OKLAHOMA SCHOOL TESTING PROGRAM

TEST BLUEPRINT AND ITEM SPECIFICATIONS

MATHEMATICS GRADE 8

OKLAHOMA Education

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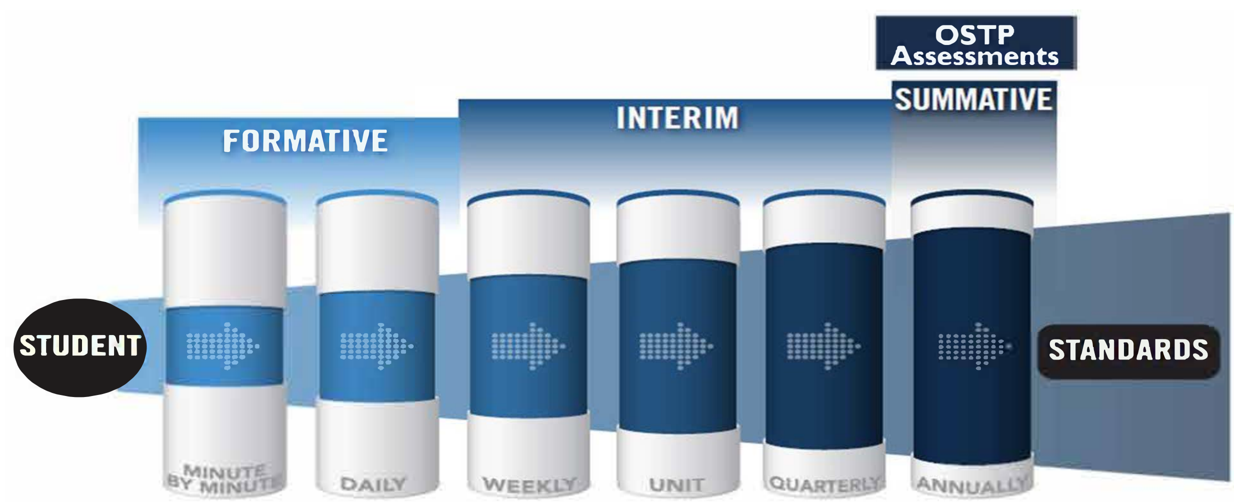
Grade 8 Mathematics Test

Purpose

A robust assessment system is predicated upon the knowledge that no one assessment is able to provide answers to all questions affecting instructional decisions. An assessment system utilizes different types of assessment to gather multiple pieces of evidence to provide timely, relevant, actionable, and reliable information about what students know and can do relative to a set of standards.

Assessments According to the Oklahoma ESSA Plan

According to page 48 of the Oklahoma ESSA Consolidated State Plan, Oklahoma recognizes that a **robust assessment system** is tied closely to students' learning and teachers' instructional practices by valuing and promoting **local, classroom-based formative assessments** that help make **student learning visible**. At the same time, that system should provide a **strong summative assessment** program that fits as a component within a multifaceted state, district, and school accountability system.



The OSDE supports an assessment system by working with Oklahoma educators and stakeholders to:

- Ensure that state and federally required annual summative assessments delivered through the Oklahoma School Testing Program (OSTP) are effective and meaningful to families, districts, educators, and members of the community;
- Develop instructional resources to support local formative and interim assessments through the curriculum frameworks projects and assessment guidance toolkit; and
- Build and deliver professional learning through face-to-face and web-based resources to support local assessment needs and interpretation of state assessment data.

Annual assessments delivered through the OSTP are aligned to the Oklahoma Academic Standards and can therefore provide point-in-time data for programmatic and curricular decisions by supporting criterion-referenced interpretations at appropriate levels and grain size (e.g., grade, student group, teacher, building/district administrator, state). Standards-based formative and interim assessments conducted at the local level can provide additional information and evidence of learning at a smaller grain size to inform instructional decisions made at the student and classroom level.

While state summative assessments are only one measure of what students know and can demonstrate, having Oklahoma students take OSTP assessments:

- ✓ Helps students, their families, and the public know how students have grown over time and how they are performing relative to the standards, their peers in Oklahoma, and the nation;
- ✓ Enables teachers to see how their students are performing against grade-level expectations communicated through the Performance Level Descriptors (PLDs) to support evaluation and enhancement of curriculum and programs for the next school year;
- ✓ Provides a standardized and reliable measure for school/district leaders, the state, policymakers, and the public to determine how well a system is meeting the goals of helping every child grow along a continuum to prepare them for careers, college, and life; and
- ✓ Provides comparable information and data to inform continuous improvement of a system and appropriately support federal and state accountability decisions.

Test Structure, Format, and Scoring

The Grade 8 Mathematics test will consist of 50 operational items and 10 field-test items, written at a reading level about two grade levels below a Grade 8 audience. The total 60 items will be divided into two test sections.

Each item is scored as correct or incorrect. Only the 50 operational items contribute to a student's scaled score on the test. Correct and incorrect field-test items do not contribute to a student's score.

The student's raw score is converted to a scaled score using the number correct scoring method.

Test Alignment with Oklahoma Academic Standards (OAS)

Criteria for Aligning the Test with the Oklahoma Academic Standards Content Strands and Standards
1. Categorical Concurrence The test is constructed so that there are at least six items measuring each OAS strand. The number of items, six, is based on estimating the number of items that could produce a reasonably reliable estimate of a student's mastery of the content measured.
2. Range-of-Knowledge Correspondence The test is constructed so that every standard for each OAS strand has at least one corresponding assessment item.
3. Source of Challenge Each test item is constructed in such a way that the major cognitive demand comes directly from the targeted OAS strand or standard being assessed, not from specialized knowledge or cultural background that the test-taker may bring to the testing situation.

