept that the volume of rectangular prisms with whole-number edge lengths can be found by counting the total number of same-sized unit cubes that fill a shape without gaps or overlaps. Use appropriate measurements such as cm^3.
	4.GM.2.4 Choose an appropriate instrument and measure the length of an object to the nearest whole centimeter or quarter-inch.
	4.GM.2.5 Solve problems that deal with measurements of length, when to use liquid volumes, when to use mass, temperatures above zero and money using addition, subtraction, multiplication, or division as appropriate (customary and metric).

4.GM.3 Determine elapsed time and convert between units of time.	**4.GM.3.1** Determine elapsed time.
	4.GM.3.2 Solve problems involving the conversion of one measure of time to another.
Data & Probability (D)	
4.D.1 Collect, organize, and analyze data.	**4.D.1.1** Represent data on a frequency table or line plot marked with whole numbers and fractions using appropriate titles, labels, and units.
	4.D.1.2 Use tables, bar graphs, timelines, and Venn diagrams to display data sets. The data may include benchmark fractions or decimals ($\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, 0.25, 0.50, 0.75).
	4.D.1.3 Solve one- and two-step problems using data in whole number, decimal, or fraction form in a frequency table and line plot.

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Number & Operations (N)

5.N.1 Divide multi-digit numbers and solve real-world and mathematical problems using arithmetic.	**5.N.1.1** Estimate solutions to division problems in order to assess the reasonableness of results.
	5.N.1.2 Divide multi-digit numbers, by one- and two-digit divisors, using efficient and generalizable procedures, based on knowledge of place value, including standard algorithms.
	5.N.1.3 Recognize that quotients can be represented in a variety of ways, including a whole number with a remainder, a fraction or mixed number, or a decimal and consider the context in which a problem is situated to select and interpret the most useful form of the quotient for the solution.
	5.N.1.4 Solve real-world and mathematical problems requiring addition, subtraction, multiplication, and division of multi-digit whole numbers. Use various strategies, including the inverse relationships between operations, the use of technology, and the context of the problem to assess the reasonableness of results.
5.N.2 Read, write, represent, and compare fractions and decimals; recognize and write equivalent fractions; convert between fractions and decimals; use fractions and decimals in real-world and mathematical situations.	**5.N.2.1** Represent decimal fractions (e.g., $\frac{1}{10}$, $\frac{1}{100}$) using a variety of models (e.g., 10 by 10 grids, rational number wheel, base-ten blocks, meter stick) and make connections between fractions and decimals.
	5.N.2.2 Represent, read and write decimals using place value to describe decimal numbers including fractional numbers as small as thousandths and whole numbers as large as millions.
	5.N.2.3 Compare and order fractions and decimals, including mixed numbers and fractions less than one, and locate on a number line.
	5.N.2.4 Recognize and generate equivalent decimals, fractions, mixed numbers, and fractions less than one in various contexts.
5.N.3 Add and subtract fractions with like and unlike denominators, mixed numbers and decimals to solve real-world and mathematical problems.	**5.N.3.1** Estimate sums and differences of fractions with like and unlike denominators, mixed numbers, and decimals to assess the reasonableness of the results.
	5.N.3.2 Illustrate addition and subtraction of fractions with like and unlike denominators, mixed numbers, and decimals using a variety of representations (e.g., fraction strips, area models, number lines, fraction rods).
	5.N.3.3 Add and subtract fractions with like and unlike denominators, mixed numbers, and decimals, using efficient and generalizable procedures, including but not limited to standard algorithms in order to solve real-world and mathematical problems including those involving money, measurement, geometry, and data.
	5.N.3.4 Find 0.1 more than a number and 0.1 less than a number. Find 0.01 more than a number and 0.01 less than a number. Find 0.001 more than a number and 0.001 less than a number.

Algebraic Reasoning & Algebra (A)	
5.A.1 Describe and graph patterns of change created through numerical patterns.	**5.A.1.1** Use tables and rules of up to two operations to describe patterns of change and make predictions and generalizations about real-world and mathematical problems.
	5.A.1.2 Use a rule or table to represent ordered pairs of whole numbers and graph these ordered pairs on a coordinate plane, identifying the origin and axes in relation to the coordinates.
5.A.2 Understand and interpret expressions, equations, and inequalities involving variables and whole numbers, and use them to represent and evaluate real-world and mathematical problems.	**5.A.2.1** Generate equivalent numerical expressions and solve problems involving whole numbers by applying the commutative, associative, and distributive properties and order of operations (no exponents).
	5.A.2.2 Determine whether an equation or inequality involving a variable is true or false for a given value of the variable.
	5.A.2.3 Evaluate expressions involving variables when values for the variables are given.
Geometry & Measurement (GM)	
5.GM.1 Describe, classify, and draw representations of two- and three-dimensional figures.	**5.GM.1.1** Describe, classify and construct triangles, including equilateral, right, scalene, and isosceles triangles. Recognize triangles in various contexts.
	5.GM.1.2 Describe and classify three-dimensional figures including cubes, rectangular prisms, and pyramids by the number of edges, faces or vertices as well as the shapes of faces.
	5.GM.1.3 Recognize and draw a net for a three-dimensional figure (e.g., cubes, rectangular prisms, pyramids).
5.GM.2 Understand how the volume of rectangular prisms and surface area of shapes with polygonal faces are determined by the dimensions of the object and that shapes with varying dimensions can have equivalent values of surface area or volume.	**5.GM.2.1** Recognize that the volume of rectangular prisms can be determined by the number of cubes (n) and by the product of the dimensions of the prism ($a \times b \times c = n$). Know that rectangular prisms of different dimensions (p, q, and r) can have the same volume if $a \times b \times c = p \times q \times r = n$.
	5.GM.2.2 Recognize that the surface area of a three-dimensional figure with rectangular faces with whole numbered edges can be found by finding the area of each component of the net of that figure. Know that three-dimensional shapes of different dimensions can have the same surface area.
	5.GM.2.3 Find the perimeter of polygons and create arguments for reasonable values for the perimeter of shapes that include curves.
5.GM.3 Understand angle and length as measurable attributes of real-world and mathematical objects. Use various tools to measure angles and lengths.	**5.GM.3.1** Measure and compare angles according to size.
	5.GM.3.2 Choose an appropriate instrument and measure the length of an object to the nearest whole centimeter or 1/16-inch.
	5.GM.3.3 Recognize and use the relationship between inches, feet, and yards to measure and compare objects.
	5.GM.3.4 Recognize and use the relationship between millimeters, centimeters, and meters to measure and compare objects.

Data & Probability (D)	
5.D.1 Display and analyze data to find the range and measures of central tendency (mean, median, and mode).	**5.D.1.1** Find the measures of central tendency (mean, median, or mode) and range of a set of data. Understand that the mean is a "leveling out" or central balance point of the data.
	5.D.1.2 Create and analyze line and double-bar graphs with whole numbers, fractions, and decimals increments.

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Number & Operations (N)

6.N.1 Read, write, and represent integers and rational numbers expressed as fractions, decimals, percents, and ratios; write positive integers as products of factors; use these representations in real-world and mathematical situations.	**6.N.1.1** Represent integers with counters and on a number line and rational numbers on a number line, recognizing the concepts of opposites, direction, and magnitude; use integers and rational numbers in real-world and mathematical situations, explaining the meaning of 0 in each situation.
	6.N.1.2 Compare and order positive rational numbers, represented in various forms, or integers using the symbols <, >, and =.
	6.N.1.3 Explain that a percent represents parts "out of 100" and ratios "to 100."
	6.N.1.4 Determine equivalencies among fractions, decimals, and percents. Select among these representations to solve problems.
	6.N.1.5 Factor whole numbers and express prime and composite numbers as a product of prime factors with exponents.
	6.N.1.6 Determine the greatest common factors and least common multiples. Use common factors and multiples to calculate with fractions, find equivalent fractions, and express the sum of two-digit numbers with a common factor using the distributive property.
6.N.2 Add and subtract integers in order to solve real-world and mathematical problems.	**6.N.2.1** Estimate solutions to addition and subtraction of integers problems in order to assess the reasonableness of results.
	6.N.2.2 Illustrate addition and subtraction of integers using a variety of representations.
	6.N.2.3 Add and subtract integers; use efficient and generalizable procedures including but not limited to standard algorithms.
6.N.3 Understand the concept of ratio and its relationship to fractions and percents and to the multiplication and division of whole numbers. Use ratios to solve real-world and mathematical problems.	**6.N.3.1** Identify and use ratios to compare quantities. Recognize that multiplicative comparison and additive comparison are different.
	6.N.3.2 Determine the unit rate for ratios.
	6.N.3.3 Apply the relationship between ratios, equivalent fractions and percents to solve problems in various contexts, including those involving mixture and concentrations.
	6.N.3.4 Use multiplicative reasoning and representations to solve ratio and unit rate problems.
6.N.4 Multiply and divide decimals, fractions, and mixed numbers; solve real-world and mathematical problems with rational numbers.	**6.N.4.1** Estimate solutions to problems with whole numbers, decimals, fractions, and mixed numbers and use the estimates to assess the reasonableness of results in the context of the problem.
	6.N.4.2 Illustrate multiplication and division of fractions and decimals to show connections to fractions, whole number multiplication, and inverse relationships.

	6.N.4.3 Multiply and divide fractions and decimals using efficient and generalizable procedures.
	6.N.4.4 Solve and interpret real-world and mathematical problems including those involving money, measurement, geometry, and data requiring arithmetic with decimals, fractions and mixed numbers.
Algebraic Reasoning & Algebra (A)	
6.A.1 Recognize and represent relationships between varying quantities; translate from one representation to another; use patterns, tables, graphs and rules to solve real-world and mathematical problems.	**6.A.1.1** Plot integer- and rational-valued (limited to halves and fourths) ordered-pairs as coordinates in all four quadrants and recognize the reflective relationships among coordinates that differ only by their signs.
	6.A.1.2 Represent relationships between two varying quantities involving no more than two operations with rules, graphs, and tables; translate between any two of these representations.
	6.A.1.3 Use and evaluate variables in expressions, equations, and inequalities that arise from various contexts, including determining when or if, for a given value of the variable, an equation or inequality involving a variable is true or false.
6.A.2 Use properties of arithmetic to generate equivalent numerical expressions and evaluate expressions involving positive rational numbers.	**6.A.2.1** Generate equivalent expressions and evaluate expressions involving positive rational numbers by applying the commutative, associative, and distributive properties and order of operations to solve real-world and mathematical problems.
6.A.3 Use equations and inequalities to represent real-world and mathematical problems and use the idea of maintaining equality to solve equations. Interpret solutions in the original context.	**6.A.3.1** Represent real-world or mathematical situations using expressions, equations and inequalities involving variables and rational numbers.
	6.A.3.2 Use number sense and properties of operations and equality to solve real-world and mathematical problems involving equations in the form $x + p = q$ and $px = q$, where x, p, and q are nonnegative rational numbers. Graph the solution on a number line, interpret the solution in the original context, and assess the reasonableness of the solution.
Geometry & Measurement (GM)	
6.GM.1 Calculate area of squares, parallelograms, and triangles to solve real-world and mathematical problems.	**6.GM.1.1** Develop and use formulas for the area of squares and parallelograms using a variety of methods including but not limited to the standard algorithm.
	6.GM.1.2 Develop and use formulas to determine the area of triangles.
	6.GM.1.3 Find the area of right triangles, other triangles, special quadrilaterals, and polygons that can be decomposed into triangles and other shapes to solve real-world and mathematical problems.
6.GM.2 Understand and use relationships between angles in geometric figures.	**6.GM.2.1** Solve problems using the relationships between the angles (vertical, complementary, and supplementary) formed by intersecting lines.
	6.GM.2.2 Develop and use the fact that the sum of the interior angles of a triangle is 180° to determine missing angle measures in a triangle.

6.GM.3 Choose appropriate units of measurement and use ratios to convert within measurement systems to solve real-world and mathematical problems.	**6.GM.3.1** Estimate weights, capacities and geometric measurements using benchmarks in customary and metric measurement systems with appropriate units.
	6.GM.3.2 Solve problems in various real-world and mathematical contexts that require the conversion of weights, capacities, geometric measurements, and time within the same measurement systems using appropriate units.
6.GM.4 Use translations, reflections, and rotations to establish congruency and understand symmetries.	**6.GM.4.1** Predict, describe, and apply translations (slides), reflections (flips), and rotations (turns) to a two-dimensional figure.
	6.GM.4.2 Recognize that translations, reflections, and rotations preserve congruency and use them to show that two figures are congruent.
	6.GM.4.3 Use distances between two points that are either vertical or horizontal to each other (not requiring the distance formula) to solve real-world and mathematical problems about congruent two-dimensional figures.
	6.GM.4.4 Identify and describe the line(s) of symmetry in two-dimensional shapes.
Data & Probability (D)	
6.D.1 Display and analyze data.	**6.D.1.1** Calculate the mean, median, and mode for a set of real-world data.
	6.D.1.2 Explain and justify which measure of central tendency (mean, median, or mode) would provide the most descriptive information for a given set of data.
	6.D.1.3 Create and analyze box and whisker plots observing how each segment contains one quarter of the data.
6.D.2 Use probability to solve real-world and mathematical problems; represent probabilities using fractions and decimals.	**6.D.2.1** Represent possible outcomes using a probability continuum from impossible to certain.
	6.D.2.2 Determine the sample space for a given experiment and determine which members of the sample space are related to certain events. Sample space may be determined by the use of tree diagrams, tables or pictorial representations.
	6.D.2.3 Demonstrate simple experiments in which the probabilities are known and compare the resulting relative frequencies with the known probabilities, recognizing that there may be differences between the two results.

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Number & Operations (N)	
7.N.1 Read, write, represent, and compare rational numbers, expressed as integers, fractions, and decimals.	**7.N.1.1** Know that every rational number can be written as the ratio of two integers or as a terminating or repeating decimal.
	7.N.1.2 Compare and order rational numbers expressed in various forms using the symbols <, >, and =.
	7.N.1.3 Recognize and generate equivalent representations of rational numbers, including equivalent fractions.
7.N.2 Calculate with integers and rational numbers, with and without positive integer exponents, to solve real-world and mathematical problems; explain the relationship between absolute value of a rational number and the distance of that number from zero.	**7.N.2.1** Estimate solutions to multiplication and division of integers in order to assess the reasonableness of results.
	7.N.2.2 Illustrate multiplication and division of integers using a variety of representations.
	7.N.2.3 Solve real-world and mathematical problems involving addition, subtraction, multiplication and division of rational numbers; use efficient and generalizable procedures including but not limited to standard algorithms.
	7.N.2.4 Raise integers to positive integer exponents.
	7.N.2.5 Solve real-world and mathematical problems involving calculations with rational numbers and positive integer exponents.
	7.N.2.6 Explain the relationship between the absolute value of a rational number and the distance of that number from zero on a number line. Use the symbol for absolute value.
Algebraic Reasoning & Algebra (A)	
7.A.1 Understand the concept of proportionality in real-world and mathematical situations, and distinguish between proportional and other relationships.	**7.A.1.1** Describe that the relationship between two variables, x and y, is proportional if it can be expressed in the form $\frac{y}{x} = k$ or $y = kx$; distinguish proportional relationships from other relationships, including inversely proportional relationships ($xy = k$ or $y = \frac{k}{x}$).
	7.A.1.2 Recognize that the graph of a proportional relationship is a line through the origin and the coordinate $(1, r)$, where both r and the slope are the unit rate (constant of proportionality, k).

7.A.2 Recognize proportional relationships in real-world and mathematical situations; represent these and other relationships with tables, verbal descriptions, symbols, and graphs; solve problems involving proportional relationships and interpret results in the original context.	**7.A.2.1** Represent proportional relationships with tables, verbal descriptions, symbols, and graphs; translate from one representation to another. Determine and compare the unit rate (constant of proportionality, slope, or rate of change) given any of these representations.
	7.A.2.2 Solve multi-step problems involving proportional relationships involving distance-time, percent increase or decrease, discounts, tips, unit pricing, similar figures, and other real-world and mathematical situations.
	7.A.2.3 Use proportional reasoning to solve real-world and mathematical problems involving ratios.
	7.A.2.4 Use proportional reasoning to assess the reasonableness of solutions.
7.A.3 Represent and solve linear equations and inequalities.	**7.A.3.1** Write and solve problems leading to linear equations with one variable in the form $px + q = r$ and $p(x + q) = r$, where $p, q, and\ r$ are rational numbers.
	7.A.3.2 Represent, write, solve, and graph problems leading to linear inequalities with one variable in the form $x + p > q$ and $x + p < q$, where p, and q are nonnegative rational numbers.
	7.A.3.3 Represent real-world or mathematical situations using equations and inequalities involving variables and rational numbers.
7.A.4 Use order of operations and properties of operations to generate equivalent numerical and algebraic expressions containing rational numbers and grouping symbols; evaluate such expressions.	**7.A.4.1** Use properties of operations (limited to associative, commutative, and distributive) to generate equivalent numerical and algebraic expressions containing rational numbers, grouping symbols and whole number exponents.
	7.A.4.2 Apply understanding of order of operations and grouping symbols when using calculators and other technologies.
Geometry & Measurement (GM)	
7.GM.1 Develop and understand the concept of surface area and volume of rectangular prisms.	**7.GM.1.1** Using a variety of tools and strategies, develop the concept that surface area of a rectangular prism with rational-valued edge lengths can be found by wrapping the figure with same-sized square units without gaps or overlap. Use appropriate measurements such as cm^2.
	7.GM.1.2 Using a variety of tools and strategies, develop the concept that the volume of rectangular prisms with rational-valued edge lengths can be found by counting the total number of same-sized unit cubes that fill a shape without gaps or overlaps. Use appropriate measurements such as cm^3.
7.GM.2 Determine the area of trapezoids and area and perimeter of composite figures.	**7.GM.2.1** Develop and use the formula to determine the area of a trapezoid to solve problems.
	7.GM.2.2 Find the area and perimeter of composite figures to solve real-world and mathematical problems.
7.GM.3 Use reasoning with proportions and ratios to determine measurements, justify formulas, and solve real-world and mathematical problems involving circles and related geometric figures.	**7.GM.3.1** Demonstrate an understanding of the proportional relationship between the diameter and circumference of a circle and that the unit rate (constant of proportionality) is π and can be approximated by rational numbers such as $\frac{22}{7}$ and 3.14.
	7.GM.3.2 Calculate the circumference and area of circles to solve problems in various contexts, in terms of π and using approximations for π.

7.GM.4 Analyze the effect of dilations, translations, and reflections on the attributes of two-dimensional figures on and off the coordinate plane.	**7.GM.4.1** Describe the properties of similarity, compare geometric figures for similarity, and determine scale factors resulting from dilations.
	7.GM.4.2 Apply proportions, ratios, and scale factors to solve problems involving scale drawings and determine side lengths and areas of similar triangles and rectangles.
	7.GM.4.3 Graph and describe translations and reflections of figures on a coordinate plane and determine the coordinates of the vertices of the figure after the transformation.
Data & Probability (D)	
7.D.1 Display and analyze data in a variety of ways.	**7.D.1.1** Design simple experiments, collect data and calculate measures of central tendency (mean, median, and mode) and spread (range). Use these quantities to draw conclusions about the data collected and make predictions.
	7.D.1.2 Use reasoning with proportions to display and interpret data in circle graphs (pie charts) and histograms. Choose the appropriate data display and know how to create the display using a spreadsheet or other graphing technology.
7.D.2 Calculate probabilities and reason about probabilities using proportions to solve real-world and mathematical problems.	**7.D.2.1** Determine the theoretical probability of an event using the ratio between the size of the event and the size of the sample space; represent probabilities as percents, fractions and decimals between 0 and 1.
	7.D.2.2 Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as percents, decimals and fractions.
	7.D.2.3 Use proportional reasoning to draw conclusions about and predict relative frequencies of outcomes based on probabilities.

Develop a Deep and Flexible Conceptual Understanding	Develop Accurate and Appropriate Procedural Fluency	Develop Strategies for Problem Solving	Develop Mathematical Reasoning	Develop a Productive Mathematical Disposition	Develop the Ability to Make Conjectures, Model, and Generalize	Develop the Ability to Communicate Mathematically

Number & Operations (N)	
PA.N.1 Read, write, compare, classify, and represent real numbers and use them to solve problems in various contexts.	**PA.N.1.1** Develop and apply the properties of integer exponents, including $a^0 = 1$ (with $a \neq 0$), to generate equivalent numerical and algebraic expressions.
	PA.N.1.2 Express and compare approximations of very large and very small numbers using scientific notation.
	PA.N.1.3 Multiply and divide numbers expressed in scientific notation, express the answer in scientific notation.
	PA.N.1.4 Classify real numbers as rational or irrational. Explain why the rational number system is closed under addition and multiplication and why the irrational system is not. Explain why the sum of a rational number and an irrational number is irrational; and the product of a non-zero rational number and an irrational number is irrational.
	PA.N.1.5 Compare real numbers; locate real numbers on a number line. Identify the square root of a perfect square to 400 or, if it is not a perfect square root, locate it as an irrational number between two consecutive positive integers.
Algebraic Reasoning & Algebra (A)	
PA.A.1 Understand the concept of function in real-world and mathematical situations, and distinguish between linear and nonlinear functions.	**PA.A.1.1** Recognize that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable.
	PA.A.1.2 Use linear functions to represent and explain real-world and mathematical situations.
	PA.A.1.3 Identify a function as linear if it can be expressed in the form $y = mx + b$ or if its graph is a straight line.
PA.A.2 Recognize linear functions in real-world and mathematical situations; represent linear functions and other functions with tables, verbal descriptions, symbols, and graphs; solve problems involving linear functions and interpret results in the original context.	**PA.A.2.1** Represent linear functions with tables, verbal descriptions, symbols, and graphs; translate from one representation to another.
	PA.A.2.2 Identify, describe, and analyze linear relationships between two variables.
	PA.A.2.3 Identify graphical properties of linear functions including slope and intercepts. Know that the slope equals the rate of change, and that the *y*-intercept is zero when the function represents a proportional relationship.
	PA.A.2.4 Predict the effect on the graph of a linear function when the slope or *y*-intercept changes. Use appropriate tools to examine these effects.
	PA.A.2.5 Solve problems involving linear functions and interpret results in the original context.

PA.A.3 Generate equivalent numerical and algebraic expressions and use algebraic properties to evaluate expressions.	**PA.A.3.1** Use substitution to simplify and evaluate algebraic expressions.
	PA.A.3.2 Justify steps in generating equivalent expressions by identifying the properties used, including the properties of operations (associative, commutative, and distributive laws) and the order of operations, including grouping symbols.
PA.A.4 Represent real-world and mathematical problems using equations and inequalities involving linear expressions. Solve and graph equations and inequalities symbolically and graphically. Interpret solutions in the original context.	**PA.A.4.1** Illustrate, write, and solve mathematical and real-world problems using linear equations with one variable with one solution, infinitely many solutions, or no solutions. Interpret solutions in the original context.
	PA.A.4.2 Represent, write, solve, and graph problems leading to linear inequalities with one variable in the form $px + q > r$ and $px + q < r$, where p, q, and r are rational numbers.
	PA.A.4.3 Represent real-world situations using equations and inequalities involving one variable.
Geometry & Measurement (GM)	
PA.GM.1 Solve problems involving right triangles using the Pythagorean Theorem.	**PA.GM.1.1** Informally justify the Pythagorean Theorem using measurements, diagrams, or dynamic software and use the Pythagorean Theorem to solve problems in two and three dimensions involving right triangles.
	PA.GM.1.2 Use the Pythagorean Theorem to find the distance between any two points in a coordinate plane.
PA.GM.2 Calculate surface area and volume of three-dimensional figures.	**PA.GM.2.1** Calculate the surface area of a rectangular prism using decomposition or nets. Use appropriate measurements such as cm^2.
	PA.GM.2.2 Calculate the surface area of a cylinder, in terms of π and using approximations for π, using decomposition or nets. Use appropriate measurements such as cm^2.
	PA.GM.2.3 Develop and use the formulas $V = lwh$ and $V = Bh$ to determine the volume of rectangular prisms. Justify why base area (B) and height (h) are multiplied to find the volume of a rectangular prism. Use appropriate measurements such as cm^3.
	PA.GM.2.4 Develop and use the formulas $V = \pi r^2 h$ and $V = Bh$ to determine the volume of right cylinders, in terms of π and using approximations for π. Justify why base area (B) and height (h) are multiplied to find the volume of a right cylinder. Use appropriate measurements such as cm^3.

Data & Probability (D)	
PA.D.1 Display and interpret data in a variety of ways, including using scatterplots and approximate lines of best fit. Use line of best fit and average rate of change to make predictions and draw conclusions about data.	**PA.D.1.1** Describe the impact that inserting or deleting a data point has on the mean and the median of a data set. Know how to create data displays using a spreadsheet and use a calculator to examine this impact.
	PA.D.1.2 Explain how outliers affect measures of central tendency.
	PA.D.1.3 Collect, display and interpret data using scatterplots. Use the shape of the scatterplot to informally estimate a line of best fit, make statements about average rate of change, and make predictions about values not in the original data set. Use appropriate titles, labels and units.
PA.D.2 Calculate experimental probabilities and reason about probabilities to solve real-world and mathematical problems.	**PA.D.2.1** Calculate experimental probabilities and represent them as percents, fractions and decimals between 0 and 1 inclusive. Use experimental probabilities to make predictions when actual probabilities are unknown.
	PA.D.2.2 Determine how samples are chosen (random, limited, biased) to draw and support conclusions about generalizing a sample to a population.
	PA.D.2.3 Compare and contrast dependent and independent events.

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Number & Operations (N)	
A1.N.1 Extend the understanding of number and operations to include square roots and cube roots.	**A1.N.1.1** Write square roots and cube roots of monomial algebraic expressions in simplest radical form.
	A1.N.1.2 Add, subtract, multiply, and simplify square roots of monomial algebraic expressions and divide square roots of whole numbers, rationalizing the denominator when necessary.
Algebraic Reasoning & Algebra (A)	
A1.A.1 Represent and solve mathematical and real-world problems using linear equations, absolute value equations, and systems of equations; interpret solutions in the original context.	**A1.A.1.1** Use knowledge of solving equations with rational values to represent and solve mathematical and real-world problems (e.g., angle measures, geometric formulas, science, or statistics) and interpret the solutions in the original context.
	A1.A.1.2 Solve absolute value equations and interpret the solutions in the original context.
	A1.A.1.3 Analyze and solve real-world and mathematical problems involving systems of linear equations with a maximum of two variables by graphing (may include graphing calculator or other appropriate technology), substitution, and elimination. Interpret the solutions in the original context.
A1.A.2 Represent and solve real-world and mathematical problems using linear inequalities, compound inequalities and systems of linear inequalities; interpret solutions in the original context.	**A1.A.2.1** Represent relationships in various contexts with linear inequalities; solve the resulting inequalities, graph on a coordinate plane, and interpret the solutions.
	A1.A.2.2 Represent relationships in various contexts with compound and absolute value inequalities and solve the resulting inequalities by graphing and interpreting the solutions on a number line.
	A1.A.2.3 Solve systems of linear inequalities with a maximum of two variables; graph and interpret the solutions on a coordinate plane.
A1.A.3 Generate equivalent algebraic expressions and use algebraic properties to evaluate expressions and arithmetic and geometric sequences.	**A1.A.3.1** Solve equations involving several variables for one variable in terms of the others.
	A1.A.3.2 Simplify polynomial expressions by adding, subtracting, or multiplying.
	A1.A.3.3 Factor common monomial factors from polynomial expressions and factor quadratic expressions with a leading coefficient of 1.
	A1.A.3.4 Evaluate linear, absolute value, rational, and radical expressions. Include applying a nonstandard operation such as $a \odot b = 2a + b$.
	A1.A.3.5 Recognize that arithmetic sequences are linear using equations, tables, graphs, and verbal descriptions. Use the pattern, find the next term.
	A1.A.3.6 Recognize that geometric sequences are exponential using equations, tables, graphs and verbal descriptions. Given the formula $f(x) = a(r)^x$, find the next term and define the meaning of a and r within the context of the problem.

A1.A.4 Analyze mathematical change involving linear equations in real-world and mathematical problems.	**A1.A.4.1** Calculate and interpret slope and the x- and y-intercepts of a line using a graph, an equation, two points, or a set of data points to solve real-world and mathematical problems.
	A1.A.4.2 Solve mathematical and real-world problems involving lines that are parallel, perpendicular, horizontal, or vertical.
	A1.A.4.3 Express linear equations in slope-intercept, point-slope, and standard forms and convert between these forms. Given sufficient information (slope and *y*-intercept, slope and one-point on the line, two points on the line, *x*- and *y*-intercept, or a set of data points), write the equation of a line.
	A1.A.4.4 Translate between a graph and a situation described qualitatively.
Functions (F)	
A1.F.1 Understand functions as descriptions of covariation (how related quantities vary together) in real-world and mathematical problems.	**A1.F.1.1** Distinguish between relations and functions.
	A1.F.1.2 Identify the dependent and independent variables as well as the domain and range given a function, equation, or graph. Identify restrictions on the domain and range in real-world contexts.
	A1.F.1.3 Write linear functions, using function notation, to model real-world and mathematical situations.
	A1.F.1.4 Given a graph modeling a real-world situation, read and interpret the linear piecewise function (excluding step functions).
A1.F.2 Recognize functions and understand that families of functions are characterized by their rate of change.	**A1.F.2.1** Distinguish between linear and nonlinear (including exponential) functions arising from real-world and mathematical situations that are represented in tables, graphs, and equations. Understand that linear functions grow by equal intervals and that exponential functions grow by equal factors over equal intervals.
	A1.F.2.2 Recognize the graph of the functions $f(x) = x$ and $f(x) = \|x\|$ and predict the effects of transformations [$f(x + c)$ and $f(x) + c$, where c is a positive or negative constant] algebraically and graphically using various methods and tools that may include graphing calculators.
A1.F.3 Represent functions in multiple ways and use the representation to interpret real-world and mathematical problems.	**A1.F.3.1** Identify and generate equivalent representations of linear equations, graphs, tables, and real-world situations.
	A1.F.3.2 Use function notation; evaluate a function, including nonlinear, at a given point in its domain algebraically and graphically. Interpret the results in terms of real-world and mathematical problems.
	A1.F.3.3 Add, subtract, and multiply functions using function notation.

Data & Probability (D)	
A1.D.1 Display, describe, and compare data. For linear relationships, make predictions and assess the reliability of those predictions.	**A1.D.1.1** Describe a data set using data displays, describe and compare data sets using summary statistics, including measures of central tendency, location, and spread. Know how to use calculators, spreadsheets, or other appropriate technology to display data and calculate summary statistics.
	A1.D.1.2 Collect data and use scatterplots to analyze patterns and describe linear relationships between two variables. Using graphing technology, determine regression lines and correlation coefficients; use regression lines to make predictions and correlation coefficients to assess the reliability of those predictions.
	A1.D.1.3 Interpret graphs as being discrete or continuous.
A1.D.2 Calculate probabilities and apply probability concepts.	**A1.D.2.1** Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.
	A1.D.2.2 Describe the concepts of intersections, unions, and complements using Venn diagrams to evaluate probabilities. Understand the relationships between these concepts and the words AND, OR, and NOT.
	A1.D.2.3 Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.
	A1.D.2.4 Apply probability concepts to real-world situations to make informed decisions.

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Geometry: Reasoning & Logic (G.RL)	
G.RL.1 Use appropriate tools and logic to evaluate mathematical arguments.	**G.RL.1.1** Understand the use of undefined terms, definitions, postulates, and theorems in logical arguments/proofs.
	G.RL.1.2 Analyze and draw conclusions based on a set of conditions using inductive and deductive reasoning. Recognize the logical relationships between a conditional statement and its inverse, converse, and contrapositive.
	G.RL.1.3 Assess the validity of a logical argument and give counterexamples to disprove a statement.
Geometry: Two-Dimensional Shapes (G.2D)	
G.2D.1 Discover, evaluate and analyze the relationships between lines, angles, and polygons to solve real-world and mathematical problems; express proofs in a form that clearly justifies the reasoning, such as two-column proofs, paragraph proofs, flow charts, or illustrations.	**G.2D.1.1** Apply the properties of parallel and perpendicular lines, including properties of angles formed by a transversal, to solve real-world and mathematical problems and determine if two lines are parallel, using algebraic reasoning and proofs.
	G.2D.1.2 Apply the properties of angles, including corresponding, exterior, interior, vertical, complementary, and supplementary angles to solve real-world and mathematical problems using algebraic reasoning and proofs.
	G.2D.1.3 Apply theorems involving the interior and exterior angle sums of polygons and use them to solve real-world and mathematical problems using algebraic reasoning and proofs.
	G.2D.1.4 Apply the properties of special quadrilaterals (square, rectangle, trapezoid, isosceles trapezoid, rhombus, kite, parallelogram) and use them to solve real-world and mathematical problems involving angle measures and segment lengths using algebraic reasoning and proofs.
	G.2D.1.5 Use coordinate geometry to represent and analyze line segments and polygons, including determining lengths, midpoints, and slopes of line segments.
	G.2D.1.6 Apply the properties of polygons to solve real-world and mathematical problems involving perimeter and area (e.g., triangles, special quadrilaterals, regular polygons up to 12 sides, composite figures).
	G.2D.1.7 Apply the properties of congruent or similar polygons to solve real-world and mathematical problems using algebraic and logical reasoning.
	G.2D.1.8 Construct logical arguments to prove triangle congruence (SSS, SAS, ASA, AAS and HL) and triangle similarity (AA, SSS, SAS).
	G.2D.1.9 Use numeric, graphic and algebraic representations of transformations in two dimensions, such as reflections, translations, dilations, and rotations about the origin by multiples of 90˚, to solve problems involving figures on a coordinate plane and identify types of symmetry.

Geometry: Three-Dimensional Shapes (G.3D)	
G.3D.1 Solve real-world and mathematical problems involving three-dimensional figures.	**G.3D.1.1** Solve real-world and mathematical problems using the surface area and volume of prisms, cylinders, pyramids, cones, spheres, and composites of these figures. Use nets, measuring devices, or formulas as appropriate.
	G.3D.1.2 Use ratios derived from similar three-dimensional figures to make conjectures, generalize, and to solve for unknown values such as angles, side lengths, perimeter or circumference of a face, area of a face, and volume.
Geometry: Circles (G.C)	
G.C.1 Solve real-world and mathematical problems using the properties of circles.	**G.C.1.1** Apply the properties of circles to solve problems involving circumference and area, approximate values and in terms of π, using algebraic and logical reasoning.
	G.C.1.2 Apply the properties of circles and relationships among angles; arcs; and distances in a circle among radii, chords, secants and tangents to solve problems using algebraic and logical reasoning.
	G.C.1.3 Recognize and write the radius r, center (h, k), and standard form of the equation of a circle $(x-h)^2+(y-k)^2=r^2$ with and without graphs.
	G.C.1.4 Apply the distance and midpoint formula, where appropriate, to develop the equation of a circle in standard form.
Geometry: Right Triangle Trigonometry (G.RT)	
G.RT.1 Develop and verify mathematical relationships of right triangles and trigonometric ratios to solve real-world and mathematical problems.	**G.RT.1.1** Apply the distance formula and the Pythagorean Theorem and its converse to solve real-world and mathematical problems, as approximate and exact values, using algebraic and logical reasoning (include Pythagorean Triples).
	G.RT.1.2 Verify and apply properties of right triangles, including properties of 45-45-90 and 30-60-90 triangles, to solve problems using algebraic and logical reasoning.
	G.RT.1.3 Use the definition of the trigonometric functions to determine the sine, cosine, and tangent ratio of an acute angle in a right triangle. Apply the inverse trigonometric functions as ratios to find the measure of an acute angle in right triangles.
	G.RT.1.4 Apply the trigonometric functions as ratios (sine, cosine, and tangent) to find side lengths in right triangles in real-world and mathematical problems.

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Number & Operations (N)	
A2.N.1 Extend the understanding of number and operations to include complex numbers, matrices, radical expressions, and expressions written with rational exponents.	**A2.N.1.1** Find the value of i^n for any whole number n.
	A2.N.1.2 Simplify, add, subtract, multiply, and divide complex numbers.
	A2.N.1.3 Use matrices to organize and represent data. Identify the order (dimension) of a matrix, add and subtract matrices of appropriate dimensions, and multiply a matrix by a scalar to create a new matrix to solve problems.
	A2.N.1.4 Understand and apply the relationship of rational exponents to integer exponents and radicals to solve problems.
Algebraic Reasoning & Algebra (A)	
A2.A.1 Represent and solve mathematical and real-world problems using nonlinear equations and systems of linear equations; interpret the solutions in the original context.	**A2.A.1.1** Represent real-world or mathematical problems using quadratic equations and solve using various methods (including graphing calculator or other appropriate technology), factoring, completing the square, and the quadratic formula. Find non-real roots when they exist.
	A2.A.1.2 Represent real-world or mathematical problems using exponential equations, such as compound interest, depreciation, and population growth, and solve these equations graphically (including graphing calculator or other appropriate technology) or algebraically.
	A2.A.1.3 Solve one-variable rational equations and check for extraneous solutions.
	A2.A.1.4 Solve polynomial equations with real roots using various methods and tools that may include factoring, polynomial division, synthetic division, graphing calculators or other appropriate technology.
	A2.A.1.5 Solve square root equations with one variable and check for extraneous solutions.
	A2.A.1.6 Solve common and natural logarithmic equations using the properties of logarithms.
	A2.A.1.7 Solve real-world and mathematical problems that can be modeled using arithmetic or finite geometric sequences or series given the n^{th} terms and sum formulas. Graphing calculators or other appropriate technology may be used.
	A2.A.1.8 Represent real-world or mathematical problems using systems of linear equations with a maximum of three variables and solve using various methods that may include substitution, elimination, and graphing (may include graphing calculators or other appropriate technology).
	A2.A.1.9 Solve systems of equations containing one linear equation and one quadratic equation using tools that may include graphing calculators or other appropriate technology.