OKLAHOMA SCHOOL TESTING PROGRAM

TEST BLUEPRINT MATHEMATICS GRADE 8

This blueprint describes the content and structure of an assessment and defines the ideal number of test items by strand and standard of the Oklahoma Academic Standards (OAS).

IDEAL % OF ITEMS	STRANDS AND STANDARDS
16–20%	**NUMBER AND OPERATIONS**
	PA.N.1 Real Number Operations
44–48%	**ALGEBRAIC REASONING AND ALGEBRA**
	PA.A.1 Linear and Non-Linear Functions
	PA.A.2 Linear Function Representations and Problem Solving
	PA.A.3 Algebraic Expressions
	PA.A.4 Equations and Inequalities
18–22%	**GEOMETRY AND MEASUREMENT**
	PA.GM.1 Pythagorean Theorem
	PA.GM.2 Surface Area and Volume
14–18%	**DATA AND PROBABILITY**
	PA.D.1 Data Analysis and Scatter Plots
	PA.D.2 Probability
100%	**TOTAL: 50 ITEMS**

Please note this blueprint does not include items that may be field-tested.
A minimum of 6 items is required to report a strand.

Depth-of-Knowledge Assessed by Test Items

The Grade 8 test will approximately reflect the following “depth-of-knowledge (DOK)” distribution of items:

Depth-of-Knowledge	Percent of Items
Level 1—Recall	10–20%
Level 2—Skills/Concept	65–75%
Level 3—Strategic Thinking	15–25%

DOK Ranges are based on the DOK of the OAS. The standards increase grade-level expectations and rigor, and set expectations for students to be college- and career-ready.

- **Level 1** (Recall and Reproduction) requires the student to recall facts, terms, definitions, or simple procedures, perform simple algorithms or apply formulas. One-step, well-defined, or straight algorithmic procedures should be included at this level.
- **Level 2** (Skills and Concepts) requires the student to make some decisions as to how to approach the problem or activity. Level 2 activities include making observations and collecting data; classifying, comparing, and organizing data; and organizing and displaying data in tables, charts, and graphs.
- **Level 3** (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking. Level 3 activities include making conjectures, drawing conclusions from observations, citing evidence and developing a logical argument for concepts, explaining phenomena in terms of concepts, and using concepts to solve nonroutine problems.

Note: These descriptions are adapted from Review Background Information and Instructions, Standards and Assessment Alignment Analysis, CCSSO TILSA Alignment Study, May 21–24, 2001, Version 2.0. For an extended description of each depth-of-knowledge level, see the web site at https://sde.ok.gov/sites/default/files/documents/files/Math%20WebbAlign_DOK_Summary_Table.pdf.

Universal Design for Learning (UDL) Considerations

Universal Design for Learning (UDL), as applied to assessments, is a framework that provides flexibility in the way information is presented and in the ways students demonstrate knowledge and skills. This reduces barriers while maintaining high expectations for all students, including students with disabilities and students who are limited English proficient. In the Oklahoma Grade 8 tests, items and instructions have been designed to provide maximum readability, comprehensibility, and legibility for all students. This includes such design aspects as reducing the language load in content areas other than Language Arts, increasing the font size, displaying fewer items per page, and boxing the items to assist visual focus.

Online Administration

Test questions will be presented one at a time.

The stimulus and question will appear on the screen at the same time.

Answers may be selected by using the mouse to click on the radio button to the left of the answer choice.

Navigation buttons appear at the bottom of the page for each question. For longer items, a scroll bar will appear on the right-hand side of the window to allow scrolling through the answer choices.

Tools (including a scientific calculator on the Grade 8 Mathematics assessment) appear at the bottom of the screen/page to aid in answering questions.

Students will be able to use scratch paper for all online assessments. This paper must be collected and destroyed by the test administrator immediately following the test. The test administrator must not look at what the student has written on the scratch paper.

Testing Schedules

This section appears in all of the test specification documents and is provided to give the reader a general sense of the overall testing program at this particular grade level.

Each Grade 8 test is meant to be administered in two sessions within one day with a break given between sessions or on consecutive days. Estimated time for scheduling purposes is given in the table below.

Grade 8 Mathematics Online Test Time Schedule	
Distributing login information	Approximately 5 minutes
Test instructions/tutorial and reviewing sample items	Approximately 15 minutes
Total:	Approximately 20 minutes
Administering Section 1 of the G8 Mathematics Online Test	Approximately 40 minutes
Administering Section 2 of the G8 Mathematics Online Test	Approximately 40 minutes

Item Types

The test will consist of a combination of multiple choice and technology enhanced items.

Most stems are positively worded—avoiding the use of the word "not." If a negative is required, it is underlined for emphasis (e.g., if a bag has the same number of red, blue, and black marbles, what is the probability that a marble randomly selected from the bag is not red?).

Multiple-Choice Item Guidelines

- All items must clearly indicate what is expected in a response and direct students to focus on their responses.
- Each multiple-choice item has a stem (question, statement, and/or graphic component) and four answer options—the correct answer and three distractors. Distractors will be developed based on the types of errors students are most likely to make.

- Multiple-choice item stems ask a question or pose a clear problem so that students will know what to do before looking at the answer choices. Students should not need to read all answer choices before knowing what is expected. A stem will seldom include an incomplete sentence.

Technology Enhanced Item Guidelines

- Technology Enhanced Items (TEIs) should be used to more authentically address some aspects of the OAS performance expectations and/or provide more opportunity for students to construct rather than select their response.
- TEI sample items have been included in this document to illustrate some of the interaction types. Each TEI contains only one interaction type per item.
- For each TEI, the interaction type used is that which is the most appropriate and enhancing to the construct to be measured.
- Each TEI is structured to contain the question (content) first followed by directions for how to complete the interaction in that item. Consistent style and language are used in these directions (e.g., "Drag the pictures," "Click the object," etc).