A2.A.2 Represent and analyze mathematical situations and structures using algebraic symbols using various strategies to write equivalent forms of expressions.	**A2.A.2.1** Factor polynomial expressions including but not limited to trinomials, differences of squares, sum and difference of cubes, and factoring by grouping using a variety of tools and strategies.
	A2.A.2.2 Add, subtract, multiply, divide, and simplify polynomial and rational expressions.
	A2.A.2.3 Recognize that a quadratic function has different equivalent representations [$f(x) = ax^2 + bx + c$, $f(x) = a(x - h)^2 + k$, and $f(x) = (x - h)(x - k)$]. Identify and use the representation that is most appropriate to solve real-world and mathematical problems.
	A2.A.2.4 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Functions (F)	
A2.F.1 Understand functions as descriptions of covariation (how related quantities vary together).	**A2.F.1.1** Use algebraic, interval, and set notations to specify the domain and range of functions of various types and evaluate a function at a given point in its domain.
	A2.F.1.2 Recognize the graphs of exponential, radical (square root and cube root only), quadratic, and logarithmic functions. Predict the effects of transformations [$f(x + c)$, $f(x) + c$, $f(cx)$, and $cf(x)$, where c is a positive or negative real-valued constant] algebraically and graphically, using various methods and tools that may include graphing calculators or other appropriate technology.
	A2.F.1.3 Graph a quadratic function. Identify the *x*- and *y*-intercepts, maximum or minimum value, axis of symmetry, and vertex using various methods and tools that may include a graphing calculator or appropriate technology.
	A2.F.1.4 Graph exponential and logarithmic functions. Identify asymptotes and *x*- and *y*-intercepts using various methods and tools that may include graphing calculators or other appropriate technology. Recognize exponential decay and growth graphically and algebraically.
	A2.F.1.5 Analyze the graph of a polynomial function by identifying the domain, range, intercepts, zeros, relative maxima, relative minima, and intervals of increase and decrease.
	A2.F.1.6 Graph a rational function and identify the *x*- and *y*-intercepts, vertical and horizontal asymptotes, using various methods and tools that may include a graphing calculator or other appropriate technology. (Excluding slant or oblique asymptotes and holes.)
	A2.F.1.7 Graph a radical function (square root and cube root only) and identify the *x*- and *y*-intercepts using various methods and tools that may include a graphing calculator or other appropriate technology.
	A2.F.1.8 Graph piecewise functions with no more than three branches (including linear, quadratic, or exponential branches) and analyze the function by identifying the domain, range, intercepts, and intervals for which it is increasing, decreasing, and constant.

A2.F.2 Analyze functions through algebraic combinations, compositions, and inverses, if they exist.	**A2.F.2.1** Add, subtract, multiply, and divide functions using function notation and recognize domain restrictions.
	A2.F.2.2 Combine functions by composition and recognize that $g(x) = f^{-1}(x)$, the inverse function of $f(x)$, if and only if $f(g(x)) = g(f(x)) = x$.
	A2.F.2.3 Find and graph the inverse of a function, if it exists, in real-world and mathematical situations. Know that the domain of a function f is the range of the inverse function f^{-1}, and the range of the function f is the domain of the inverse function f^{-1}.
	A2.F.2.4 Apply the inverse relationship between exponential and logarithmic functions to convert from one form to another.
Data & Probability (D)	
A2.D.1 Display, describe, and compare data. For linear and nonlinear relationships, make predictions and assess the reliability of those predictions.	**A2.D.1.1** Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve).
	A2.D.1.2 Collect data and use scatterplots to analyze patterns and describe linear, exponential or quadratic relationships between two variables. Using graphing calculators or other appropriate technology, determine regression equation and correlation coefficients; use regression equations to make predictions and correlation coefficients to assess the reliability of those predictions.
	A2.D.1.3 Based upon a real-world context, recognize whether a discrete or continuous graphical representation is appropriate and then create the graph.
A2.D.2 Analyze statistical thinking to draw inferences, make predictions, and justify conclusions.	**A2.D.2.1** Evaluate reports based on data published in the media by identifying the source of the data, the design of the study, and the way the data are analyzed and displayed. Given spreadsheets, tables, or graphs, recognize and analyze distortions in data displays. Show how graphs and data can be distorted to support different points of view.
	A2.D.2.2 Identify and explain misleading uses of data. Recognize when arguments based on data confuse correlation and causation.

Sample of Consulted Works

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Mathematical Glossary Terms and Tables

Whenever possible a reference was identified for glossary terms from the following resources:

(DPI) *http://dpi.wi.gov/standards*
(H) *http://www.hbschool.com/glossary/math2/*
(M) *http://www.merriam-webster.com/*
(MW) *http://www.mathwords.com*
(MA) *http://www.doe.mass.edu/frameworks/current.html*
(NCTM) *http://www.nctm.org*
(PASS) *http://www.ok.gov./sde/sites/ok.gov.sde/files/C3%20PASS%20math.pdf*

AA similarity (Angle-Angle similarity) If two triangles have two pairs of corresponding angles that are congruent, then the triangles are similar. (MW)

ASA congruence (Angle-Side-Angle congruence) If two triangles have two corresponding angles and the side adjacent to both angles congruent, then the triangles themselves are congruent. (MW)

Absolute value The absolute value of a real number is its (non-negative) distance from 0 on a number line. Formally,

$$|k| = \begin{cases} k \text{ if } k \geq 0 \\ -k \text{ if } k < 0 \end{cases}$$

Addend In the addition problem 3+2+6 = 11, the addends are 3, 2, and 6. (PASS)

Addition and subtraction within 5, 10, 20, 100, or 1,000 Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0–5, 0–10, 0–20, or 0–100, respectively. *Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100. (MA)*

Additive inverses Two numbers whose sum is 0 are additive inverses of one another. *Example: 3/4 and –3/4 are additive inverses of one another because 3/4 + (–3/4) = (–3/4) + 3/4 = 0. (MA)*

Algorithm A finite set of steps for completing a procedure, e.g., long division. (H)

Analog Having to do with data represented by continuous variables, e.g., a clock with hour, minute, and second hands. (M)

Arc (minor and major) A portion of the circumference of a circle with ending points A and B. Unless stated otherwise, arc AB always refers to the shorter segment of the two (the minor arc). Together with the major arc the two portions beginning and ending at points A and B form the entire circumference of a circle.

Arc length The distance along the curved line forming the arc.

Arc measure The angle formed by the arc at the center of the circle.

Area A measurement of the amount of space within a closed two-dimensional shape. Area is usually measured in terms of "square units", in which 1 square unit is the amount of space within a square that measures 1 unit by 1 unit (for a given unit of length). For example, area may be measured in "square centimeters", 1 square centimeter being the amount of space within a 1cm by 1cm square.

Arithmetic sequence (progression) A sequence in which successive terms exhibit a common difference.

Array (rectangular) An orderly arrangement of objects into a rectangular configuration (e.g., take six tiles and arrange two long and three wide to form a rectangle). (PASS)

Associative property of addition See Table 1 in this Glossary.

Associative property of multiplication See Table 1 in this Glossary.

Assumption A fact or statement (as a proposition, axiom, postulate, or notion) taken for granted. (M)

Attribute Characteristic (e.g., size, shape, color, weight). (PASS)

Benchmark fraction A common fraction against which other fractions can be measured, such as ½. (MA)

Bar graph A display of categorical data in which vertical or horizontal bars represent the count of a category. The relative lengths of the various bars in the graph are commensurate with the relative sizes of the counts of the data.

Bivariate data Pairs of linked numerical observations. *Example: a list of heights and weights for each player on a football team*. (MA)

Box plot A graphic method that shows the distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data. (DPI)

Capacity The maximum amount or number that can be contained or accommodated, e.g., a jug with a one-gallon *capacity;* the auditorium was filled to *capacity*. (MA)

Cardinal number A number (such as 1, 5, 15) that is used in simple counting and that indicates how many elements there are in a set. (MA)

Cardinality The cardinality of a finite collection of objects is the number of objects in the set. (For example, in PK-Grade 1 students are still learning that "5" represents the number of objects in any group of "five" objects.)

Categorical data Data that measures the number of occurrences of a discrete set of outcomes (e.g., noticing the different colors of shoes in the class and then recording the number of each color).

Chord A chord is a line that connects two points on a circle.

Circle The set of all points that are equidistant from a given point, called the center of the circle. The set of all points that lie inside the circle is called the *interior* of the circle.

> **Radius of a circle** Both a segment with one endpoint on the center of the circle and the other endpoint on the circle, and the length of this segment (which is necessarily the same for any point on the circle).
>
> **Diameter of a circle** Both a segment with endpoints on the circle that contains the center, and the length of this segment.
>
> **Circumference of a circle** The length of the circle if cut and opened up to make a straight line segment, which can be found with $C = 2\pi r$ where r is the radius and π is the irrational number "pi". (Can be thought of as the perimeter of the circle.)
>
> **Area of a circle** The area of the interior of the circle, which can be found with $A = \pi r^2$ where r is the radius and π the irrational number "pi".

Combinations A selection of objects without regard to order. (PASS)

Coefficient Any of the factors of a product considered in relation to a specific factor. Often, this will be a numerical factor in a product of numbers and variables, e.g., $3x^2$ has coefficient 3. (W)

Commutative property See Table 1 in this Glossary.

Complement (of a set) A set *A* is typically considered to be a subset of an understood "universal set." The complement of *A,* denoted by *A/C* is the set of all elements of the universal set that are not members of *A*.

Complementary angles Two angles whose measures have a sum of 90 degrees. (PASS)

Complex fraction A fraction *A/B* where *A* and/or *B* are fractions ($B \neq 0$). (MA)

Complex number Numbers of the form $a + bi$, where a, a real number, is the "real part" and b, also a real number, is the "imaginary part," and i is the imaginary number. *See also:* ***imaginary number***.

Complex plane A Cartesian plane in which the point (a,b) is used to represent $a + bi$.

Compose numbers To compose numbers is to create new numbers using any of the four operations with other numbers. For example, students compose 10 in many ways (9+1, 8+2, … , 5+5, …). Also, each place in the base ten place value is composed of ten units of the place to the left, i.e., one hundred is composed of ten bundles of ten, one ten is composed of ten ones, etc.

Compose shapes Join geometric shapes without overlaps to form new shapes. (MA)

Composite number Any positive integer divisible by one or more positive integers other than itself and 1. (PASS)

Computation algorithm A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* ***algorithm***; ***computation strategy***. (MA)

Computation strategy Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* ***computation algorithm***. (MA)

Conditional statement A statement of the form, "If P, then Q," where each of P and Q are themselves statements. For example, "If *it rains*, then *the streets get wet*," is a conditional statement. If the conditional statement "If P, then Q," is *true,* then this means that it is never the case that the statement P is true while the statement Q is false. For example, it will never be the case that "it rained" but "the streets are not wet".

Related statements are:

> **Converse:** "If Q, then P." This may or may not be true if the original statement is true.
>
> **Inverse:** "If NOT P, then NOT Q." This may or may not be true if the original statement is true.
>
> **Contrapositive:** "If NOT Q, then NOT P." This is always true if the original statement is true, and vice versa. For an example, notice that, "If the streets are NOT wet, then it did NOT rain," is logically equivalent to the example statement above.

Congruent Two geometric objects are congruent if one can be mapped onto the other using a sequence of rigid motions (*rigid motions* are geometric transformations that preserve lengths and angles).

Conjugate The result of writing a sum of two terms as a difference, or vice versa. *For example, the conjugate of x – 2 is x + 2.* (MW)

Conjecture A statement believed to be true but not yet proved. (PASS)

Constant A number on its own, or sometimes a letter such as a, b or c to stand for a fixed number. Example: in "x + 5 = 9", 5 and 9 are constants. If it is not a constant it is called a variable.

Constant of proportionality Given a proportional relationship expressed as $y=kx$, the number k is often called the constant of proportionality.

Coordinate plane A plane in which a point is represented using two coordinates that determine the precise location of the point. In the Cartesian plane, two perpendicular number lines are used to determine the locations of points. In the polar coordinate plane, points are determined by their distance along a ray through that point and the origin, and the angle that ray makes with a pre-determined horizontal axis.

Cosine (of an acute angle) In a right triangle, the cosine of an acute angle is the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse. (PASS)

Counterexample An example to show that a given statement is false. For example, to disprove the statement "All right triangles are isosceles," all one needs to do is produce a right triangle that is scalene.

Counting number A number used in counting objects, i.e., a number from the set

$$\{1, 2, 3, 4, 5, \ldots\}.$$

See also: ***Natural number***.

Counting on A strategy for finding the number of objects in a group without having to count every member of the group. *For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again; one can find the total by counting on—pointing to the top book and saying "eight," following this with, "nine, ten, eleven. There are eleven books now."* (MA)

Continuous graph (of data) A graph is continuous if it contains intervals of data points.

Decimal expansion The resulting decimal number found when dividing a rational number in fraction form. May include terminating and repeating decimals.

Decimal fraction A fraction (as 0.25 = 25/100 or 0.025 = 25/1000) or mixed number (as 3.025 = 3 25/1000) in which the denominator is a power of ten, usually expressed by the use of the decimal point. (M)

Decimal number Any real number expressed in base 10 notation, such as 2.673. (MA)

Decompose numbers Given a number, identify pairs, triples, etc. of numbers that combine to form the given number.

Decompose shapes. Given a geometric shape, identify geometric shapes that meet without overlap to form the given shape. (MA)

Deductive reasoning Informally, the process of using known facts and relationships to derive new facts and relationships.

Dependent events. Events that influence each other. If one of the events occurs, it changes the probability of the other event. (PASS)

Dependent variable The output of a function. The quantity that is affected when the input is changed.

Digit a) Any of the Arabic numerals 1 to 9 and usually the symbol 0; b) One of the elements that combine to form numbers in a system other than the decimal system. (MA)

Digital Having to do with data that is represented in the form of numerical digits; providing a readout in numerical digits, e.g., a digital watch. (MA)

Dilation A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. (MA)

Discrete graph (of data) A graph is discrete if it consists of separated data points and contains no intervals of data.

Divisible A non-zero integer p is said to be divisible by a non-zero integer q if there exists an integer r such that $q \times r = p$.

Domain of a relation The set of all the first elements or x-coordinates of a relation. (PASS)

Dot plot *See:* ***line plot***.

Equivalent expressions Two expressions (numerical or otherwise) are said to be equivalent if one can be obtained from the other using the properties of operations, such as the commutative, associative and distributive properties, as well as by representing numbers in the expressions in different but equivalent forms.

Equivalent fractions Two fractions a/b and c/d are said to be equivalent if there exists a non-zero number n such that $na/nb=c/d$. Equivalent fractions represent the same amount by changing both the size and the number of parts of a given fraction.

Equivalent ratios Two ratios $a:b$ and $c:d$ are equivalent if there is a non-zero number k such that $ka=c$ and $kb=d$. Equivalent ratios can be shown to have the same unit rate.

Expanded form A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. *For example, 643 = 600 + 40 + 3. (MA)*

Expected value For a random variable, the weighted average of its possible values, with weights given by their respective probabilities. (MA)

Experimental probability When trials of a probability experiment are run and data is collected, the experimental probability of a desired outcome is the relative frequency of that outcome as a ratio of the number of such outcomes to the total number of outcomes. For example, if a coin is flipped 100 times, and heads comes up 45 times, then the experimental probability of heads is 45/100 or 0.45. (The *theoretical probability* is 0.50, and if the number of trials is increased the experimental probability will get closer and closer to 0.50.)

Exponent (Integer) A negative integer exponent denotes the reciprocal of the base raised to the corresponding opposite integer. Thus $x^{-2} = \frac{1}{x^2}$.

Exponent (Whole Number) The number that indicates how many times the base is used as a factor, e.g., in $4^3 = 4 \times 4 \times 4 = 64$, the exponent is 3, indicating that 4 is repeated as a factor three times. (MA)

Exponential function An exponential function with base b is defined by $y = b^x$, where $b > 0$ and b is not equal to 1. (PASS).

Expression A mathematical phrase that combines operations, numbers, and/or variables (e.g., $3^2 \div a$). (H)

Exterior angles (of a polygon) The supplement of an interior angle of a polygon that is formed by extending one of the line segments determining the interior angle at a given vertex.

Extraneous solution A solution, such as that to an equation, that emerges from the process of solving the problem but is not a valid solution to the original problem. For example, consider the equation $\sqrt{2x+12} - 2 = x$. After adding 2 to both sides and squaring both sides of the equation, we obtain $2x + 12 = x^2 + 4x + 4$. We can subtract $2x$ and 12 to both sides to obtain the quadratic equation $x^2 + 2x - 8 = 0$. Solving this quadratic equation, we obtain two possible solutions, $x = 2$ and $x = -4$. While the original equation is true when evaluated at $x = 2$, -4 is considered an extraneous solution because it is false when evaluated at $x = -4$.

$$\sqrt{2x+12} - 2 = x$$
$$\sqrt{2(-4)+12} - 2 = -4$$
$$\sqrt{-8+12} - 2 = -4$$
$$\sqrt{4} - 2 = -4$$
$$2 - 2 = -4$$
$$0 \neq -4$$

Fluency Easily and accurately responding to calculations (Van de Walle). See Table 4 in this Glossary.

First quartile[1] For a data set with median *M*, the first quartile is the median of the data values less than *M*. *Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6. See also:* ***median, third quartile, interquartile range***. (MA)

Fraction A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* ***rational number*** *and* ***complex fraction***. (MA)

[1] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

Frequency table A representation of data in which categories are listed in one column (row) of a table and the number of occurrences (frequency) of each category is indicated in another column (row).

Function A rule that assigns to every element of one set (the domain) exactly one element of another set (the range). A function is often thought of as an "input/output" rule, as in every input determines an output (usually according to mathematical operations performed on the input).

Function machine An input/output model (often made with milk cartons, boxes, or drawn on the board) to show one number entering and a different number exiting. Students guess the rule that produced the second number (e.g., enter 3, exit 5, rule: add 2). (PASS)

Function notation A notation that describes a function. For a function f, when x is a member of the domain, the symbol $f(x)$ denotes the corresponding member of the range (e.g., $f(x) = x + 3$).

Geometric sequence (progression) An ordered list of numbers that has a common ratio between consecutive terms, e.g., 2, 6, 18, 54. (H)

Histogram A type of bar graph used to display the distribution of measurement data across a continuous range. (MA)

Hypotenuse The longest side of a right triangle, necessarily opposite to the right angle. The other sides are called the *legs* of the right triangle (*longer* and *shorter* if applicable).

HL (Hypotenuse-Leg) congruence If two right triangles have hypotenuse and one corresponding leg congruent, then the triangles are congruent.

Identity property of 0 See Table 1 in this Glossary.

Imaginary number A number i is considered imaginary if $i^2 = -1$. *See also:* ***complex number***.

Independent events Events that do not influence one another. Each event occurs without changing the probability of the other event. Specifically, two events *A* and *B* are independent if $P(A \text{ AND } B) = P(A) \cdot P(B)$. (PASS)

Independent variable The input of a function. The quantity whose value is changed to affect the output.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair. (MA)

Inductive reasoning Informally, the process of examining patterns and making conclusions based on observed patterns.

Input/Output table Usually a two-column table (or two-row table) with one column (row) listing the inputs of a rule and the other column (row) listing the corresponding outputs for each input.

Integer The set of numbers that contains the whole numbers and their additive inverses (opposites). I.e., $\{\dots, -2, -1, 0, 1, 2, 3, \dots\}$.

Intercepts (of a graph) Geometrically, where a graph intersects an axis in a Cartesian plane.

Interquartile range A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* ***first quartile, third quartile***. (MA)

Intersection (of sets) For two sets A and B, the intersection $A \cap B$ is the set of all elements that are members of both sets simultaneously.