In summary, Grade 8 test items assess whether students understand algebraic concepts and procedures, whether they can communicate their understandings effectively in mathematical terms, and whether they can approach problems and develop viable solutions.

Stimulus Materials

Stimulus materials are the tables, charts, graphs, passages, and illustrations students must use in order to respond to items. The following characteristics are necessary for stimulus materials:

1. A stimulus that gives information must precede a question or a set of questions.
2. When students are given information to evaluate, they should know the question and the purpose of the information.
3. Passages, graphics, tables, etc., provide sufficient information for assessment of multiple objectives.
4. Stimulus materials for a set of items may be a combination of multiple stimuli.
5. Information in stimulus materials is based on situations students would encounter in or beyond school.
6. For conceptual items, stimulus materials are necessary but not conceptually sufficient for student response.
7. There is a balance of graphic and textual stimulus materials within a test form. Approximately 50 percent of the items will have appropriate pictorial or graphical representations. Graphs, tables, or figures are clearly associated with their intended items. Graphics appear on the same page as the textual stimulus or on the facing page.

General Considerations—Oklahoma School Testing Program

1. Items deal with issues and details that are of consequence in the stimulus and central to students' understanding and interpretation of the stimulus.
2. Test items are varied and address all OAS standards listed in the Test Blueprint.
3. To the greatest extent possible, no item or response choice clues the answer to any other item.
4. All items reviewed and approved by the Oklahoma Item Review Committee are assigned an OAS strand, standard, and/or objective. The Test Blueprints and score reports reflect the degree to which each OAS strand and standard is represented on the test.
5. Test items are tied closely and particularly to the stimuli from which they derive, so that the impact of outside (prior) knowledge, while never wholly avoidable, is minimized.
6. Each multiple-choice item contains a question and four answer options, only one of which is correct. Correct answers will be approximately equally distributed among A, B, C, and D responses.
7. Distractors adopt the language and sense of the material in the stimuli so that students must think their way to the correct answer rather than simply identify incorrect responses by virtue of a distractor's obviously inappropriate nature.
8. Distractors should always be plausible (but, of course, incorrect) in the context of the stimulus. Students should not be able to rule out a wrong answer or identify a correct response solely because it looks different from the other answer choices.
9. Order of presentation of item types is dictated by logic (chronologically, spatially, etc.).
10. Items are worded precisely and clearly. The better focused an item, the more reliable and fair it is certain to be, and the more likely all students will understand it in the same way.
11. The range of items measuring an OAS standard consisting of more than one skill will provide a balanced representation of those skills.
12. Items should be focused on what all students should know and be able to do as they complete their Grade 8 coursework.
13. The responses "Both of the above," "All of the above," "None of the above," and "Neither of the above" will not be used.
14. The material presented is balanced, culturally diverse, well written, and of interest to Grade 8 test level students. The stimuli and items are fairly presented in order to gain a true picture of students' skills.
15. Across all forms, a balance of gender and active/passive roles by gender is maintained.
16. Forms attempt to represent the ethnic diversity of Oklahoma students.
17. Approved calculators and the formula sheet on page 9 may be used on the Grade 8 Mathematics test. No other resource materials may be used by students during the test. More information regarding the calculator policy can be found at http://sde.ok.gov/sde/assessment-administrator-resources-administrators.
18. The stimuli avoid subject matter that might prompt emotional distress on the part of the students.
19. Permission to use stimuli from copyrighted material is obtained as necessary by testing vendor.

Considerations Specific to the Grade 8 Mathematics Test

It is necessary to create test items that are reliable, fair, and targeted to the Oklahoma Academic Standards listed on the following pages. There are some general considerations and procedures for effective item development.

These considerations include, but are not limited to, the following:

1. Each test form contains items assessing all content standards.
2. Test items that assess each standard are not limited to one particular type of response format.
3. Test questions attempt to focus on content that is authentic and that Grade 8 level students can relate to and understand.
4. Test items are worded precisely and clearly. The better focused an item, the more reliable and fair it is likely to be, and the more likely all students will understand what is required of them.
5. All items are reviewed to eliminate language that shows bias or that would otherwise likely disadvantage a particular group of students. That is, items do not display unfair representations of gender, race, ethnicity, disability, culture, or religion; nor do items contain elements that are offensive to any such groups.
6. Items are written so that calculations are kept to a minimum, and numbers are selected to minimize the time spent on computations.
7. All test items and answer choices have appropriate labels and units.
8. Most graphs are placed on a gray grid, with the x- and y-axes labeled and marked.

All items developed using these specifications are reviewed annually by Oklahoma educators and approved by the Oklahoma State Department of Education. The distribution of newly developed items is based on difficulty, cognitive ability, percentage of art/graphics, and grade-level appropriateness as determined by an annual Item Development Plan approved by the Oklahoma State Department of Education.