Inverse function. A function g that satisfies $g(f(x)) = x$ and $f(g(x)) = x$ is said to be an inverse function for f. The inverse of f is often denoted by f^{-1}.

Inverse operations Operations that undo each other (e.g., addition and subtraction are inverse operations; multiplication and division are inverse operations). (PASS)

Irrational number Numbers that are not rational. Irrational numbers have nonterminating, nonrepeating decimal expansions (e.g., square root of 2, pi). (MA)

Length (of a segment) The length of a (straight) line segment is a measurement of the distance from one endpoint of the object to the other. Once a unit of length is specified, the length of a segment is found by placing such units end-to-end without gaps or overlaps and counting how many such units are used.

Line Plot A representation of data in which categories are listed underneath points on a number line, and in which the number of occurrences (frequency) of each category is represented by a corresponding number of marks (X's, dots) above each category's point.

Linear association A set of bivariate data exhibits a linear association if a scatter plot of the data can be well-approximated by a line. (MA)

Linear equation Any equation that can be written in the form $Ax + By + C = 0$ where A and B cannot both be 0. The graph of such an equation is a line. (MA)

Linear function A function f is linear if it can be written in the form $f(x) = mx + b$.

Literal equation An equation involving multiple variables and numbers, often that cannot be solved for an explicit numerical value of any of the individual variables. In such a case one may solve for one variable as an expression of the others.

Logarithm The exponent that indicates the power to which a base number is raised to produce a given number. *For example, the logarithm of 100 to the base 10 is 2.* (M)

Logarithmic function Any function in which an independent variable appears in the form of a logarithm; they are the inverse functions of exponential functions. (MA)

Manipulatives Concrete materials (e.g., buttons, beans, egg and milk cartons, counters, attribute and pattern blocks, interlocking cubes, base-10 blocks, geometric models, geo-boards, fractions pieces, rulers, balances, spinners, dot paper) used to represent mathematical concepts, operations, and relationships. (PASS)

Matrix (pl. matrices) A rectangular array of numbers or variables. (MA)

Mean (arithmetic) A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. *Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.* (MA)

Mean absolute deviation A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.* (MA)

Measure of central tendency A determination of the center of a data set meant to describe a set of data. *See also:* ***mean***, ***median***, ***mode***, *and* ***percentile***.

Measure of spread (or variability) A determination of how much the data in a set deviates from a measure of center .The most frequently used measure is standard deviation. *See also:* ***standard deviation***, ***range***.

Median A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list; or the mean of the two central values, if the list contains an even number of values. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.* (MA)

Midline In the graph of a sine or cosine function, the horizontal line halfway between its maximum and minimum values. (MA)

Mixed number A number written in the form $A\frac{b}{c}$, which is a shorthand way to represent the quantity $A+\frac{b}{c}$. A mixed number may be written as a fraction greater than 1 by writing $A\frac{b}{c}=A+\frac{b}{c}=\frac{Ac}{c}+\frac{b}{c}=\frac{Ac+b}{c}$.

Model A mathematical representation (e.g., number, graph, matrix, equation(s), geometric figure) for real-world or mathematical objects, properties, actions, or relationships. (DPI)

Modulus of a complex number The distance between a complex number and the origin on the complex plane. The modulus of a complex number, $a+bi$ is written $|a+bi|$ and is found by finding the hypotenuse of the triangle with legs a and b. Thus, $|a+bi|=\sqrt{a^2+b^2}$. For a complex number in polar form, $r(\cos\theta+i\sin\theta)$, the modulus is $|r|$.

Multiplication and division within 100 Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0–100. *Example:* $72 \div 8 = 9$. (MA)

Multiplication counting principle If *k* actions can be taken in N_1, N_2, … , N_k different ways, then there are a total of N_1, N_2, … , N_k different ways to perform those actions in sequence.

Multiplicative inverses Two numbers whose product is 1 are multiplicative inverses of one another. *Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 ´ 4/3 = 4/3 ´ 3/4 = 1.* (MA)

Natural number A number used in counting objects, i.e., a number from the set

$$\{1, 2, 3, 4, 5, \ldots\}.$$

See also: ***Counting number***.

Net A two-dimensional representation of a three-dimensional figure constructed of polygons, such that if folds were made on certain edges of the net and appropriate sides were "glued" together, the resulting figure would be the original three-dimensional figure.

Network a) A figure consisting of vertices and edges that shows how objects are connected, b) A collection of points (vertices), with certain connections (edges) between them. (MA)

Non-linear association The relationship between two variables is nonlinear if the change in the second is not simply proportional to the change in the first, independent of the value of the first variable. (MA)

Nonstandard measurement A measurement determined by the use of nonstandard units such as hands, paper clips, beans, cotton balls, etc. (PASS)

Number line diagram A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity. (MA)

Number sense The understanding of number size (relative magnitude), number representations, number operations, referents for quantities and measurement used in everyday situations, etc. (PASS)

Numeral A symbol or mark used to represent a number. (MA)

One-to-one correspondence A matching of the elements of two sets such that each element from the first set is matched with one and only one element of the second set, and such that each element of the second set is matched with some element of the first. Early grades students use this to establish the concept of cardinal use of numbers (as in "5" can represent any collection of five objects; if I can match the fingers on one hand to all the elements of a given set then that set has "5" objects.)

Operation General term for any one of addition, subtraction, multiplication, and division. (PASS)

Order of operations Convention adopted to perform mathematical operations in a consistent order.

Step 1. Perform all operations inside grouping symbols, and/or above and below a fraction bar in the order specified in Steps 2, 3 and 4.

Step 2. Find the value of any powers or roots;

Step 3. Multiply, including division, from left to right;

Step 4. Add, including subtraction, from left to right. (NCTM)

Ordinal number A number designating the place (as first, second, or third) occupied by an item in an ordered sequence. (M)

Outlier A data point that is far outside a representative range of the data set. For example, once the inter-quartile range (IQR) is computed, one might calculate the interval of 1.5× IQR above the median and 1.5× IQR below the median and decide that any data point that lies outside this range is considered an outlier.

Parallel lines Lines that do not intersect. Distinct lines can be shown to be parallel if and only if they have equal slopes.

Partition A process of dividing an object into parts or a set into (smaller) subsets. (MA)

Pascal's triangle A triangular arrangement of numbers in which each row starts and ends with 1, and each other number is the sum of the two numbers above it. (H)

Piecewise function A function that is defined differently on different intervals.

Percent rate of change A rate of change expressed as a percent. *Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.* (MA)

Perfect square A number that is a whole number squared, that is, a number that can be expressed as n^2 for n a whole number.

Perimeter (of a polygon) The total length of all the edges of a polygon. Often, perimeter is thought of as the distance around an object, traversed once along the edges starting from one vertex and ending at the same vertex.

Periodic phenomena Events that recur over regular intervals, for example, ocean tides, machine cycles. (MA)

Perpendicular lines Lines that intersect such that all four angles that are created are congruent. Two lines can be shown to be perpendicular if and only if the product of their slopes is -1.

Pi (π) The irrational number that is derived by finding the ratio of the circumference to the diameter of circles. That this ratio is constant and an irrational number are important concepts and challenging to prove, so they are often arrived at empirically by students.

Picture graph A graph that uses pictures to show and compare information. (MA)

Place value The concept that the order in which digits are written in the base-10 number system determines the value of that digit. Thus, in the number 245, the digit 2 is in the "hundreds place", indicating that the value of that particular 2 is actually 2 hundreds or 200.

Polygon A closed, two-dimensional figure comprised of line segments connected end-to-end, and such that no two segments cross each other. The segments are typically called sides or edges, and the common endpoints of adjacent segments are called vertices (sing. vertex). The space within the polygon is called its *interior*. The angles formed by adjacent sides that lie in the interior of a polygon are called its *interior* angles.

Polynomial The sum or difference of terms which have variables raised to positive integer powers and which have coefficients that may be real or complex. The following are all polynomials: $5x^3 - 2x^2 + x - 13$, $x^2y^3 + xy$, and $(1 + i)a^2 + ib^2$. (MW)

Polynomial function Any function whose output is given by a polynomial expression of the input.

Postulate A statement accepted as true without proof. (MA)

Prime factorization A number written as the product of all its prime factors. (H)

Prime number A whole number greater than 1 whose only factors are 1 and itself. (MA)

Probability distribution The set of possible values of a random variable with a probability assigned to each. (MA)

Properties of equality See Table 2 in this Glossary.

Properties of inequality See Table 3 in this Glossary.

Properties of operations See Table 1 in this Glossary.

Probability The study and measure of the likelihood of an event happening. (PASS)

Probability model A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* ***uniform probability model***. (MA)

Proof A method of constructing a valid argument using deductive reasoning. (MA)

Proportion An equation that states that two ratios are equivalent, e.g., 4/8 = ½ or 4 : 8 = 1 : 2. (MA)

Pyramid A three-dimensional shape constructed from a polygon (called the *base*) and triangles that have one edge matching the edges of the base and such that the triangles share a common vertex.

Pythagorean theorem For any right triangle, the sum of the squares of the lengths of the legs equals the square of the lengths of the hypotenuse. (MA)

Quadratic equation An equation that is equivalent to $ax^2 + bx + c = 0$, where $a \neq 0$.

Quadratic expression An expression that contains variables raised to whole number exponents no higher than 2.

Quadratic function A function that can be represented by an equation of the form $y = ax^2 + bx + c$, where a, b, and c are arbitrary, but fixed, numbers and $a \neq 0$. The graph of this function is a parabola. (DPI)

Quadratic polynomial A polynomial where the highest degree of any of its terms is 2. (MA)

Quadrilateral A polygon with 4 sides. Important classes of quadrilaterals:

Trapezoid A quadrilateral in which at least two sides are parallel.

Parallelogram A quadrilateral in which opposite sides are parallel.

Rhombus A parallelogram in which opposite sides are congruent (have the same length).

Rectangle A parallelogram that has at least one right interior angle.

Square A rectangle that has all sides congruent.

Kite A quadrilateral that has two pairs of congruent adjacent sides.

Quotient The result of a division problem. Also, given whole numbers n and m with $n>m$, if we write $n=mq+r$ with $0\leq r<m$, then we say q is the quotient and r is the remainder.

Radical The $\sqrt{\ }$ symbol, which is used to indicate square roots or n^{th} roots. (MW)

Random sampling A smaller group of people or objects chosen from a larger group or population by a process giving equal chance of selection to all possible people or objects. (H)

Random variable An assignment of a numerical value to each outcome in a sample space. (M)

Range (of a relation) The set of all the second elements or y-coordinates of a relation is called the range. (PASS)

Range (of a data set) The difference between the maximum and minimum values of a data set, a measure of the spread of the data.

Ratio A relationship between quantities such that for every a units of one quantity there are b units of the other. A ratio is often denoted by $a:b$, and read "a to b."

Rational expression A quotient of two polynomials with a non-zero denominator. (MA)

Rational number A number expressible in the form a/b or $-a/b$ for some fraction a/b. The rational numbers include the integers. (MA)

Real number An element of the set of numbers consisting of all rational and all irrational numbers. (MA)

Rectangular array An arrangement of mathematical elements into rows and columns.(MA)

Rectangular prism A three-dimensional object constructed from three pairs of parallel rectangles (called *faces* in this context) that share common edges so as to form an enclosed space and such that opposite rectangles are congruent. The vertices of the rectangles are the vertices of the prism, and the sides of the rectangles are called edges. A **cube** is a rectangular prism in which each face is a square of the same size as the other faces.

Rectilinear figure A polygon, all angles of which are right angles. (MA)

Recursive pattern (or sequence) Patterns in which each number is found from the previous number by repeating a process (e.g. Fibonacci numbers). (PASS)

Reflection A type of transformation that flips points about a line, called the *line of reflection*. Taken together, the image and the pre-image have the line of reflection as a line of symmetry. (MA)

Real numbers (set of) The set of all rational and irrational numbers (PASS)

Relation A collection of ordered pairs of real numbers.

Relative frequency The empirical counterpart of probability. If an event occurs N' times in N trials, its relative frequency is N'/N. (M)

Remainder Theorem If f(x) is a polynomial in x then the remainder on dividing f(x) by x − a is f(a). (M)

Repeating decimal. A decimal in which, after a certain point, a particular digit or sequence of digits repeats itself indefinitely. (M) *See also:* ***terminating decimal***. (MA)

Right angle Informally, an angle whose measure is 90 degrees. Formally, if two congruent copies of a given angle are supplementary (that is, they form a straight line when one matches an edge of one copy with one edge of the other), then the given angle is said to be a right angle. (We can then define the measure of this angle to be 90 degrees and measure other angles in terms of a right angle.)

Rigid motion A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures. (MA)

Rotation A type of transformation that turns a figure about a fixed point, called the *center of rotation*. (MA)

SAS congruence (Side-Angle-Side congruence) If in two triangles two corresponding sides and the angles formed by those sides are congruent, then the triangles are congruent. (MW)

SSS congruence (Side-Side-Side congruence) If two triangles have corresponding sides that are congruent, then the triangles are congruent. (MW)

Sample space In a probability model for a random process, a list of the individual outcomes that are to be considered. (MA)

Scale factor For similar shapes, the common ratio of corresponding side lengths is called the scale factor. Informally, it is the multiplicative amount by which the lengths of one shape are "blown up" or "shrunk down" to obtain the other shape to which it is similar.

Scatter plot A graph in the coordinate plane representing a set of bivariate data. *For example, the heights and weights of a group of people could be displayed on a scatter plot.* (DPI)

Scientific notation A widely used floating-point system in which numbers are expressed as products consisting of a number between 1 and 10 multiplied by an appropriate power of 10, e.g., $562 = 5.62 \times 10^2$. (MW)

Secant (of a circle) A line that intersects a circle at two points.

Sequence A set of elements ordered so that they can be labeled with consecutive positive integers starting with 1, e.g., 1, 3, 9, 27, 81. In this sequence, 1 is the *first term*, 3 is the *second term*, 9 is the *third term*, and so on. (MA)

Set model (for fractions) The use of a discrete set of objects to represent the whole and a subset of those objects to represent a fraction. For example, since 3 of the 15 students in class are wearing blue shirts, 3/15 of the students are wearing blue shirts.

Significant figures (digits) Digits included in a measurement that purposely indicate the precision of the measurement. For example, writing a measurement as 3.50 seconds instead of 3.5 seconds indicates that the measurement is accurate to the hundredths place.

Similar (shapes) Two geometric shapes are said to be similar (to each other) if one can be mapped onto the other by a sequence of similarity transformations.

Similarity transformation A rigid motion followed by a dilation. (MA)

Simultaneous equations Two or more equations containing common variables. (MW)

Sine (of an acute angle) The trigonometric function that for an acute angle is the ratio between the leg opposite the angle when the angle is considered part of a right triangle and the hypotenuse. (M)

Slope (of a line) A measure of the steepness of a line in a Cartesian plane, found by determining the constant change in the y-coordinate per 1-unit change in the x-coordinate.

Spatial sense The ability to build and manipulate mental representations of 2- and 3-dimensional objects and ideas. (PASS)

Standard deviation A measurement of how much each value in the data differs from the mean of the data. (PASS)

Statistics The study of data. (PASS)

Stem-and-leaf plot A frequency distribution made by arranging data in the following way (e.g., student scores on a test were 96, 87, 77, 93, 85, 85, and 75 would be displayed as:

9] 6,3

8] 7,5,5

7] 7,5

Subitize Instantly knowing "how many." Recognizing a number without using other mathematical processes. (Clements)

Substitution The substitution of one expression for an equivalent expression, used when rewriting expressions as equivalent ones or solving equations. It is based on the *transitive property of equality*, which states, "If $A=B$,and $B=C$, then $A=C$."

Summary statistics A collection of statistics (measurements based on data) that describe the data set. For example, the range, mean, and standard deviation of a given data set indicate certain features of the data set and hence are summary statistics.

Supplementary angles Two angles whose measures have a sum of 180 degrees. (PASS)

Supposition (act of supposing) Making a statement or assumption without proof. (PASS)

Surface area (of a rectangular prism) The total measure of the area of the faces of a rectangular prism. Equivalently, the total area of a net for the prism.

Tangent a) Meeting a curve or surface in a single point if a sufficiently small interval is considered. b) (of an acute angle) The trigonometric function that, for an acute angle, is the ratio between the leg opposite the angle and the leg adjacent to the angle when the angle is considered part of a right triangle. (MW)

Tape diagram A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model. (MA)

Terminating decimal A decimal is called terminating if its repeating digit is 0. Every terminating decimal is the decimal form of some rational number. *See also:* ***repeating decimal***. (MA)

Third quartile For a data set with median *M*, the third quartile is the median of the data values greater than *M*. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also:* ***median, first quartile, interquartile range***. (MA)

Transformation A prescription, or rule, that sets up a one-to-one correspondence between the points in a geometric object (the *pre-image*) and the points in another geometric object (the *image*). Reflections, rotations, translations, and dilations are particular examples of transformations. (MA)

Transitivity principle for indirect measurement If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well. (MA)

Translation A type of transformation that moves every point in a graph or geometric figure by the same distance in the same direction without a change in orientation or size. (MW)

Transversal line A line that crosses two or more other lines is called a transversal.

Triangle A polygon with three sides. Important classes of triangles:

Equilateral triangle A triangle with all sides congruent.

Right triangle Contains an interior angle that is a right angle.

Scalene triangle A triangle with no side congruent to another.

Isosceles triangle A triangle with two congruent sides.

Trigonometric function Trigonometric functions (sine, cosine, tangent, and their reciprocals) are commonly defined as ratios of two sides of a right triangle containing the angle, and can equivalently be defined as the lengths of various line segments from a unit circle.

Trigonometry The study of trigonometric functions.

Uniform probability model A probability model which assigns equal probability to all outcomes. *See also:* ***probability model***.

Unit fraction A fraction with a numerator of 1, such as 1/3 or 1/5. (MA)

Unit of measurement When measuring a given attribute of an object, a "unit" is defined in terms of which all other measurements are determined. That a given unit is fixed is a concept to be learned by young students (e.g. we wouldn't measure the length of a room in hands because your hand is different from mine, and we wouldn't measure the length of a room using cm and inches at the same time).

Union (of sets) For two sets A and B, the union $A \cup B$ is the set of all elements that are members of one or both of the sets.

Variable (a) A quantity that can change or that may take on different values. (b) A symbol (often a letter of the alphabet, sometimes including the Greek alphabet) that represents a number in a mathematical expression.

Venn diagram A data display in which (typically) circles are used to represent categories and in which the overlapping of two (or more) circles indicates data that lies in each category in the overlap.

Visual fraction model A diagram or representation to show the relative size of a fraction, for example, a tape diagram, number line diagram, or area model. (MA)

Volume (of a 3D object) A measurement of the amount of space within a closed three-dimensional shape. Volume is often measured in terms of "cubic units", in which 1 cubic unit is the amount of space within a cube that measures 1 unit by 1 unit by 1 unit (for a given unit of length). For example, volume may be measured in "cubic centimeters", 1 cubic centimeter being the amount of space within a 1cm by 1cm by 1cm cube. Note that since one can measure the volume of a liquid by placing said liquid into a 3D shape, volume has historically been measured in various units such as cups, fluid ounces, and liters. Note that 1 cubic centimeter is equal to 1 milliliter, one way to connect such fluid units to cubic units.