Oklahoma State Testing Program
8th Grade Mathematics Formula Sheet

UNIT CONVERSIONS

1 foot = 12 inches

1 yard = 3 feet

1 mile = 5280 feet

1 mile = 1760 yards

1 meter = 100 centimeters

1 meter = 1000 millimeters

1 pound = 16 ounces

1 ton = 2000 pounds

1 kilogram = 1000 grams

1 cup = 8 fluid ounces

1 pint = 2 cups

1 quart = 2 pints

1 gallon = 4 quarts

AREA

Square	$A = s^2$	Parallelogram	$A = bh$
Rectangle	$A = lw$	Circle	$A = \pi r^2$
Triangle	$A = \frac{1}{2}bh$	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$

CIRCUMFERENCE

Circle $C = \pi d$ or $C = 2\pi r$

VOLUME

Rectangular Prism	$V = Bh$ or $V = lwh$	Right Cylinder	$V = Bh$ or $V = \pi r^2 h$

SURFACE AREA

Rectangular Prism $S = 2B + Ph$ or $S = 2lw + 2lh + 2wh$

Cylinder $S = 2\pi rh + 2\pi r^2$

LINEAR EQUATIONS

Slope-intercept	$y = mx + b$	Direct Variation	$y = kx$
Slope formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$		

OTHER

$d = rt$

Pythagorean Theorem $a^2 + b^2 = c^2$

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Overview of Item Specifications

For each OAS strand, item specifications are organized under the following headings:

- OAS Strand
- OAS Standard
- OAS Objectives
- Item Specifications
    a. Emphasis
    b. Stimulus Attributes
    c. Format
    d. Content Limits
    e. Primary Process Standard(s)
    f. Distractor Domain
    g. Sample Test Items

The headings "OAS Strands" and "OAS Standards" state the OAS strand followed by the OAS standard being measured in the mathematics section of the Oklahoma Academic Standards document.

For each standard, the information under the heading "Item Specifications" highlights important points about a test item's emphasis, format, content limits, and distractor domain. Sample test items are provided with each strand to illustrate these specifications. Although it is sometimes possible to score single items for more than one concept, all items in these tests are written to address a single objective as the primary concept.

Note: With the exception of content limits, the Item Specifications offer suggestions of what might be included and do not provide an exhaustive list of what can be included. For this reason, Item Specifications are only meant to be a supplemental resource for classroom instruction.

In addition, the sample test items are not intended to be definitive in nature or construction—the stimuli and the test items that follow them may differ from test form to test form, as may their presentations. Sample test items are not intended to predict a student's performance on the actual test, but rather to allow students to familiarize themselves with the item types and formats that they may see on the test.

STANDARDS & SAMPLE ITEMS

OAS STRAND—NUMBER & OPERATIONS (N): STANDARD PA.N.1

OAS STANDARD

PA.N.1 Read, write, compare, classify, and represent real numbers and use them to solve problems in various contexts.

OAS OBJECTIVES

PA.N.1.1 Develop and apply the properties of integer exponents, including $a^0 = 1$ (with $a \neq 0$), to generate equivalent numerical and algebraic expressions.

PA.N.1.2 Express and compare approximations of very large and very small numbers using scientific notation.

PA.N.1.3 Multiply and divide numbers expressed in scientific notation, express the answer in scientific notation.

PA.N.1.4 Classify real numbers as rational or irrational. Explain why the rational number system is closed under addition and multiplication and why the irrational system is not. Explain why the sum of a rational number and an irrational number is irrational; and the product of a non-zero rational number and an irrational number is irrational.

PA.N.1.5 Compare real numbers; locate real numbers on a number line. Identify the square root of a perfect square to 400 or, if it is not a perfect square root, locate it as an irrational number between two consecutive positive integers.

ITEM SPECIFICATIONS

Emphasis:

- Demonstrate an understanding of the properties of integer exponents and generate equivalent expressions that involve integer exponents.
- Demonstrate an understanding of scientific notation.
- Demonstrate an ability to multiply and divide numbers expressed in scientific notation.
- Demonstrate an understanding of the rational number system and the irrational number system.
- Demonstrate an understanding of perfect squares and square roots, and integers and approximate square roots.
- Demonstrate an ability to classify real numbers as rational or irrational.
- Demonstrate an ability to locate real numbers on a number line.

Stimulus Attributes:

- Test items may include calculator displays, tables, graphs, charts, maps, scale drawings, data sets, other diagrams, number lines, two-dimensional geometric figures, coordinate graphs, 10-by-10 grids, and counting manipulatives.

Format:

- Use the rules of exponents in mathematical and real-life contexts, including $a^0 = 1$ (with $a \neq 0$)
- Demonstrate an understanding of the magnitude of very large and very small numbers and the role of scientific and exponential notation in the representation of these numbers
- Use the rules of exponents in mathematical and real-life contexts to generate equivalent numerical and algebraic expressions
- Demonstrate the concepts of positive and negative exponents using patterns
- Demonstrate an understanding of scientific notation
- Compare, order, and translate among representations of rational numbers

ITEM SPECIFICATIONS

Format (continued):

- Classify real numbers as rational or irrational
- Demonstrate an understanding of the rational and irrational number system
- Use numbers to explore, estimate, and identify square roots and perfect squares in mathematical, geometric, and real-world contexts
- Use graphic representations including arrays and models of multiples to explore, estimate, and identify square roots and perfect squares in mathematical, geometric, and real-world contexts
- Use a number line to locate real numbers

Content Limits:

- Limit mathematical and real-life contexts to age-appropriate situations
- Limit to no more than two operations on exponential or scientific numbers
- Limit to integer exponents with the same base
- Limit computations involving only fractions to halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths
- Limit graphic representations to common two-dimensional geometric figures
- Limit classifications to rational or irrational
- Limit square roots to up to the square root of 400

Primary Process Standards:

- Develop Strategies for Problem Solving
- Develop the Ability to Communicate Mathematically
- Develop Mathematical Reasoning
- Develop a Deep and Flexible Conceptual Understanding
- Develop the Ability to Make Conjectures, Model, and Generalize

Distractor Domain:

- Computational errors
- Incorrect use of rules or properties
- Comparison errors
- Common errors
- Rounding errors
- Use of incorrect equivalencies
- Incorrect interpretation of data display

1 **Which of the following is equivalent to the expression below?**

$$\frac{4^8}{4^2}$$

A 4^4

B 4^6

C 4^{10}

D 4^{16}

Standard: PA.N.1.1 Develop and apply the properties of integer exponents, including $a^0 = 1$ (with $a \neq 0$), to generate equivalent numerical and algebraic expressions.

Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to perform a simple one-step procedure, dividing with exponents.

Distractor Rationale:

A. The student divided the exponents.

B. Correct. The student demonstrated an ability to apply the properties of integer exponents.

C. The student added the exponents.

D. The student multiplied the exponents.

2 **A space shuttle travels at 2.6×10^4 feet per second. An hour is 3.6×10^3 seconds. This expression can be used to find the number of feet the space shuttle travels in an hour.**

$$\boxed{(2.6 \times 10^4)\ (3.6 \times 10^3)}$$

How many feet does the shuttle travel in an hour?

A 6.2×10^1 feet

B 6.2×10^{12} feet

C 9.36×10^7 feet

D 9.36×10^{12} feet

Standard: PA.N.1.3 Multiply and divide numbers expressed in scientific notation, express the answer in scientific notation.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to multiply numbers expressed in scientific notation.

Distractor Rationale:

A. The student computed 2.6 + 3.6 to get the base number and then computed 4 − 1 to get the exponent.

B. The student computed 2.6 + 3.6 to get the base number and then computed 4 × 3 to get the exponent.

C. Correct. The student demonstrated an ability to multiply numbers expressed in scientific notation.

D. The student computed the base number correctly, but then multiplied instead of added to find the exponent.

3

Complete the statements to describe the outcomes of operations with the following numbers.

- a and b are non-zero rational numbers.
- x and y are irrational numbers.

Select the word that best completes each statement. To select a word, click the menu and then click the desired word. To choose a different word, click the menu and click the new word.

$a + b$ is [-Select an Answer- ▼] rational.

$x \cdot y$ is [-Select an Answer- ▼] irrational.

$a + x$ is [-Select an Answer- ▼] rational.

$b \cdot x$ is [-Select an Answer- ▼] irrational.

Standard: PA.N.1.4 Classify real numbers as rational or irrational. Explain why the rational number system is closed under addition and multiplication and why the irrational system is not. Explain why the sum of a rational number and an irrational number is irrational; and the product of a non-zero rational number and an irrational number is irrational.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to classify expressions as rational or irrational.

Sample Distractor Rationales:

Correct

$a + b$ is [always] rational. $x \cdot y$ is [sometimes] irrational.
$a + x$ is [never] rational. $b \cdot x$ is [always] irrational.

Incorrect

$a + b$ is [always] rational. $x \cdot y$ is [always] irrational.
$a + x$ is [never] rational. $b \cdot x$ is [always] irrational.

The student thought that whenever an irrational number is involved, the answer must be irrational.

$a + b$ is [always] rational. $x \cdot y$ is [sometimes] irrational.
$a + x$ is [always] rational. $b \cdot x$ is [never] irrational.

The student confused rational and irrational numbers.

OAS STRAND—ALGEBRAIC REASONING & ALGEBRA (A): STANDARD PA.A.1

OAS STANDARD

PA.A.1 Understand the concept of function in real-world and mathematical situations, and distinguish between linear and nonlinear functions.

OAS OBJECTIVES

PA.A.1.1 Recognize that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable.

PA.A.1.2 Use linear functions to represent and explain real-world and mathematical situations.

PA.A.1.3 Identify a function as linear if it can be expressed in the form $y = mx + b$ or if its graph is a straight line.

ITEM SPECIFICATIONS

Emphasis:

- Identify a relationship that forms a function.
- Determine the dependent and independent variables of various relations and functions represented mathematically and in real-world contexts.
- Use linear functions to represent real-world and mathematical situations.
- Identify characteristics of linear functions expressed graphically and numerically.

Stimulus Attributes:

- Test items may include graphs, diagrams, tables, and situations.

Format:

- Identify the dependent and independent variables of various relations and functions represented algebraically, graphically, numerically, and in tables
- Identify rates of constant change
- Identify the meaning of rates of change in real-world contexts
- Distinguish between linear and nonlinear data represented graphically, numerically, in equation form, and in tables, etc.
- Identify the appropriate situation that corresponds to an algebraic equation
- Identify the appropriate algebraic equation that represents a situation described graphically or verbally

Content Limits:

- When classifying functions as linear or nonlinear, exclude relations that include xy
- Limit real-world and mathematical contexts to age appropriate situations

OAS STRAND—ALGEBRAIC REASONING & ALGEBRA (A): STANDARD PA.A.1

ITEM SPECIFICATIONS

Primary Process Standards:

- Develop Strategies for Problem Solving
- Develop the Ability to Communicate Mathematically
- Develop Mathematical Reasoning
- Develop a Deep and Flexible Conceptual Understanding
- Develop the Ability to Make Conjectures, Model, and Generalize

Distractor Domain:

- Computational errors
- Interchange range and domain
- Common algebraic misconceptions
- Conceptual errors

4 **A smoothie shop makes $3.00 for every smoothie sold after subtracting the costs of ingredients and packaging. They have additional costs of $150.00 per day. Which linear model expresses how much the shop makes, *y*, for selling *x* smoothies in a day?**

A $y = 3x$

B $y = 3x - 150$

C $y = 150x - 3$

D $y = 3x + 150$

Standard: PA.A.1.2 Use linear functions to represent and explain real-world and mathematical situations.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to translate a verbal description into an equation.

Distractor Rationale:

A. The student ignored the fixed costs.

B. Correct. The student demonstrated an ability to write a linear function to represent a real-world problem.

C. The student reversed the slope and *y*-intercept.

D. The student added the fixed costs instead of subtracting.

5 **A taxi company charges a base fee of $2.00 plus $0.50 per mile. Which of these represents the cost of a taxi ride, *y* for the distance of *x* miles?**

A $y = 2.5x$

B $y = 50x + 2$

C $y = 0.50x + 2$

D $y = 2x + 0.50$

Standard: PA.A.1.2 Use linear functions to represent and explain real-world and mathematical situations.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to translate a verbal description into an equation.