Whole numbers The numbers 0, 1, 2, 3, …

Table 1: The Properties of Operations	
Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.	
Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every a (where $a \neq 0$) there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$.
Distributive property of multiplication over addition	$a \times (b \times c) = a \times b + a \times c$

Table 2: The Properties of Equality	
Here a, b, and c stand for arbitrary numbers in the rational, real, or complex number systems.	
Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 3: The Properties of Inequality	
Here a, b, and c stand for arbitrary numbers in the rational or real number systems.	
Law of Trichotomy	Exactly one of the following is true: $a < b$, $a = b$, or $a > b$
Reversal Property	If $a > b$, then $a < b$.
Additive Inverse	If $a > b$, then $-a < -b$.
Addition and Subtraction Property of Inequality	If $a > b$, then $a \pm c > b \pm c$.
Positive Multiplication Property of Inequality	If $a > b$ and $c > 0$, then $a \times c > b \times c$.
Negative Multiplication Property of Inequality	If $a > b$ and $c < 0$, then $a \times c < b \times c$.
Positive Division Property of Inequality	If $a > b$ and $c > 0$, then $a \div c > b \div c$.
Negative Division Property of Inequality	If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Table 4: Fluency Expectations				
Grade level fluency expectations apply to operations of whole numbers.	**Addition**	**Subtraction**	**Multiplication**	**Division**
1st Grade	Through 10	Through 10		
2nd Grade	Through 20	Through 20		
3rd Grade			Through factors of 10	
4th Grade			Through factors of 12	Through factors of 12

Number & Operations (N)			
Topic	**Pre-Kindergarten (PK)**	**Kindergarten (K)**	**First Grade (1)**
Quantity	**PK.N.1 Know number names and count in sequence.** **PK.N.1.1** Count aloud forward in sequence by 1's to 20. **PK.N.1.2** Recognize and name written numerals 0-10. **PK.N.1.3** Recognize that zero represents the count of no objects. **PK.N.2 Count to tell the number of objects.** **PK.N.2.1** Identify the number of objects, up to 10, in a row or column. **PK.N.2.2** Use one-to-one correspondence in counting objects and matching groups of objects. **PK.N.2.3** Understand the last numeral spoken, when counting aloud, tells how many total objects are in a set. **PK.N.2.4** Count up to 5 items in a scattered configuration; not in a row or column. **PK.N.3 Compare sets using number.** **PK.N.3.1** Compare two sets of 1-5 objects using comparative language such as same, more, or fewer.	**K.N.1 Understand the relationship between quantities and whole numbers.** **K.N.1.1** Count aloud forward in sequence to 100 by 1's and 10's. **K.N.1.2** Recognize that a number can be used to represent how many objects are in a set up to 10. **K.N.1.3** Use ordinal numbers to represent the position of an object in a sequence up to 10. **K.N.1.4** Recognize without counting (subitize) the quantity of a small group of objects in organized and random arrangements up to 10. **Clarification statement:** Subitizing is defined as instantly recognizing the quantity of a set without having to count. "Subitizing" is not a vocabulary word and is not meant for student discussion at this age. **K.N.1.5** Count forward, with and without objects, from any given number up to 10. **K.N.1.6** Read, write, discuss, and represent whole numbers from 0 to at least 10. Representations may include numerals, pictures, real objects and picture graphs, spoken words, and manipulatives. **K.N.1.7** Find a number that is 1 more or 1 less than a given number up to 10. **K.N.1.8** Using the words more than, less than or equal to compare and order whole numbers, with and without objects, from 0 to 10.	**1.N.1 Count, compare and represent whole numbers up to 100, with an emphasis on groups of tens and ones.** **1.N.1.1** Recognize numbers to 20 without counting (subitize) the quantity of structured arrangements. **Clarification statement:** Subitizing is defined as instantly recognizing the quantity of a set without having to count. "Subitizing" is not a vocabulary word and is not meant for student discussion at this age. **1.N.1.2** Use concrete representations to describe whole numbers between 10 and 100 in terms of tens and ones. **1.N.1.3** Read, write, discuss, and represent whole numbers up to 100. Representations may include numerals, addition and subtraction, pictures, tally marks, number lines and manipulatives, such as bundles of sticks and base 10 blocks. **1.N.1.4** Count forward, with and without objects, from any given number up to 100 by 1s, 2s, 5s and 10s. **1.N.1.5** Find a number that is 10 more or 10 less than a given number up to 100. **1.N.1.6** Compare and order whole numbers from 0 to 100. **1.N.1.7** Use knowledge of number relationships to locate the position of a given whole number on an open number line up to 20. **1.N.1.8** Use objects to represent and use words to describe the relative size of numbers, such as more than, less than, and equal to.
Operations	Topic addressed at other grade levels.	**K.N.2 Develop conceptual fluency with addition and subtraction (up to 10) using objects and pictures.** **K.N.2.1** Compose and decompose numbers up to 10 with objects and pictures.	**1.N.2 Solve addition and subtraction problems up to 10 in real-world and mathematical contexts.** **1.N.2.1** Represent and solve real-world and mathematical problems using addition and subtraction up to ten. **1.N.2.2** Determine if equations involving addition and subtraction are true. **1.N.2.3** Demonstrate fluency with basic addition facts and related subtraction facts up to 10.

Number & Operations (N)			
Topic	**Pre-Kindergarten (PK)**	**Kindergarten (K)**	**First Grade (1)**
Fractions	Topic addressed at other grade levels.	**K.N.3 Understand the relationship between whole numbers and fractions through fair share.** **K.N.3.1** Distribute equally a set of objects into at least two smaller equal sets.	**1.N.3 Develop foundational ideas for fractions.** **1.N.3.1** Partition a regular polygon using physical models and recognize when those parts are equal. **1.N.3.2** Partition (fair share) sets of objects into equal groupings.
Money	Topic addressed at other grade levels.	**K.N.4 Identify coins by name.** **K.N.4.1** Identify pennies, nickels, dimes, and quarters by name.	**1.N.4 Identify coins and their values.** **1.N.4.1** Identify pennies, nickels, dimes, and quarters by name and value. **1.N.4.2** Write a number with the cent symbol to describe the value of a coin. **1.N.4.3** Determine the value of a collection of pennies, nickels, or dimes up to one dollar counting by ones, fives, or tens.
Algebraic Reasoning & Algebra (A)			
Topic	**Pre-Kindergarten (PK)**	**Kindergarten (K)**	**First Grade (1)**
Patterns	**PK.A.1 Recognize, duplicate, and extend patterns.** **PK.A.1.1** Sort and group up to 5 objects into a set based upon characteristics such as color, size, and shape and explain verbally what the objects have in common. **PK.A.1.2** Recognize, duplicate, and extend repeating patterns involving manipulatives, sound, movement, and other contexts.	**K.A.1 Duplicate patterns in a variety of contexts.** **K.A.1.1** Sort and group up to 10 objects into a set based upon characteristics such as color, size, and shape. Explain verbally what the objects have in common. **K.A.1.2** Recognize, duplicate, complete, and extend repeating, shrinking and growing patterns involving shape, color, size, objects, sounds, movement, and other contexts.	**1.A.1 Identify patterns found in real-world and mathematical situations.** **1.A.1.1** Identify, create, complete, and extend repeating, growing, and shrinking patterns with quantity, numbers, or shapes in a variety of real-world and mathematical contexts.
Number Sentences	Topic addressed at other grade levels.	Topic addressed at other grade levels.	Topic addressed at other grade levels.

Geometry & Measurement (GM)			
Topic	**Pre-Kindergarten (PK)**	**Kindergarten (K)**	**First Grade (1)**
Geometry	**PK.GM.1 Identify common shapes.** **PK.GM.1.1** Identify circles, squares, rectangles, and triangles by pointing to the shape when given the name.	**K.GM.1 Recognize and sort basic two-dimensional shapes and use them to represent real-world objects.** **K.GM.1.1** Recognize squares, circles, triangles, and rectangles. **K.GM.1.2** Sort two-dimensional objects using characteristics such as shape, size, color, and thickness. **K.GM.1.3** Identify attributes of two-dimensional shapes using informal and formal geometric language interchangeably. **K.GM.1.4** Use smaller shapes to form a larger shape when there is an outline to follow. **K.GM.1.5** Compose free-form shapes with blocks. **K.GM.1.6** Use basic shapes and spatial reasoning to represent objects in the real world.	**1.GM.1 Recognize, compose, and decompose two- and three-dimensional shapes.** **1.GM.1.1** Identify trapezoids and hexagons by pointing to the shape when given the name. **1.GM.1.2** Compose and decompose larger shapes using smaller two-dimensional shapes. **1.GM.1.3** Compose structures with three-dimensional shapes. **1.GM.1.4** Recognize three-dimensional shapes such as cubes, cones, cylinders, and spheres.
Measurement	**PK.GM.2 Describe and compare measureable attributes.** **PK.GM.2.**1 Identify measurable attributes of objects. Describe them as little, big, long, short, tall, heavy, light, or other age appropriate vocabulary. **PK.GM.2.2** Directly compare two objects with a common measurable attribute using words such as longer/shorter; heavier/lighter; or taller/shorter. **PK.GM.2.3** Sort objects into sets by one or more attributes.	**K.GM.2 Compare and order objects according to location and measurable attributes.** **K.GM.2.1** Use words to compare objects according to length, size, weight, position, and location. **K.GM.2.2** Order up to 6 objects using measurable attributes, such as length and weight. **K.GM.2.3** Sort objects into sets by more than one attribute. **K.GM.2.4** Compare the number of objects needed to fill two different containers.	**1.GM.2 Select and use nonstandard and standard units to describe length and volume/capacity.** **1.GM.2.1** Use nonstandard and standard measuring tools to measure the length of objects to reinforce the continuous nature of linear measurement. **1.GM.2.2** Illustrate that the length of an object is the number of same-size units of length that, when laid end-to-end with no gaps or overlaps, reach from one end of the object to the other. **1.GM.2.3** Measure the same object/distance with units of two different lengths and describe how and why the measurements differ. **1.GM.2.4** Describe a length to the nearest whole unit using a number and a unit. **1.GM.2.5** Use standard and nonstandard tools to identify volume/capacity. Compare and sort containers that hold more, less, or the same amount.
Time	Topic addressed at other grade levels.	**K.GM.3 Tell time as it relates to daily life.** **K.GM.3.1** Develop an awareness of simple time concepts using words such as yesterday, today, tomorrow, morning, afternoon, and night within his/her daily life.	**1.GM.3 Tell time to the half and full hour.** **1.GM.3.1** Tell time to the hour and half-hour (analog and digital).

Data & Probability (D)			
Topic	**Pre-Kindergarten (PK)**	**Kindergarten (K)**	**First Grade (1)**
Data Analysis	**PK.D.1 Collect and organize categorical data.** **PK.D.1.1** Collect and organize information about objects and events in the environment. **PK.D.1.2** Use categorical data to create real-object graphs.	**K.D.1 Collect, organize, and interpret categorical data.** **K.D.1.1** Collect and sort information about objects and events in the environment. **K.D.1.2** Use categorical data to create real-object and picture graphs. **K.D.1.3** Draw conclusions from real-object and picture graphs.	**1.D.1 Collect, organize, and interpret categorical and numerical data.** **1.D.1.1** Collect, sort, and organize data in up to three categories using representations (e.g., tally marks, tables, Venn diagrams). **1.D.1.2** Use data to create picture and bar-type graphs to demonstrate one-to-one correspondence. **1.D.1.3** Draw conclusions from picture and bar-type graphs.

Number & Operations (N)			
Topic	**Second Grade (2)**	**Third Grade (3)**	**Fourth Grade (4)**
Quantity	**2.N.1 Compare and represent whole numbers up to 1,000 with an emphasis on place value and equality.** **2.N.1.1** Read, write, discuss, and represent whole numbers up to 1,000. Representations may include numerals, words, pictures, tally marks, number lines and manipulatives. **2.N.1.2** Use knowledge of number relationships to locate the position of a given whole number on an open number line up to 100. **2.N.1.3** Use place value to describe whole numbers between 10 and 1,000 in terms of hundreds, tens and ones. Know that 100 is 10 tens, and 1,000 is 10 hundreds. **2.N.1.4** Find 10 more or 10 less than a given three-digit number. Find 100 more or 100 less than a given three-digit number. **2.N.1.5** Recognize when to round numbers to the nearest 10 and 100. **2.N.1.6** Use place value to compare and order whole numbers up to 1,000 using comparative language, numbers, and symbols (e.g., 425 > 276, 73 < 107, page 351 comes after page 350, 753 is between 700 and 800).	**3.N.1 Compare and represent whole numbers up to 10,000 with an emphasis on place value and equality.** **3.N.1.1** Read, write, discuss, and represent whole numbers up to 10,000. Representations may include numerals, expressions with operations, words, pictures, number lines, and manipulatives. **3.N.1.2** Use place value to describe whole numbers between 1,000 and 10,000 in terms of ten thousands, thousands, hundreds, tens and ones, including expanded form. **3.N.1.3** Find 1,000 more or 1,000 less than a given four- or five-digit number. Find 100 more or 100 less than a given four- or five-digit number. **3.N.1.4** Use place value to compare and order whole numbers up to 10,000, using comparative language, numbers, and symbols.	Topic addressed at other grade levels.

Number & Operations (N)			
Topic	**Second Grade (2)**	**Third Grade (3)**	**Fourth Grade (4)**
Operations	**2.N.2 Add and subtract one- and two-digit numbers in real-world and mathematical problems.** **2.N.2.1** Use the relationship between addition and subtraction to generate basic facts up to 20. **2.N.2.2** Demonstrate fluency with basic addition facts and related subtraction facts up to 20. **2.N.2.3** Estimate sums and differences up to 100. **2.N.2.4** Use strategies and algorithms based on knowledge of place value and equality to add and subtract two-digit numbers. **2.N.2.5** Solve real-world and mathematical addition and subtraction problems involving whole numbers up to 2 digits. **2.N.2.6** Use concrete models and structured arrangements, such as repeated addition, arrays and ten frames to develop understanding of multiplication.	**3.N.2 Add and subtract multi-digit whole numbers; multiply with factors up to 10; represent multiplication and division in various ways; Solve real-world and mathematical problems through the representation of related operations.** **3.N.2.1** Represent multiplication facts by using a variety of approaches, such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line and skip counting. **3.N.2.2** Demonstrate fluency of multiplication facts with factors up to 10. **3.N.2.3** Use strategies and algorithms based on knowledge of place value and equality to fluently add and subtract multi-digit numbers. **3.N.2.4** Recognize when to round numbers and apply understanding to round numbers to the nearest ten thousand, thousand, hundred, and ten and use compatible numbers to estimate sums and differences. **3.N.2.5** Use addition and subtraction to solve real-world and mathematical problems involving whole numbers. Use various strategies, including the relationship between addition and subtraction, the use of technology, and the context of the problem to assess the reasonableness of results. **3.N.2.6** Represent division facts by using a variety of approaches, such as repeated subtraction, equal sharing and forming equal groups. **3.N.2.7** Recognize the relationship between multiplication and division to represent and solve real-world problems. **3.N.2.8** Use strategies and algorithms based on knowledge of place value, equality and properties of addition and multiplication to multiply a two-digit number by a one-digit number.	**4.N.1 Solve real-world and mathematical problems using multiplication and division.** **4.N.1.1** Demonstrate fluency with multiplication and division facts with factors up to 12. **4.N.1.2** Use an understanding of place value to multiply or divide a number by 10, 100 and 1,000. **4.N.1.3** Multiply 3-digit by 1-digit or a 2-digit by 2-digit whole numbers, using efficient and generalizable procedures and strategies, based on knowledge of place value, including but not limited to standard algorithms. **4.N.1.4** Estimate products of 3-digit by 1-digit or 2-digit by 2-digit whole numbers using rounding, benchmarks and place value to assess the reasonableness of results. Explore larger numbers using technology to investigate patterns. **4.N.1.5** Solve multi-step real-world and mathematical problems requiring the use of addition, subtraction, and multiplication of multi-digit whole numbers. Use various strategies, including the relationship between operations, the use of appropriate technology, and the context of the problem to assess the reasonableness of results. **4.N.1.6** Use strategies and algorithms based on knowledge of place value, equality and properties of operations to divide 3-digit dividend by 1-digit whole number divisors. (e.g., mental strategies, standard algorithms, partial quotients, repeated subtraction, the commutative, associative, and distributive properties). **4.N.1.7** Determine the unknown addend or factor in equivalent and non-equivalent expressions. (e.g., 5 + 6 = 4 + ☐ , 3 x 8 < 3 x ☐).

Number & Operations (N)			
Topic	**Second Grade (2)**	**Third Grade (3)**	**Fourth Grade (4)**
Fractions	**2.N.3 Explore the foundational ideas of fractions.** **2.N.3.1** Identify the parts of a set and area that represent fractions for halves, thirds, and fourths. **2.N.3.2** Construct equal-sized portions through fair sharing including length, set, and area models for halves, thirds, and fourths.	**3.N.3 Understand meanings and uses of fractions in real-world and mathematical situations**. **3.N.3.1** Read and write fractions with words and symbols. **3.N.3.2** Construct fractions using length, set, and area models. **3.N.3.3** Recognize unit fractions and use them to compose and decompose fractions related to the same whole. Use the numerator to describe the number of parts and the denominator to describe the number of partitions. **3.N.3.4** Use models and number lines to order and compare fractions that are related to the same whole.	**4.N.2 Represent and compare fractions and decimals in real-world and mathematical situations; use place value to understand how decimals represent quantities**. **4.N.2.**1 Represent and rename equivalent fractions using fraction models (e.g. parts of a set, area models, fraction strips, number lines). **4.N.2.2** Use benchmark fractions (0, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, 1) to locate additional fractions on a number line. Use models to order and compare whole numbers and fractions less than and greater than one using comparative language and symbols. **4.N.2.3** Decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations (e.g., $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$). **4.N.2.4** Use fraction models to add and subtract fractions with like denominators in real-world and mathematical situations. **4.N.2.5** Represent tenths and hundredths with concrete models, making connections between fractions and decimals. **4.N.2.6** Represent, read and write decimals up to at least the hundredths place in a variety of contexts including money. **4.N.2.7** Compare and order decimals and whole numbers using place value, a number line and models such as grids and base 10 blocks. **4.N.2.8** Compare benchmark fractions ($\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$) and decimals (0.25, 0.50, 0.75) in real-world and mathematical situations.
Money	**2.N.4 Determine the value of a set of coins**. **2.N.4.1** Determine the value of a collection(s) of coins up to one dollar using the cent symbol. **2.N.4.2** Use a combination of coins to represent a given amount of money up to one dollar.	**3.N.4 Determine the value of a set of coins or bills.** **3.N.4.1** Use addition to determine the value of a collection of coins up to one dollar using the cent symbol and a collection of bills up to twenty dollars. **3.N.4.2** Select the fewest number of coins for a given amount of money up to one dollar.	**4.N.3 Determine the value of coins in order to solve monetary transactions.** **4.N.3.1** Given a total cost (whole dollars up to $20 or coins) and amount paid (whole dollars up to $20 or coins), find the change required in a variety of ways. Limited to whole dollars up to $20 or sets of coins.