Distractor Rationale:

A. The student added the $2 and the $0.50.

B. The student confused 50 and 0.50.

C. Correct. The student demonstrated an ability to use a linear function to represent a real-world situation.

D. The student swapped the slope and the *y*-intercept.

OAS STRAND—ALGEBRAIC REASONING & ALGEBRA (A): STANDARD PA.A.2

OAS STANDARD

PA.A.2 Recognize linear functions in real-world and mathematical situations; represent linear functions and other functions with tables, verbal descriptions, symbols, and graphs; solve problems involving linear functions and interpret results in the original context.

OAS OBJECTIVES

PA.A.2.1 Represent linear functions with tables, verbal descriptions, symbols, and graphs; translate from one representation to another.

PA.A.2.2 Identify, describe, and analyze linear relationships between two variables.

PA.A.2.3 Identify graphical properties of linear functions including slope and intercepts. Know that the slope equals the rate of change, and that the y-intercept is zero when the function represents a proportional relationship.

PA.A.2.4 Predict the effect on the graph of a linear function when the slope or y-intercept changes. Use appropriate tools to examine these effects.

PA.A.2.5 Solve problems involving linear functions and interpret results in the original context.

ITEM SPECIFICATIONS

Emphasis:

- Use a variety of representations of linear functions and translate from one representation to another.
- Identify, describe, and analyze linear relationships between two variables.
- Identify graphical properties of linear functions.
- Predict the effect on the graph of a linear function when the slope or y-intercept changes.
- Solve real-world and mathematical problems involving linear functions and interpret the results.

Stimulus Attributes:

- Test items may include tables, graphs, data sets, algebraic equations, situations, coordinate graphs, number lines, balances, other diagrams, and two- and three-dimensional geometric figures.

Format:

- Identify the appropriate situation that corresponds to a linear equation
- Identify the appropriate linear equation that represents a situation described graphically or verbally
- Identify, write, and solve multi-step linear equations involved in mathematical and real-world situations
- Translate between algebraic and geometric representations of linear equations
- Analyze attributes of algebraic and geometric representations of linear equations
- Select and apply appropriate formulas for mathematical and real-world situations
- Formulas may or may not be given
- Analyze the effect on the graph of a linear function when the slope or y-intercept changes

OAS STRAND—ALGEBRAIC REASONING & ALGEBRA (A): STANDARD PA.A.2

ITEM SPECIFICATIONS

Content Limits:

- Limit real-world and mathematical contexts to age appropriate situations
- Limit to linear equations
- Limit linear equations to rational coefficients
- Limit equations to slope-intercept form
- Limit change of slope to opposites, integers, and common fractions, or simple identification of steepness, or closer to vertical or horizontal (without calculation)
- Limit multistep processes to no more than two steps for each component stage

Primary Process Standards:

- Develop Strategies for Problem Solving
- Develop the Ability to Communicate Mathematically
- Develop Mathematical Reasoning
- Develop a Deep and Flexible Conceptual Understanding
- Develop the Ability to Make Conjectures, Model, and Generalize

Distractor Domain:

- Computational errors
- Interchange range and domain
- Common algebraic misconceptions
- Common errors
- Incorrect procedures
- Inappropriate operations with variables
- Use of inappropriate formulas

6 **Match the equation in the left column to the description of the slope and *y*-intercept of its graph in the right column.** To connect an equation to a description, click an equation in the left column and then a description in the right column, and a line will automatically be drawn between them. To remove a connection, hold the pointer over the line until it turns red, and then click it. Each equation in the left column matches to exactly two descriptions in the right column. Each description in the right column can connect to one and only one equation in the left column.

	slope $= -\frac{2}{3}$
$3x - 2y = 8$	slope $= \frac{2}{3}$
slope $= \frac{3}{2}$	
$2x + 3y = 12$	y-intercept $= -4$
	y-intercept $= -2$
$3y + 6 = 2x$	y-intercept $= 4$

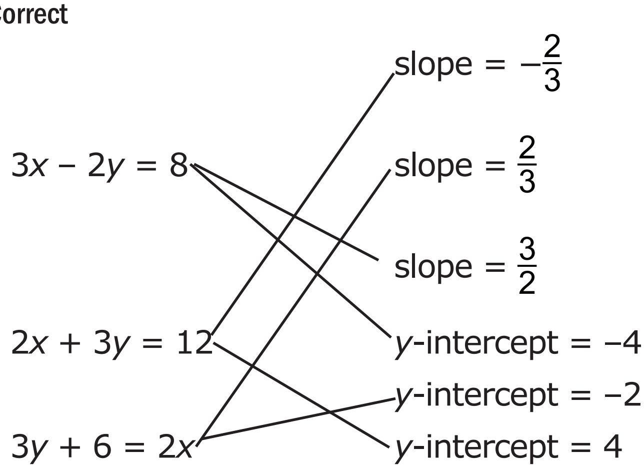
Standard: PA.A.2.3 Identify graphical properties of linear functions including slope and intercepts. Know that the slope equals the rate of change, and that the y-intercept is zero when the function represents a proportional relationship.

Depth-of-Knowledge: 2

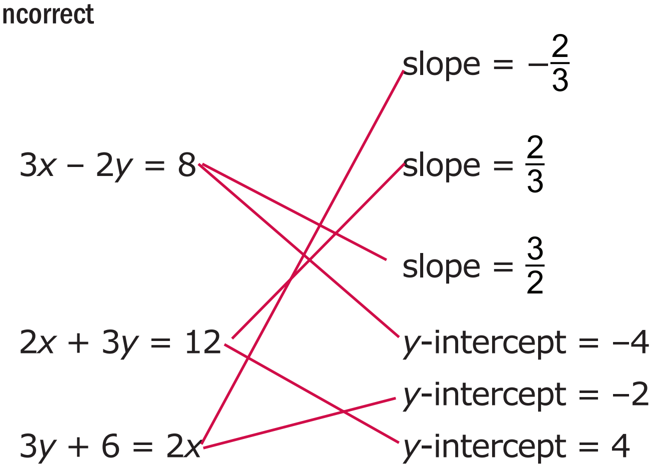
This item is a DOK 2 because it requires the student to identify the slope and *y*-intercept of equations that are not presented in slope-intercept form.

Sample Distractor Rationales:

Correct

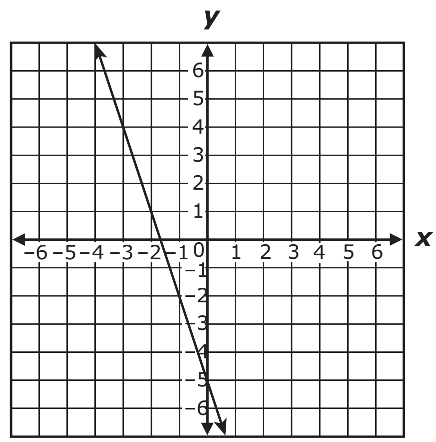


Incorrect



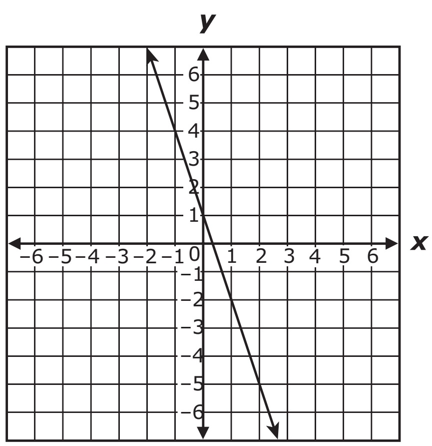
The student confused –2/3 and 2/3.

7 **The graph of the equation $y = -3x - 5$ is shown below.**

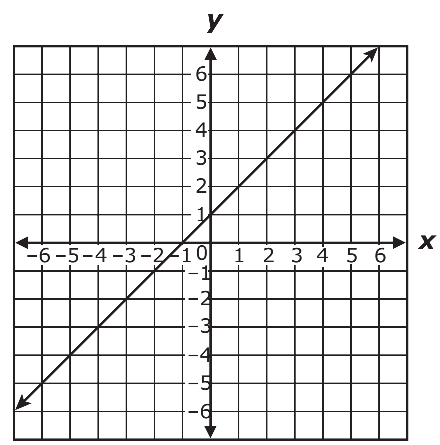


Which best represents the graph of the equation $y = -3x - 5$ when the slope is changed to 1 and the y-intercept remains the same?

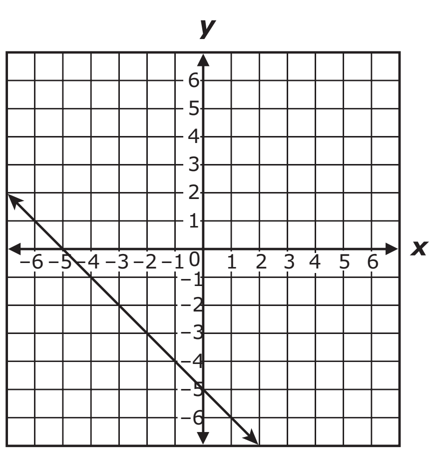
A



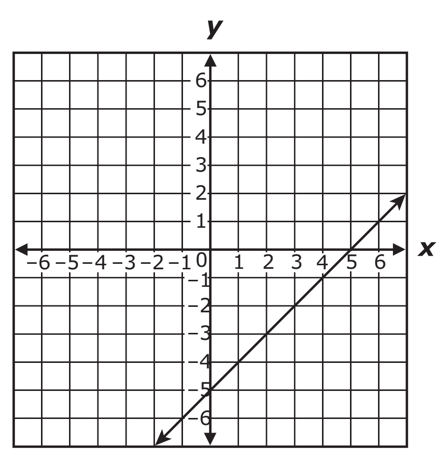
B



C



D



Standard: PA.A.2.4 Predict the effect on the graph of a linear function when the slope or y-intercept changes. Use appropriate tools to examine these effects.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to use an understanding of graphs and equations to find a graph that matches an equation with a different slope, but same *y*-intercept.

Distractor Rationale:

A. The student changed the *y*-intercept to 1.

B. The student changed both the slope and the *y*-intercept to 1.

C. The student changed the slope to –1.

D. Correct. The student demonstrated an ability to predict the effect on the graph when the slope changes.

8

x	*y*
−7	−12
−5	−8
−3	−4
−1	0
1	4

Which statement best describes the values of *x* and *y* in the table?

A As the value of *x* increases by 2, the value of *y* increases by 4.

B As the value of *x* increases by 2, the value of *y* decreases by 4.

C As the value of *x* increases by 4, the value of *y* increases by 2.

D As the value of *x* increases by 4, the value of *y* decreases by 2.

Standard: PA.A.2.1 Represent linear functions with tables, verbal descriptions, symbols, and graphs; translate from one representation to another.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to analyze the data presented in the table and then describe the relationship between *x* and *y*.