Algebraic Reasoning & Algebra (A)			
Topic	**Second Grade (2)**	**Third Grade (3)**	**Fourth Grade (4)**
Patterns	**2.A.1 Describe the relationship found in patterns to solve real-world and mathematical problems.** **2.A.1.1** Represent, create, describe, complete, and extend growing and shrinking patterns with quantity and numbers in a variety of real-world and mathematical contexts. **2.A.1.2** Represent and describe repeating patterns involving shapes in a variety of contexts.	**3.A.1 Describe and create representations of numerical and geometric patterns.** **3.A.1.1** Create, describe, and extend patterns involving addition, subtraction, or multiplication to solve problems in a variety of contexts. **3.A.1.2** Describe the rule (single operation) for a pattern from an input/output table or function machine involving addition, subtraction, or multiplication. **3.A.1.3** Explore and develop visual representations of growing geometric patterns and construct the next steps.	**4.A.1 Use multiple representations of patterns to solve real-world and mathematical problems.** **4.A.1.1** Create an input/output chart or table to represent or extend a numerical pattern. **4.A.1.2** Describe the single operation rule for a pattern from an input/output table or function machine involving any operation of a whole number. **4.A.1.3** Create growth patterns involving geometric shapes and define the single operation rule of the pattern.
Number Sentences	**2.A.2 Use number sentences involving unknowns to represent and solve real-world and mathematical problems.** **2.A.2.1** Use objects and number lines to represent number sentences. **2.A.2.2** Generate real-world situations to represent number sentences and vice versa. **2.A.2.3** Apply commutative and identity properties and number sense to find values for unknowns that make number sentences involving addition and subtraction true or false.	**3.A.2 Use number sentences involving multiplication and unknowns to represent and solve real-world and mathematical problems.** **3.A.2.1** Find unknowns represented by symbols in arithmetic problems by solving one-step open sentences (equations) and other problems involving addition, subtraction, and multiplication. Generate real-world situations to represent number sentences. **3.A.2.2** Recognize, represent and apply the number properties (commutative, identity, and associative properties of addition and multiplication) using models and manipulatives to solve problems**.**	**4.A.2 Use multiplication and division with unknowns to create number sentences representing a given problem situation.** **4.A.2.1** Use number sense, properties of multiplication and the relationship between multiplication and division to solve problems and find values for the unknowns represented by letters and symbols that make number sentences true. **4.A.2.2** Solve for unknowns in problems by solving open sentences (equations) and other problems involving addition, subtraction, multiplication, or division with whole numbers. Use real-world situations to represent number sentences and vice versa.

Geometry & Measurement (GM)			
Topic	**Second Grade (2)**	**Third Grade (3)**	**Fourth Grade (4)**
Geometry	**2.GM.1 Analyze attributes of two-dimensional figures and develop generalizations about their properties.** **2.GM.1.1** Recognize trapezoids and hexagons. **2.GM.1.2** Describe, compare, and classify two-dimensional figures according to their geometric attributes. **2.GM.1.3** Compose two-dimensional shapes using triangles, squares, hexagons, trapezoids, and rhombi. **2.GM.1.4** Recognize right angles and classify angles as smaller or larger than a right angle.	**3.GM.1 Use geometric attributes to describe and create shapes in various contexts.** **3.GM.1.1** Sort three-dimensional shapes based on attributes. **3.GM.1.2** Build a three-dimensional figure using unit cubes when picture/shape is shown. **3.GM.1.3** Classify angles as acute, right, obtuse, and straight.	**4.GM.1 Name, describe, classify, and construct polygons and three-dimensional figures.** **4.GM.1.1** Identify points, lines, line segments, rays, angles, endpoints, and parallel and perpendicular lines in various contexts. **4.GM.1.2** Describe, classify, and sketch quadrilaterals, including squares, rectangles, trapezoids, rhombuses, parallelograms, and kites. Recognize quadrilaterals in various contexts. **4.GM.1.3** Given two three-dimensional shapes, identify similarities, and differences.
Measurement	**2.GM.2 Understand length as a measurable attribute and explore capacity.** **2.GM.2.1** Explain the relationship between the size of the unit of measurement and the number of units needed to measure the length of an object. **2.GM.2.2** Explain the relationship between length and the numbers on a ruler by using a ruler to measure lengths to the nearest whole unit. **2.GM.2.3** Explore how varying shapes and styles of containers can have the same capacity.	**3.GM.2 Understand measurable attributes of real-world and mathematical objects using various tools.** **3.GM.2.1** Find perimeter of polygon, given whole number lengths of the sides, in real-world and mathematical situations. **3.GM.2.2** Develop and use formulas to determine the area of rectangles. Justify why length and width are multiplied to find the area of a rectangle by breaking the rectangle into one unit by one unit squares and viewing these as grouped into rows and columns. **3.GM.2.3** Choose an appropriate measurement instrument and measure the length of objects to the nearest whole centimeter or meter. **3.GM.2.4** Choose an appropriate measurement instrument and measure the length of objects to the nearest whole yard, whole foot, or half inch. **3.GM.2.5** Using common benchmarks, estimate the lengths (customary and metric) of a variety of objects. **3.GM.2.6** Use an analog thermometer to determine temperature to the nearest degree in Fahrenheit and Celsius. **3.GM.2.7** Count cubes systematically to identify the number of cubes needed to pack the whole or half of a three-dimensional structure. **3.GM.2.8** Find the area of two-dimensional figures by counting total number of same size unit squares that fill the shape without gaps or overlaps.	**4.GM.2 Understand angle, length, and area as measurable attributes of real-world and mathematical objects. Use various tools to measure angles, length, area, and volume.** **4.GM.2.1** Measure angles in geometric figures and real-world objects with a protractor or angle ruler. **4.GM.2.2** Find the area of polygons that can be decomposed into rectangles. **4.GM.2.3** Using a variety of tools and strategies, develop the concept that the volume of rectangular prisms with whole-number edge lengths can be found by counting the total number of same-sized unit cubes that fill a shape without gaps or overlaps. Use appropriate measurements such as cm^3. **4.GM.2.4** Choose an appropriate instrument and measure the length of an object to the nearest whole centimeter or quarter-inch. **4.GM.2.5** Solve problems that deal with measurements of length, when to use liquid volumes, when to use mass, temperatures above zero and money using addition, subtraction, multiplication, or division as appropriate (customary and metric).

Geometry & Measurement (GM)			
Topic	**Second Grade (2)**	**Third Grade (3)**	**Fourth Grade (4)**
Time	**2.GM.3 Tell time to the quarter hour.** **2.GM.3.1** Read and write time to the quarter-hour on an analog and digital clock. Distinguish between a.m. and p.m.	**3.GM.3 Tell time to the nearest 5-minutes and solve problems.** **3.GM.3.1** Read and write time to the nearest 5-minute (analog and digital). **3.GM.3.2** Determine the solutions to problems involving addition and subtraction of time in intervals of 5 minutes, up to one hour, using pictorial models, number line diagrams, or other tools.	**4.GM.3 Determine elapsed time and convert between units of time.** **4.GM.3.1** Determine elapsed time. **4.GM.3.2** Solve problems involving the conversion of one measure of time to another.

Data & Probability (D)			
Topic	**Second Grade (2)**	**Third Grade (3)**	**Fourth Grade (4)**
Data Analysis	**2.D.1 Collect, organize, and interpret data.** **2.D.1.1** Explain that the length of a bar in a bar graph or the number of objects in a picture graph represents the number of data points for a given category. **2.D.1.2** Organize a collection of data with up to four categories using pictographs and bar graphs with intervals of 1s, 2s, 5s or 10s. **2.D.1.3** Write and solve one-step word problems involving addition or subtraction using data represented within pictographs and bar graphs with intervals of one. **2.D.1.4** Draw conclusions and make predictions from information in a graph.	**3.D.1 Summarize, construct, and analyze data.** **3.D.1.1** Summarize and construct a data set with multiple categories using a frequency table, line plot, pictograph, and/or bar graph with scaled intervals. **3.D.1.2** Solve one- and two-step problems using categorical data represented with a frequency table, pictograph, or bar graph with scaled intervals.	**4.D.1 Collect, organize, and analyze data.** **4.D.1.1** Represent data on a frequency table or line plot marked with whole numbers and fractions using appropriate titles, labels, and units. **4.D.1.2** Use tables, bar graphs, timelines, and Venn diagrams to display data sets. The data may include benchmark fractions or decimals ($\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, 0.25, 0.50, 0.75). **4.D.1.3** Solve one- and two-step problems using data in whole number, decimal, or fraction form in a frequency table and line plot.

Number & Operations (N)

Fifth Grade (5)	Sixth Grade (6)	Seventh Grade (7)	Pre-Algebra (PA)
5.N.1 Divide multi-digit numbers and solve real-world and mathematical problems using arithmetic. **5.N.1.1** Estimate solutions to division problems in order to assess the reasonableness of results. **5.N.1.2** Divide multi-digit numbers, by one- and two-digit divisors, using efficient and generalizable procedures, based on knowledge of place value, including standard algorithms. **5.N.1.3** Recognize that quotients can be represented in a variety of ways, including a whole number with a remainder, a fraction or mixed number, or a decimal and consider the context in which a problem is situated to select and interpret the most useful form of the quotient for the solution. **5.N.1.4** Solve real-world and mathematical problems requiring addition, subtraction, multiplication, and division of multi-digit whole numbers. Use various strategies, including the inverse relationships between operations, the use of technology, and the context of the problem to assess the reasonableness of results. **5.N.2 Read, write, represent, and compare fractions and decimals; recognize and write equivalent fractions; convert between fractions and decimals; use fractions and decimals in real-world and mathematical situations.** **5.N.2.1** Represent decimal fractions (e.g., $\frac{1}{10}$, $\frac{1}{100}$) using a variety of models (e.g., 10 by 10 grids, rational number wheel, base-ten blocks, meter stick) and make connections between fractions and decimals.	**6.N.1 Read, write, and represent integers and rational numbers expressed as fractions, decimals, percents, and ratios; write positive integers as products of factors; use these representations in real-world and mathematical situations**. **6.N.1.1** Represent integers with counters and on a number line and rational numbers on a number line, recognizing the concepts of opposites, direction, and magnitude; use integers and rational numbers in real-world and mathematical situations, explaining the meaning of 0 in each situation. **6.N.1.2** Compare and order positive rational numbers, represented in various forms, or integers using the symbols <, >, and =. **6.N.1.3** Explain that a percent represents parts "out of 100" and ratios "to 100." **6.N.1.4** Determine equivalencies among fractions, decimals, and percents. Select among these representations to solve problems. **6.N.1.5** Factor whole numbers and express prime and composite numbers as a product of prime factors with exponents. **6.N.1.6** Determine the greatest common factors and least common multiples. Use common factors and multiples to calculate with fractions, find equivalent fractions, and express the sum of two-digit numbers with a common factor using the distributive property. **6.N.2 Add and subtract integers in order to solve real-world and mathematical problems.** **6.N.2.1** Estimate solutions to addition and subtraction of integers problems in order to assess the reasonableness of results.	**7.N.1 Read, write, represent, and compare rational numbers, expressed as integers, fractions, and decimals.** **7.N.1.1** Know that every rational number can be written as the ratio of two integers or as a terminating or repeating decimal. **7.N.1.2** Compare and order rational numbers expressed in various forms using the symbols <, >, and =. **7.N.1.3** Recognize and generate equivalent representations of rational numbers, including equivalent fractions. **7.N.2 Calculate with integers and rational numbers, with and without positive integer exponents, to solve real-world and mathematical problems; explain the relationship between absolute value of a rational number and the distance of that number from zero.** **7.N.2.1** Estimate solutions to multiplication and division of integers in order to assess the reasonableness of results. **7.N.2.2** Illustrate multiplication and division of integers using a variety of representations. **7.N.2.3** Solve real-world and mathematical problems involving addition, subtraction, multiplication and division of rational; use efficient and generalizable procedures including but not limited to standard algorithms. **7.N.2.4** Raise integers to positive integer exponents. **7.N.2.5** Solve real-world and mathematical problems involving calculations with rational numbers and positive integer exponents. **7.N.2.6** Explain the relationship between the absolute value of a rational number	**PA.N.1 Read, write, compare, classify, and represent real numbers and use them to solve problems in various contexts.** **PA.N.1.1** Develop and apply the properties of integer exponents, including $a^0 = 1$ (with $a \neq 0$), to generate equivalent numerical and algebraic expressions. **PA.N.1.2** Express and compare approximations of very large and very small numbers using scientific notation. **PA.N.1.3** Multiply and divide numbers expressed in scientific notation, express the answer in scientific notation. **PA.N.1.4** Classify real numbers as rational or irrational. Explain why the rational number system is closed under addition and multiplication and why the irrational system is not. Explain why the sum of a rational number and an irrational number is irrational; and the product of a non-zero rational number and an irrational number is irrational. **PA.N.1.5** Compare real numbers; locate real numbers on a number line. Identify the square root of a perfect square to 400 or, if it is not a perfect square root, locate it as an irrational number between two consecutive positive integers.

Number & Operations (N)			
Fifth Grade (5)	**Sixth Grade (6)**	**Seventh Grade (7)**	**Pre-Algebra (PA)**
5.N.2.2 Represent, read and write decimals using place value to describe decimal numbers including fractional numbers as small as thousandths and whole numbers as large as millions. **5.N.2.3** Compare and order fractions and decimals, including mixed numbers and fractions less than one, and locate on a number line. **5.N.2.4** Recognize and generate equivalent decimals, fractions, mixed numbers, and fractions less than one in various contexts. **5.N.3 Add and subtract fractions with like and unlike denominators, mixed numbers and decimals to solve real-world and mathematical problems.** **5.N.3.1** Estimate sums and differences of fractions with like and unlike denominators, mixed numbers, and decimals to assess the reasonableness of the results. **5.N.3.2** Illustrate addition and subtraction of fractions with like and unlike denominators, mixed numbers, and decimals using a variety of representations (e.g., fraction strips, area models, number lines, fraction rods). **5.N.3.3** Add and subtract fractions with like and unlike denominators, mixed numbers, and decimals, using efficient and generalizable procedures, including but not limited to standard algorithms in order to solve real-world and mathematical problems including those involving money, measurement, geometry, and data. **5.N.3.4** Find 0.1 more than a number and 0.1 less than a number. Find 0.01 more than a number and 0.01 less than a	**6.N.2.2** Illustrate addition and subtraction integers using a variety of representations. **6.N.2.3** Add and subtract integers; use efficient and generalizable procedures including but not limited to standard algorithms. **6.N.3 Understand the concept of ratio and its relationship to fractions and percents and to the multiplication and division of whole numbers. Use ratios to solve real-world and mathematical problems.** **6.N.3.1** Identify and use ratios to compare quantities. Recognize that multiplicative comparison and additive comparison are different. **6.N.3.2** Determine the unit rate for ratios. **6.N.3.3** Apply the relationship between ratios, equivalent fractions and percents to solve problems in various contexts, including those involving mixture and concentrations. **6.N.3.4** Use multiplicative reasoning and representations to solve ratio and unit rate problems. **6.N.4 Multiply and divide decimals, fractions, and mixed numbers; solve real-world and mathematical problems with rational numbers.** **6.N.4.1** Estimate solutions to problems with whole numbers, decimals, fractions, and mixed numbers and use the estimates to assess the reasonableness of results in the context of the problem. **6.N.4.2** Illustrate multiplication and division of fractions and decimals to show connections to fractions, whole number multiplication, and inverse relationships. **6.N.4.3** Multiply and divide fractions and decimals, using efficient and generalizable	and the distance of that number from zero on a number line. Use the symbol for absolute value.	

Number & Operations (N)

Fifth Grade (5)	Sixth Grade (6)	Seventh Grade (7)	Pre-Algebra (PA)
number. Find 0.001 more than a number and 0.001 less than a number.	procedures. **6.N.4.4** Solve and interpret real-world and mathematical problems including those involving money, measurement, geometry, and data requiring arithmetic with decimals, fractions and mixed numbers.		

Algebraic Reasoning & Algebra (A)

Fifth Grade (5)	Sixth Grade (6)	Seventh Grade (7)	Pre-Algebra (PA)
5.A.1 Describe and graph patterns of change created through numerical patterns. **5.A.1.1** Use tables and rules of up to two operations to describe patterns of change and make predictions and generalizations about real-world and mathematical problems. **5.A.1.2** Use a rule or table to represent ordered pairs of whole numbers and graph these ordered pairs on a coordinate plane, identifying the origin and axes in relation to the coordinates. **5.A.2 Understand and interpret expressions, equations, and inequalities involving variables and whole numbers, and use them to represent and evaluate real-world and mathematical problems.** **5.A.2.1** Generate equivalent numerical expressions and solve problems involving whole numbers by applying the commutative, associative, and distributive properties and order of operations (no exponents). **5.A.2.2** Determine whether an equation or inequality involving a variable is true or false for a given value of the variable.	**6.A.1 Recognize and represent relationships between varying quantities; translate from one representation to another; use patterns, tables, graphs and rules to solve real-world and mathematical problems.** **6.A.1.1** Plot integer- and rational-valued (limited to halves and fourths) ordered-pairs as coordinates in all four quadrants and recognize the reflective relationships among coordinates that differ only by their signs. **6.A.1.2** Represent relationships between two varying quantities involving no more than two operations with rules, graphs, and tables; translate between any two of these representations. **6.A.1.3** Use and evaluate variables in expressions, equations, and inequalities that arise from various contexts, including determining when or if, for a given value of the variable, an equation or inequality involving a variable is true or false. **6.A.2 Use properties of arithmetic to generate equivalent numerical expressions and evaluate expressions involving positive rational numbers.** **6.A.2.1** Generate equivalent expressions and evaluate expressions involving	**7.A.1 Understand the concept of proportionality in real-world and mathematical situations, and distinguish between proportional and other relationships.** **7.A.1.1** Describe that the relationship between two variables, x and y, is proportional if it can be expressed in the form $\frac{y}{x} = k$ or $y = kx$; distinguish proportional relationships from other relationships, including inversely proportional relationships ($xy = k$ or $y = \frac{k}{x}$). **7.A.1.2** Recognize that the graph of a proportional relationship is a line through the origin and the coordinate $(1, r)$, where both r and the slope are the unit rate (constant of proportionality, k). **7.A.2 Recognize proportional relationships in real-world and mathematical situations; represent these and other relationships with tables, verbal descriptions, symbols, and graphs; solve problems involving proportional relationships and interpret results in the original context.** **7.A.2.1** Represent proportional relationships with tables, verbal descriptions, symbols, and graphs;	**PA.A.1 Understand the concept of function in real-world and mathematical situations, and distinguish between linear and nonlinear functions.** **PA.A.1.1** Recognize that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable. **PA.A.1.2** Use linear functions to represent and explain real-world and mathematical situations. **PA.A.1.3** Identify a function as linear if it can be expressed in the form $y = mx + b$ or if its graph is a straight line. **PA.A.2 Recognize linear functions in real-world and mathematical situations; represent linear functions and other function with tables, verbal descriptions, symbols, and graphs; solve problems involving linear functions and interpret results in the original context.** **PA.A.2.1** Represent linear functions with tables, verbal descriptions, symbols, and graphs; translate from one representation to another. **PA.A.2.2** Identify, describe, and analyze linear relationships between two variables.

Algebraic Reasoning & Algebra (A)			
Fifth Grade (5)	**Sixth Grade (6)**	**Seventh Grade (7)**	**Pre-Algebra (PA)**
5.A.2.3 Evaluate expressions involving variables when values for the variables are given.	positive rational numbers by applying the commutative, associative, and distributive properties and order of operations to solve real-world and mathematical problems. **6.A.3 Use equations and inequalities to represent real-world and mathematical problems and use the idea of maintaining equality to solve equations. Interpret solutions in the original context.** **6.A.3.1** Represent real-world or mathematical situations using expressions, equations and inequalities involving variables and rational numbers. **6.A.3.2** Use number sense and properties of operations and equality to solve real-world and mathematical problems involving equations in the form $x + p = q$ and $px = q$, where x, p, and q are nonnegative rational numbers. Graph the solution on a number line, interpret the solution in the original context, and assess the reasonableness of the solution.	translate from one representation to another. Determine and compare the unit rate (constant of proportionality, slope, or rate of change) given any of these representations. **7.A.2.2** Solve multi-step problems involving proportional relationships involving distance-time, percent increase or decrease, discounts, tips, unit pricing, similar figures, and other real-world and mathematical situations. **7.A.2.3** Use proportional reasoning to solve real-world and mathematical problems involving ratios. **7.A.2.4** Use proportional reasoning to assess the reasonableness of solutions. **7.A.3 Represent and solve linear equations and inequalities.** **7.A.3.1** Write and solve problems leading to linear equations with one variable in the form $px + q = r$ and $p(x+q) = r$, where p, q, and r are rational numbers. **7.A.3.**2 Represent, write, solve, and graph problems leading to linear inequalities with one variable in the form $x+p>q$ and $x+p<q$, where p, and q are nonnegative rational numbers. **7.A.3.3** Represent real-world or mathematical situations using equations and inequalities involving variables and rational numbers. **7.A.4 Use order of operations and properties to generate equivalent numerical and algebraic expressions containing rational numbers and grouping symbols; evaluate such expressions.** **7.A.4.1** Use properties of operations (limited to associative, commutative, and distributive) to generate equivalent	**PA.A.2.3** Identify graphical properties of linear functions including slope and intercepts. Know that the slope equals the rate of change, and that the y-intercept is zero when the function represents a proportional relationship. **PA.A.2.4** Predict the effect on the graph of a linear function when the slope or y-intercept changes. Use appropriate tools to examine these effects. **PA.A.2.5** Solve problems involving linear functions and interpret results in the original context. **PA.A.3 Generate equivalent numerical and algebraic expressions and use algebraic properties to evaluate expressions**. **PA.A.3.1** Use substitution to simplify and evaluate algebraic expressions. **PA.A.3.2** Justify steps in generating equivalent expressions by identifying the properties used, including the properties of operations (associative, commutative, and distributive laws) and the order of operations, including grouping symbols. **PA.A.4 Represent real-world and mathematical problems using equations and inequalities involving linear expressions. Solve and graph equations and inequalities symbolically and graphically. Interpret solutions in the original context.** **PA.A.4.1** Illustrate, write, and solve mathematical and real-world problems using linear equations with one variable with one solution, infinitely many solutions, or no solutions. Interpret solutions in the original context. **PA.A.4.2** Represent, write, solve, and graph problems leading to linear

Algebraic Reasoning & Algebra (A)

Fifth Grade (5)	Sixth Grade (6)	Seventh Grade (7)	Pre-Algebra (PA)
		numerical and algebraic expressions containing rational numbers, grouping symbols and whole number exponents. **7.A.4.2** Apply understanding of order of operations and grouping symbols when using calculators and other technologies.	inequalities with one variable in the form $px + q > r$ and $px + q < r$, where p, q, and r are rational numbers. **PA.A.4.3** Represent real-world situations using equations and inequalities involving one variable.

Geometry & Measurement (GM)

Fifth Grade (5)	Sixth Grade (6)	Seventh Grade (7)	Pre-Algebra (PA)
5.GM.1 Describe, classify, and draw representations of two- and three-dimensional figures. **5.GM.1.1** Describe, classify and construct triangles, including equilateral, right, scalene, and isosceles triangles. Recognize triangles in various contexts. **5.GM.1.2** Describe and classify three-dimensional figures including cubes, rectangular prisms, and pyramids by the number of edges, faces or vertices as well as the shapes of faces. **5.GM.1.3** Recognize and draw a net for a three-dimensional figure (e.g., cubes, rectangular prisms, pyramids). **5.GM.2 Understand how the volume of rectangular prisms and surface area of shapes with polygonal faces are determined by the dimensions of the object and that shapes with varying dimensions can have equivalent values of surface area or volume.** **5.GM.2.1** Recognize that the volume of rectangular prisms can be determined by the number of cubes (n) and by the product of the dimensions of the prism ($a \times b \times c = n$). Know that rectangular prisms of different dimensions (p, q, and r) can have the same volume if $a \times b \times c = p \times q \times r = n$.	**6.GM.1 Calculate area of squares, parallelograms, and triangles to solve real-world and mathematical problems.** **6.GM.1.1** Develop and use formulas for the area of squares and parallelograms using a variety of methods including but not limited to the standard algorithm. **6.GM.1.2** Develop and use formulas to determine the area of triangles. **6.GM.1.3** Find the area of right triangles, other triangles, special quadrilaterals, and polygons that can be decomposed into triangles and other shapes to solve real-world and mathematical problems. **6.GM.2 Understand and use relationships between angles in geometric figures**. **6.GM.2.1** Solve problems using the relationships between the angles (vertical, complementary, and supplementary) formed by intersecting lines. **6.GM.2.2** Develop and use the fact that the sum of the interior angles of a triangle is 180° to determine missing angle measures in a triangle. **6.GM.3 Choose appropriate units of measurement and use ratios to convert within measurement systems to solve real-world and mathematical problems.** **6.GM.3.1** Estimate weights, capacities	**7.GM.1 Develop and understand the concept of surface area and volume of rectangular prisms.** **7.GM.1.1** Using a variety of tools and strategies, develop the concept that surface area of a rectangular prism with rational-valued edge lengths can be found by wrapping the figure with same-sized square units without gaps or overlap. Use appropriate measurements such as cm^2. **7.GM.1.2** Using a variety of tools and strategies, develop the concept that the volume of rectangular prisms with rational-valued edge lengths can be found by counting the total number of same-sized unit cubes that fill a shape without gaps or overlaps. Use appropriate measurements such as cm^3. **7.GM.2 Determine the area of trapezoids and area and perimeter of composite figures.** **7.GM.2.1** Develop and use the formula to determine the area of a trapezoid to solve problems. **7.GM.2.2** Find the area and perimeter of composite figures to solve real-world and mathematical problems. **7.GM.3 Use reasoning with proportions**	**PA.GM.1 Solve problems involving right triangles using the Pythagorean Theorem**. **PA.GM.1.1** Informally justify the Pythagorean Theorem using measurements, diagrams or dynamic software and use the Pythagorean Theorem to solve problems in two and three dimensions involving right triangles. **PA.GM.1.2** Use the Pythagorean Theorem to find the distance between any two points in a coordinate plane. **PA.GM.2 Calculate surface area and volume of three-dimensional figures.** **PA.GM.2.1** Calculate the surface area of a rectangular prism using decomposition or nets. Use appropriate measurements such as cm^2. **PA.GM.2.2** Calculate the surface area of a cylinder, in terms of π and using approximations for π, using decomposition or nets. Use appropriate measurements such as cm^2. **PA.GM.2.3** Develop and use the formulas $V = lwh$ and $V = Bh$ to determine the volume of rectangular prisms. Justify why base area (B) and height (h) are multiplied to find the volume of a rectangular prism. Use appropriate measurements such as cm^3. **PA.GM.2.4** Develop and use the formulas

Geometry & Measurement (GM)			
Fifth Grade (5)	**Sixth Grade (6)**	**Seventh Grade (7)**	**Pre-Algebra (PA)**
5.GM.2.2 Recognize that the surface area of a three-dimensional figure with rectangular faces with whole numbered edges can be found by finding the area of each component of the net of that figure. Know that three-dimensional shapes of different dimensions can have the same surface area. **5.GM.2.3** Find the perimeter of polygons and create arguments for reasonable values for the perimeter of shapes that include curves. **5.GM.3 Understand angle and length as measurable attributes of real-world and mathematical objects. Use various tools to measure angles and lengths.** **5.GM.3.1** Measure and compare angles according to size. **5.GM.3.2** Choose an appropriate instrument and measure the length of an object to the nearest whole centimeter or 1/16-inch. **5.GM.3.3** Recognize and use the relationship between inches, feet, and yards to measure and compare objects. **5.GM.3.4** Recognize and use the relationship between millimeters, centimeters, and meters to measure and compare objects.	and geometric measurements using benchmarks in customary and metric measurement systems with appropriate units. **6.GM.3.2** Solve problems in various real-world and mathematical contexts that require the conversion of weights, capacities, geometric measurements, and time within the same measurement systems using appropriate units. **6.GM.4 Use translations, reflections, and rotations to establish congruency and understand symmetries.** **6.GM.4.1** Predict, describe, and apply translations (slides), reflections (flips), and rotations (turns) to a two-dimensional figure. **6.GM.4.2** Recognize that translations, reflections, and rotations preserve congruency and use them to show that two figures are congruent. **6.GM.4.3** Use distances between two points that are either vertical or horizontal to each other (not requiring the distance formula) to solve real-world and mathematical problems about congruent two-dimensional figures. **6.GM.4.4** Identify and describe the line(s) of symmetry in two-dimensional shapes.	**and ratios to determine measurements, justify formulas, and solve real-world and mathematical problems involving circles and related geometric figures.** **7.GM.3.1** Demonstrate an understanding of the proportional relationship between the diameter and circumference of a circle and that the unit rate (constant of proportionality) is π and can be approximated by rational numbers such as $\frac{22}{7}$ and 3.14. **7.GM.3.2** Calculate the circumference and area of circles to solve problems in various contexts, in terms of π and using approximations for π. **7.GM.4 Analyze the effect of dilations, translations, and reflections on the attributes of two-dimensional figures on and off the coordinate plane.** **7.GM.4.1** Describe the properties of similarity, compare geometric figures for similarity, and determine scale factors resulting from dilations. **7.GM.4.2** Apply proportions, ratios, and scale factors to solve problems involving scale drawings and determine side lengths and areas of similar triangles and rectangles. **7.GM.4.3** Graph and describe translations and reflections of figures on a coordinate plane and determine the coordinates of the vertices of the figure after the transformation.	$V = \pi r^2 h$ and $V = Bh$ to determine the volume of right cylinders, in terms of π and using approximations for π. Justify why base area (B) and height (h) are multiplied to find the volume of a right cylinder. Use appropriate measurements such as cm^3.

Data & Probability (D)			
Fifth Grade (5)	**Sixth Grade (6)**	**Seventh Grade (7)**	**Pre-Algebra (PA)**
5.D.1 Display and analyze data to find the range and measures of central tendency (mean, median, and mode). **5.D.1.1** Find the measures of central tendency (mean, median, or mode) and range of a set of data. Understand that the mean is a "leveling out" or central balance point of the data. **5.D.1.2** Create and analyze line and double-bar graphs with whole numbers, fractions, and decimals increments.	**6.D.1 Display and analyze data.** **6.D.1.1** Calculate the mean, median, and mode for a set of real-world data. **6.D.1.2** Explain and justify which measure of central tendency (mean, median, or mode) would provide the most descriptive information for a given set of data. **6.D.1.3** Create and analyze box and whisker plots observing how each segment contains one quarter of the data. **6.D.2 Use probability to solve real-world and mathematical problems; represent probabilities using fractions and decimals.** **6.D.2.1** Represent possible outcomes using a probability continuum from impossible to certain. **6.D.2.2** Determine the sample space for a given experiment and determine which members of the sample space are related to certain events. Sample space may be determined by the use of tree diagrams, tables or pictorial representations. **6.D.2.3** Demonstrate simple experiments in which the probabilities are known and compare the resulting relative frequencies with the known probabilities, recognizing that there may be differences between the two results.	**7.D.1 Display and analyze data in a variety of ways.** **7.D.1.1** Design simple experiments, collect data and calculate measures of central tendency (mean, median, and mode) and spread (range). Use these quantities to draw conclusions about the data collected and make predictions. **7.D.1.2** Use reasoning with proportions to display and interpret data in circle graphs (pie charts) and histograms. Choose the appropriate data display and know how to create the display using a spreadsheet or other graphing technology. **7.D.2 Calculate probabilities and reason about probabilities using proportions to solve real-world and mathematical problems.** **7.D.2.1** Determine the theoretical probability of an event using the ratio between the size of the event and the size of the sample space; represent probabilities as percents, fractions and decimals between 0 and 1. **7.D.2.2** Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as percents, decimals and fractions. **7.D.2.3** Use proportional reasoning to draw conclusions about and predict relative frequencies of outcomes based on probabilities.	**PA.D.1 Display and interpret data in a variety of ways, including using scatterplots and approximate lines of best fit. Use line of best fit and average rate of change to make predictions and draw conclusions about data.** **PA.D.1.1** Describe the impact that inserting or deleting a data point has on the mean and the median of a data set. Know how to create data displays using a spreadsheet and use a calculator to examine this impact. **PA.D.1.2** Explain how outliers affect measures of central tendency. **PA.D.1.3** Collect, display and interpret data using scatterplots. Use the shape of the scatterplot to informally estimate a line of best fit, make statements about average rate of change, and make predictions about values not in the original data set. Use appropriate titles, labels and units. **PA.D.2 Calculate experimental probabilities and reason about probabilities to solve real-world and mathematical problems.** **PA.D.2.1** Calculate experimental probabilities and represent them as percents, fractions and decimals between 0 and 1 inclusive. Use experimental probabilities to make predictions when actual probabilities are unknown. **PA.D.2.2** Determine how samples are chosen (random, limited, biased) to draw and support conclusions about generalizing a sample to a population. **PA.D.2.3** Compare and contrast dependent and independent events.

Number & Operations (N)

Pre-Algebra (PA)

PA.N.1 Read, write, compare, classify, and represent real numbers and use them to solve problems in various contexts.

PA.N.1.1 Develop and apply the properties of integer exponents, including $a^0 = 1$ (with $a \neq 0$), to generate equivalent numerical and algebraic expressions.

PA.N.1.2 Express and compare approximations of very large and very small numbers using scientific notation.

PA.N.1.3 Multiply and divide numbers expressed in scientific notation, express the answer in scientific notation.

PA.N.1.4 Classify real numbers as rational or irrational. Explain why the rational number system is closed under addition and multiplication and why the irrational system is not. Explain why the sum of a rational number and an irrational number is irrational; and the product of a non-zero rational number and an irrational number is irrational.

PA.N.1.5 Compare real numbers; locate real numbers on a number line. Identify the square root of a perfect square to 400 or, if it is not a perfect square root, locate it as an irrational number between two consecutive positive integers.

Algebra 1 (A1)

A1.N.1 Extend the understanding of number and operations to include square roots and cube roots.

A1.N.1.1 Write square roots and cube roots of monomial algebraic expressions in simplest radical form.

A1.N.1.2 Add, subtract, multiply, and simplify square roots of monomial algebraic expressions and divide square roots of whole numbers, rationalizing the denominator when necessary.

Algebra 2 (A2)

A2.N.1 Extend the understanding of number and operations to include complex numbers, matrices, radical expressions, and expressions written with rational exponents.

A2.N.1.1 Find the value of i^n for any whole number n.

A2.N.1.2 Simplify, add, subtract, multiply, and divide complex numbers.

A2.N.1.3 Use matrices to organize and represent data. Identify the order (dimension) of a matrix, add and subtract matrices of appropriate dimensions, and multiply a matrix by a scalar to create a new matrix to solve problems.

A2.N.1.4 Understand and apply the relationship of rational exponents to integer exponents and radicals to solve problems.

Algebraic Reasoning & Algebra (A)

Pre-Algebra (PA)

PA.A.1 Understand the concept of function in real-world and mathematical situations, and distinguish between linear and nonlinear functions.

PA.A.1.1 Recognize that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable.

PA.A.1.2 Use linear functions to represent and explain real-world and mathematical situations.

PA.A.1.3 Identify a function as linear if it can be expressed in the form $y = mx + b$ or if its graph is a straight line.

PA.A.2 Recognize linear functions in real-world and mathematical situations; represent linear functions and other function with tables, verbal descriptions, symbols, and graphs; solve problems involving linear functions and interpret results in the original context.

Algebra 1 (A1)

A1.A.1 Represent and solve mathematical and real-world problems using linear equations, absolute value equations, and systems of equations; interpret solutions in the original context.

A1.A.1.1 Use knowledge of solving equations with rational values to represent and solve mathematical and real-world problems (e.g., angle measures, geometric formulas, science, or statistics) and interpret the solutions in the original context.

A1.A.1.2 Solve absolute value equations and interpret the solutions in the original context.

A1.A.1.3 Analyze and solve real-world and mathematical problems involving systems of linear equations with a maximum of two variables by graphing (may include graphing calculator or other appropriate technology), substitution, and elimination. Interpret the solutions in the original context.

Algebra 2 (A2)

A2.A.1 Represent and solve mathematical and real-world problems using nonlinear equations and systems of linear equations; interpret the solutions in the original context.

A2.A.1.1 Represent real-world or mathematical problems using quadratic equations and solve using various methods (including graphing calculator or other appropriate technology), factoring, completing the square, and the quadratic formula. Find non-real roots when they exist.

A2.A.1.2 Represent real-world or mathematical problems using exponential equations, such as compound interest, depreciation, and population growth, and solve these equations graphically (including graphing calculator or other appropriate technology) or algebraically.

A2.A.1.3 Solve one-variable rational equations and check for extraneous solutions.

A2.A.1.4 Solve polynomial equations with real roots using

Algebraic Reasoning & Algebra (A)

Pre-Algebra (PA)

PA.A.2.1 Represent linear functions with tables, verbal descriptions, symbols, and graphs; translate from one representation to another.
PA.A.2.2 Identify, describe, and analyze linear relationships between two variables.
PA.A.2.3 Identify graphical properties of linear functions including slope and intercepts. Know that the slope equals the rate of change, and that the *y*-intercept is zero when the function represents a proportional relationship.
PA.A.2.4 Predict the effect on the graph of a linear function when the slope or *y*-intercept changes. Use appropriate tools to examine these effects.
PA.A.2.5 Solve problems involving linear functions and interpret results in the original context.

PA.A.3 Generate equivalent numerical and algebraic expressions and use algebraic properties to evaluate expressions.
PA.A.3.1 Use substitution to simplify and evaluate algebraic expressions.
PA.A.3.2 Justify steps in generating equivalent expressions by identifying the properties used, including the properties of operations (associative, commutative, and distributive laws) and the order of operations, including grouping symbols.

PA.A.4 Represent real-world and mathematical problems using equations and inequalities involving linear expressions. Solve and graph equations and inequalities symbolically and graphically. Interpret solutions in the original context.
PA.A.4.1 Illustrate, write, and solve mathematical and real-world problems using linear equations with one variable with one solution, infinitely many solutions, or no solutions. Interpret solutions in the original context.
PA.A.4.2 Represent, write, solve, and graph problems leading to linear inequalities with one variable in the form $px + q > r$ and $px + q < r$, where p, q, and r are rational numbers.
PA.A.4.3 Represent real-world situations using equations and inequalities involving one variable.

Algebra 1 (A1)

A1.A.2 Represent and solve real-world and mathematical problems using linear inequalities, compound inequalities and systems of linear inequalities; interpret solutions in the original context.
A1.A.2.1 Represent relationships in various contexts with linear inequalities; solve the resulting inequalities, graph on a coordinate plane, and interpret the solutions.
A1.A.2.2 Represent relationships in various contexts with compound and absolute value inequalities and solve the resulting inequalities by graphing, and interpreting the solutions on a number line.
A1.A.2.3 Solve systems of linear inequalities with a maximum of two variables; graph and interpret the solutions on a coordinate plane.

A1.A.3 Generate equivalent algebraic expressions and use algebraic properties to evaluate expressions and arithmetic and geometric sequences.
A1.A.3.1 Solve equations involving several variables for one variable in terms of the others.
A1.A.3.2 Simplify polynomial expressions by adding, subtracting, or multiplying.
A1.A.3.3 Factor common monomial factors from polynomial expressions and factor quadratic expressions with a leading coefficient of 1.
A1.A.3.4 Evaluate linear, absolute value, rational, and radical expressions. Include applying a nonstandard operation such as $a \odot b = 2a + b$.
A1.A.3.5 Recognize that arithmetic sequences are linear using equations, tables, graphs, and verbal descriptions. Using the pattern, find the next term.
A1.A.3.6 Recognize that geometric sequences are exponential using equations, tables, graphs and verbal descriptions. Given the formula $f(x) = a(r)^x$, find the next term and define the meaning of a and r within the context of the problem.

A1.A.4 Analyze mathematical change involving linear equations in real-world and mathematical problems.
A1.A.4.1 Calculate and interpret slope and the *x*- and *y*-intercepts of a line using a graph, an equation, two points, or a set of data points to solve real-world and mathematical

Algebra 2 (A2)

various methods and tools that may include factoring, polynomial division, synthetic division, graphing calculators or other appropriate technology.
A2.A.1.5 Solve square root equations with one variable and check for extraneous solutions.
A2.A.1.6 Solve common and natural logarithmic equations using the properties of logarithms.
A2.A.1.7 Solve real-world and mathematical problems that can be modeled using arithmetic or finite geometric sequences or series given the n^{th} terms and sum formulas. Graphing calculators or other appropriate technology may be used.
A2.A.1.8 Represent real-world or mathematical problems using systems of linear equations with a maximum of three variables and solve using various methods that may include substitution, elimination, and graphing (may include graphing calculators or other appropriate technology).
A2.A.1.9 Solve systems of equations containing one linear equation and one quadratic equation using tools that may include graphing calculators or other appropriate technology.

A2.A.2 Represent and analyze mathematical situations and structures using algebraic symbols using various strategies to write equivalent forms of expressions.
A2.A.2.1 Factor polynomial expressions including but not limited to trinomials, differences of squares, sum and difference of cubes, and factoring by grouping using a variety of tools and strategies.
A2.A.2.2 Add, subtract, multiply, divide, and simplify polynomial and rational expressions.
A2.A.2.3 Recognize that a quadratic function has different equivalent representations [$f(x) = ax^2 + bx + c$, $f(x) = a(x - h)^2 + k$, and $f(x) = (x - h)(x - k)$]. Identify and use the representation that is most appropriate to solve real-world and mathematical problems.
A2.A.2.4 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Algebraic Reasoning & Algebra (A)		
Pre-Algebra (PA)	**Algebra 1 (A1)**	**Algebra 2 (A2)**
	problems. **A1.A.4.2** Solve mathematical and real-world problems involving lines that are parallel, perpendicular, horizontal, or vertical. **A1.A.4.3** Express linear equations in slope-intercept, point-slope, and standard forms and convert between these forms. Given sufficient information (slope and *y*-intercept, slope and one-point on the line, two points on the line, *x*- and *y*-intercept, or a set of data points), write the equation of a line. **A1.A.4.4** Translate between a graph and a situation described qualitatively.	
Functions (F)		
Pre-Algebra (PA)	**Algebra 1 (A1)**	**Algebra 2 (A2)**
Strand addressed at other grade levels.	**A1.F.1 Understand functions as descriptions of covariation (how related quantities vary together) in real-world and mathematical problems.** **A1.F.1.1** Distinguish between relations and functions. **A1.F.1.2** Identify the dependent and independent variables as well as the domain and range given a function, equation, or graph. Identify restrictions on the domain and range in real-world contexts. **A1.F.1.3** Write linear functions, using function notation, to model real-world and mathematical situations. **A1.F.1.4** Given a graph modeling a real-world situation, read and interpret the linear piecewise function (excluding step functions). **A1.F.2 Recognize functions and understand that families of functions are characterized by their rate of change.** **A1.F.2.1** Distinguish between linear and nonlinear (including exponential) functions arising from real-world and mathematical situations that are represented in tables, graphs, and equations. Understand that linear functions grow by equal intervals and that exponential functions grow by equal factors over equal intervals. **A1.F.2.2** Recognize the graph of the functions $f(x) = x$ and $f(x) = \|x\|$ and predict the effects of transformations [$f(x + c)$ and $f(x) + c$, where c is a positive or negative constant] algebraically and graphically using various	**A2.F.1 Understand functions as descriptions of covariation (how related quantities vary together).** **A2.F.1.1** Use algebraic, interval, and set notations to specify the domain and range of functions of various types and evaluate a function at a given point in its domain. **A2.F.1.2** Recognize the graphs of exponential, radical (square root and cube root only), quadratic, and logarithmic functions. Predict the effects of transformations [$f(x + c)$, $f(x) + c$, $f(cx)$, and $cf(x)$, where c is a positive or negative real-valued constant] algebraically and graphically, using various methods and tools that may include graphing calculators or other appropriate technology. **A2.F.1.3** Graph a quadratic function. Identify the *x*- and *y*-intercepts, maximum or minimum value, axis of symmetry, and vertex using various methods and tools that may include a graphing calculator o appropriate technology. **A2.F.1.4** Graph exponential and logarithmic functions. Identify asymptotes and *x*- and *y*-intercepts using various methods and tools that may include graphing calculators or other appropriate technology. Recognize exponential decay and growth graphically and algebraically. **A2.F.1.5** Analyze the graph of a polynomial function by identifying the domain, range, intercepts, zeros, relative maxima, relative minima, and intervals of increase and

Functions (F)

Pre-Algebra (PA)	Algebra 1 (A1)	Algebra 2 (A2)
	methods and tools that may include graphing calculators. **A1.F.3 Represent functions in multiple ways and use the representation to interpret real-world and mathematical problems.** **A1.F.3.1** Identify and generate equivalent representations of linear equations, graphs, tables, and real-world situations. **A1.F.3.2** Use function notation; evaluate a function, including nonlinear, at a given point in its domain algebraically and graphically. Interpret the results in terms of real-world and mathematical problems. **A1.F.3.3** Add, subtract, and multiply functions using function notation.	decrease. **A2.F.1.6** Graph a rational function and identify the *x*- and *y*-intercepts, vertical and horizontal asymptotes, using various methods and tools that may include a graphing calculator or other appropriate technology. (Excluding slant or oblique asymptotes and holes.) **A2.F.1.7** Graph a radical function (square root and cube root only) and identify the *x*- and *y*-intercepts using various methods and tools that may include a graphing calculator or other appropriate technology. **A2.F.1.8** Graph piecewise functions with no more than three branches (including linear, quadratic, or exponential branches) and analyze the function by identifying the domain, range, intercepts, and intervals for which it is increasing, decreasing, and constant. **A2.F.2 Analyze functions through algebraic combinations, compositions, and inverses, if they exist.** **A2.F.2.1** Add, subtract, multiply, and divide functions using function notation and recognize domain restrictions. **A2.F.2.2** Combine functions by composition and recognize that $g(x) = f^{-1}(x)$, the inverse function of $f(x)$, if and only if $f\big(g(x)\big) = g\big(f(x)\big) = x$. **A2.F.2.3** Find and graph the inverse of a function, if it exists, in real-world and mathematical situations. Know that the domain of a function f is the range of the inverse function f^{-1}, and the range of the function f is the domain of the inverse function f^{-1}. **A2.F.2.4** Apply the inverse relationship between exponential and logarithmic functions to convert from one form to another.

Data & Probability (D)

Pre-Algebra (PA)	Algebra 1 (A1)	Algebra 2 (A2)
PA.D.1 Display and interpret data in a variety of ways, including using scatterplots and approximate lines of best fit. Use line of best fit and average rate of change to make predictions and draw conclusions about data. **PA.D.1.1** Describe the impact that inserting or deleting a data point has on the mean and the median of a data set. Know how to create data displays using a spreadsheet and	**A1.D.1 Display, describe, and compare data. For linear relationships, make predictions and assess the reliability of those predictions.** **A1.D.1.1** Describe a data set using data displays, describe and compare data sets using summary statistics, including measures of central tendency, location, and spread. Know how to use calculators, spreadsheets, or other appropriate	**A2.D.1 Display, describe, and compare data. For linear and nonlinear relationships, make predictions and assess the reliability of those predictions.** **A2.D.1.1** Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve). **A2.D.1.2** Collect data and use scatterplots to analyze patterns and describe linear, exponential or quadratic

Data & Probability (D)		
Pre-Algebra (PA)	**Algebra 1 (A1)**	**Algebra 2 (A2)**
use a calculator to examine this impact. **PA.D.1.2** Explain how outliers affect measures of central tendency. **PA.D.1.3** Collect, display and interpret data using scatterplots. Use the shape of the scatterplot to informally estimate a line of best fit, make statements about average rate of change, and make predictions about values not in the original data set. Use appropriate titles, labels and units. **PA.D.2 Calculate experimental probabilities and reason about probabilities to solve real-world and mathematical problems.** **PA.D.2.1** Calculate experimental probabilities and represent them as percents, fractions and decimals between 0 and 1 inclusive. Use experimental probabilities to make predictions when actual probabilities are unknown. **PA.D.2.2** Determine how samples are chosen (random, limited, biased) to draw and support conclusions about generalizing a sample to a population. **PA.D.2.3** Compare and contrast dependent and independent events.	technology to display data and calculate summary statistics. **A1.D.1.2** Collect data and use scatterplots to analyze patterns and describe linear relationships between two variables. Using graphing technology, determine regression lines and correlation coefficients; use regression lines to make predictions and correlation coefficients to assess the reliability of those predictions. **A1.D.1.3** Interpret graphs as being discrete or continuous. **A1.D.2 Calculate probabilities and apply probability concepts.** **A1.D.2.1** Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities. **A1.D.2.2** Describe the concepts of intersections, unions, and complements using Venn diagrams to evaluate probabilities. Understand the relationships between these concepts and the words AND, OR, and NOT. **A1.D.2.3** Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes. **A1.D.2.4** Apply probability concepts to real-world situations to make informed decisions.	relationships between two variables. Using graphing calculators or other appropriate technology, determine regression equation and correlation coefficients; use regression equations to make predictions and correlation coefficients to assess the reliability of those predictions. **A2.D.1.3** Based upon a real-world context, recognize whether a discrete or continuous graphical representation is appropriate and then create the graph. **A2.D.2 Analyze statistical thinking to draw inferences, make predictions, and justify conclusions.** **A2.D.2.1** Evaluate reports based on data published in the media by identifying the source of the data, the design of the study, and the way the data are analyzed and displayed. Given spreadsheets, tables, or graphs, recognize and analyze distortions in data displays. Show how graphs and data can be distorted to support different points of view. **A2.D.2.2** Identify and explain misleading uses of data. Recognize when arguments based on data confuse correlation and causation.

Reasoning & Logic (G.RL)		
Seventh Grade (7)	**Pre-Algebra (PA)**	**Geometry (G)**
Topic addressed at other grade levels.	Topic addressed at other grade levels.	**G.RL.1 Use appropriate tools and logic to evaluate mathematical arguments.** **G.RL.1.1** Understand the use of undefined terms, definitions, postulates, and theorems in logical arguments/proofs. **G.RL.1.2** Analyze and draw conclusions based on a set of conditions using inductive and deductive reasoning. Recognize the logical relationships between a conditional statement and its inverse, converse, and contrapositive. **G.RL.1.3** Assess the validity of a logical argument and give counterexamples to disprove a statement.
Two-Dimensional Shapes (G.2D)		
Seventh Grade (7)	**Pre-Algebra (PA)**	**Geometry (G)**
7.GM.2 Determine the area of trapezoids and area and perimeter of composite figures. **7.GM.2.1** Develop and use the formula to determine the area of a trapezoid to solve problems. **7.GM.2.2** Find the area and perimeter of composite figures to solve real-world and mathematical problems. **7.GM.4 Analyze the effect of dilations, translations, and reflections on the attributes of two-dimensional figures on and off the coordinate plane**. **7.GM.4.1** Describe the properties of similarity, compare geometric figures for similarity, and determine scale factors resulting from dilations. **7.GM.4.2** Apply proportions, ratios, and scale factors to solve problems involving scale drawings and determine side lengths and areas of similar triangles and rectangles. **7.GM.4.3** Graph and describe translations and reflections of figures on a coordinate plane and determine the coordinates of the vertices of the figure after the transformation.	Topic addressed at other grade levels.	**G.2D.1 Discover, evaluate, and analyze the relationships between lines, angles, and polygons to solve real-world and mathematical problems; express proofs in a form that clearly justifies the reasoning, such as two-column proofs, paragraph proofs, flow charts, or illustrations.** **G.2D.1.1** Apply the properties of parallel and perpendicular lines, including properties of angles formed by a transversal, to solve real-world and mathematical problems and determine if two lines are parallel, using algebraic reasoning and proofs. **G.2D.1.2** Apply the properties of angles, including corresponding, exterior, interior, vertical, complementary, and supplementary angles to solve real-world and mathematical problems using algebraic reasoning and proofs. **G.2D.1.3** Apply theorems involving the interior and exterior angle sums of polygons and use them to solve real-world and mathematical problems using algebraic reasoning and proofs. **G.2D.1.4** Apply the properties of special quadrilaterals (square, rectangle, trapezoid, isosceles trapezoid, rhombus, kite, parallelogram) and use them to solve real-world and mathematical problems involving angle measures and segment lengths using algebraic reasoning and proofs. **G.2D.1.5** Use coordinate geometry to represent and

		analyze line segments and polygons, including determining lengths, midpoints, and slopes of line segments. **G.2D.1.6** Apply the properties of polygons to solve real-world and mathematical problems involving perimeter and area (e.g., triangles, special quadrilaterals, regular polygons up to 12 sides, composite figures). **G.2D.1.7** Apply the properties of congruent or similar polygons to solve real-world and mathematical problems using algebraic and logical reasoning. **G.2D.1.8** Construct logical arguments to prove triangle congruence (SSS, SAS, ASA, AAS and HL) and triangle similarity (AA, SSS, SAS). **G.2D.1.9** Use numeric, graphic and algebraic representations of transformations in two dimensions, such as reflections, translations, dilations, and rotations about the origin by multiples of 90˚, to solve problems involving figures on a coordinate plane and identify types of symmetry.
Three-Dimensional Shapes (G.3D)		
Seventh Grade (7)	**Pre-Algebra (PA)**	**Geometry (G)**
7.GM.1 Develop and understand the concept of surface area and volume of rectangular prisms. **7.GM.1.1** Using a variety of tools and strategies, develop the concept that surface area of a rectangular prism with rational-valued edge lengths can be found by wrapping the figure with same-sized square units without gaps or overlap. Use appropriate measurements such as cm^2. **7.GM.1.2** Using a variety of tools and strategies, develop the concept that the volume of rectangular prisms with rational-valued edge lengths can be found by counting the total number of same-sized unit cubes that fill a shape without gaps or overlaps. Use appropriate measurements such as cm^3.	**PA.GM.2 Calculate surface area and volume of three-dimensional figures.** **PA.GM.2.1** Calculate the surface area of a rectangular prism using decomposition or nets. Use appropriate measurements such as cm^2. **PA.GM.2.2** Calculate the surface area of a cylinder, in terms of π and using approximations for π, using decomposition or nets. Use appropriate measurements such as cm^2. **PA.GM.2.3** Develop and use the formulas $V = lwh$ and $V = Bh$ to determine the volume of rectangular prisms. Justify why base area (B) and height (h) are multiplied to find the volume of a rectangular prism. Use appropriate measurements such as cm^3. **PA.GM.2.4** Develop and use the formulas $V = \pi r^2 h$ and $V = Bh$ to determine the volume of right cylinders, in terms of π and using approximations for π. Justify why base area (B) and height (h) are multiplied to find the volume of a right cylinder. Use appropriate measurements such as cm^3.	**G.3D.1 Solve real-world and mathematical problems involving three-dimensional figures.** **G.3D.1.1** Solve real-world and mathematical problems using the surface area and volume of prisms, cylinders, pyramids, cones, spheres, and composites of these figures. Use nets, measuring devices, or formulas as appropriate. **G.3D.1.2** Use ratios derived from similar three-dimensional figures to make conjectures, generalize, and to solve for unknown values such as angles, side lengths, perimeter or circumference of a face, area of a face, and volume.

Circles (G.C)

Seventh Grade (7)	Pre-Algebra (PA)	Geometry (G)
7.GM.3 Use reasoning with proportions and ratios to determine measurements, justify formulas, and solve real-world and mathematical problems involving circles and related geometric figures. **7.GM.3.1** Demonstrate an understanding of the proportional relationship between the diameter and circumference of a circle and that the unit rate (constant of proportionality) is π and can be approximated by rational numbers such as $\frac{22}{7}$ and 3.14. **7.GM.3.2** Calculate the circumference and area of circles to solve problems in various contexts, in terms of π and using approximations for π.	Topic addressed at other grade levels.	**G.C.1 Solve real-world and mathematical problems using the properties of circles.** **G.C.1.1** Apply the properties of circles to solve problems involving circumference and area, approximate values and in terms of π, using algebraic and logical reasoning. **G.C.1.2** Apply the properties of circles and relationships among angles; arcs; and distances in a circle among radii, chords, secants and tangents to solve problems using algebraic and logical reasoning. **G.C.1.3** Recognize and write the radius r, center (h, k), and standard form of the equation of a circle $(x-h)^2 + (y-k)^2 = r^2$ with and without graphs. **G.C.1.4** Apply the distance and midpoint formula, where appropriate, to develop the equation of a circle in standard form.

Right Triangle Trigonometry (G.RT)

Seventh Grade (7)	Pre-Algebra (PA)	Geometry (G)
Topic addressed at other grade levels.	**PA.GM.1 Solve problems involving right triangles using the Pythagorean Theorem.** **PA.GM.1.1** Informally justify the Pythagorean Theorem using measurements, diagrams, or dynamic software and use the Pythagorean Theorem to solve problems in two and three dimensions involving right triangles. **PA.GM.1.2** Use the Pythagorean Theorem to find the distance between any two points in a coordinate plane.	**G.RT.1 Develop and verify mathematical relationships of right triangles and trigonometric ratios to solve real-world and mathematical problems.** **G.RT.1.1** Apply the distance formula and the Pythagorean Theorem and its converse to solve real-world and mathematical problems, as approximate and exact values, using algebraic and logical reasoning (include Pythagorean Triples). **G.RT.1.2** Verify and apply properties of right triangles, including properties of 45-45-90 and 30-60-90 triangles, to solve problems using algebraic and logical reasoning. **G.RT.1.3** Use the definition of the trigonometric functions to determine the sine, cosine, and tangent ratio of an acute angle in a right triangle. Apply the inverse trigonometric functions as ratios to find the measure of an acute angle in right triangles. **G.RT.1.4** Apply the trigonometric functions as ratios (sine, cosine, and tangent) to find side lengths in right triangles in real-world and mathematical problems.