Distractor Rationale:

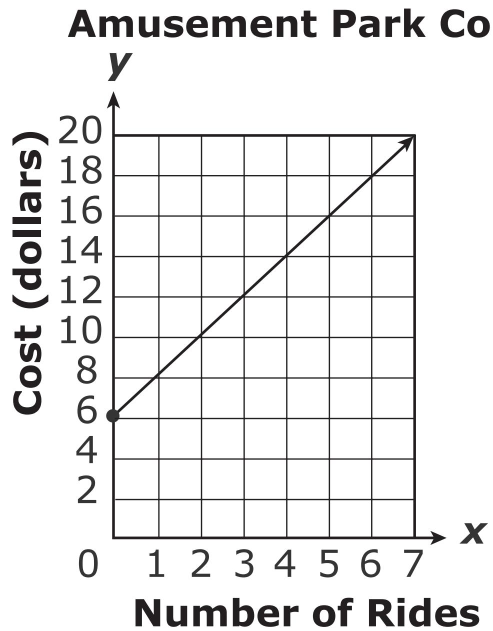
A. **Correct. The student demonstrated an ability to represent a linear function with a verbal description.**

B. The student identified the correct relationship for *x* only.

C. The student reversed *x* and *y*.

D. The student reversed *x* and *y* and thought *y* was decreasing.

9 **When planning a trip to the local amusement park, Leah drew a graph to show her possible costs.**



Based on the graph, what is the cost per ride?

A \$1.00

B \$2.00

C \$2.60

D \$6.00

Standard: PA.A.2.3 Identify graphical properties of linear functions including slope and intercepts. Know that the slope equals the rate of change, and that the *y*-intercept is zero when the function represents a proportional relationship.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to analyze the graph to find the cost per ride.

Distractor Rationale:

A. The student saw that the values on the *x*-axis increase by 1 but did not recognize the scale of the *y*-axis is 2.

B. Correct. The student demonstrated an ability to identify graphical properties of a linear function.

C. Balance distractor

D. The student saw that the first data point has a cost of \$6.

OAS STRAND—ALGEBRAIC REASONING & ALGEBRA (A): STANDARD PA.A.3

OAS STANDARD	**PA.A.3**	Generate equivalent numerical and algebraic expressions and use algebraic properties to evaluate expressions.
OAS OBJECTIVES	**PA.A.3.1**	Use substitution to simplify and evaluate algebraic expressions.
	PA.A.3.2	Justify steps in generating equivalent expressions by identifying the properties used, including the properties of operations (associative, commutative, and distributive laws) and the order of operations, including grouping symbols.

ITEM SPECIFICATIONS

Emphasis:
- Simplify and evaluate algebraic expressions using substitution.
- Justify steps in generating equivalent expressions by identifying the properties used.

Stimulus Attributes:
- Test items may include illustrations of the following: tables, graphs, charts, data sets, equivalency statements, and algebraic expressions.

Format:
- Use variables as unknowns
- Substitute numerical values for variables in algebraic expressions
- Use the rules for order of operations with rational numbers to find the values of numerical and algebraic expressions
- Items may include parentheses and other grouping symbols
- Identify applications of the properties of operations

Content Limits:
- Properties of operations to include associative, commutative, and distributive laws
- Limit exponents to integers

Primary Process Standards:
- Develop Accurate and Appropriate Procedural Fluency
- Develop Mathematical Reasoning
- Develop the Ability to Communicate Mathematically

Distractor Domain:
- Common errors
- Incorrect procedures
- Computational errors
- Incorrect use of rules or properties
- Order of operations errors

10 **The total amount of money, in dollars, Sandy earns for working *h* hours is represented by this expression.**

$$15h$$

How much money does Sandy earn for working 35 hours?

A \$20

B \$50

C \$525

D \$1535

Standard: PA.A.3.1 Use substitution to simplify and evaluate algebraic expressions.

Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to complete a simple procedure, substituting 35 for *h* in a single step expression.

Distractor Rationale:

A. The student subtracted instead of multiplying.

B. The student added instead of multiplying.

C. Correct. The student demonstrated an ability to use substitution to evaluate an algebraic expression.

D. The student substituted 35 for *h* and then read left to right, not understanding the implied multiplication.

11

Complete these equations. Select the numbers you want to choose and drag and drop them into the boxes. To drag a number, click and hold it, and then drag to the desired box. To change a number, click and hold it, then drag it back to the desired box. You may use each number once, more than once, or not at all.

If $x = 3$, then $5x + 2 = \boxed{}$.

If $x = 1$, then $2(x-4) = \boxed{}$.

If $x = -2$, then $-3x^2 + 4x + 15 = \boxed{}$.

19 17 11 6 −5 −6

Standard: PA.A.3.1 Use substitution to simplify and evaluate algebraic expressions.

Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to complete a simple procedure, substituting values for x expressions.

Sample Distractor Rationales:

Correct

If $x = 3$, then $5x + 2 = 17$.
If $x = 1$, then $2(x - 4) = -6$.
If $x = -2$, then $-3x^2 + 4x + 15 = -5$.

Incorrect

If $x = 3$, then $5x + 2 = 17$.
If $x = 1$, then $2(x - 4)= -6$.
If $x = -2$, then $-3x^2 + 4x + 15 =$ **19**.

The student thought $(-2)^2$ was -4.

If $x = 3$, then $5x + 2 = 17$.
If $x = 1$, then $2(x - 4) =$ **6**.
If $x = -2$, then $-3x^2 + 4x + 15 = 19$.

The student computed $(1 - 4)$ as $(4 - 1)$ for the second equation.

OAS STRAND—ALGEBRAIC REASONING & ALGEBRA (A): STANDARD PA.A.4

OAS STANDARD	**PA.A.4**	Represent real-world and mathematical problems using equations and inequalities involving linear expressions. Solve and graph equations and inequalities symbolically and graphically. Interpret solutions in the original context.
OAS OBJECTIVES	**PA.A.4.1**	Illustrate, write, and solve mathematical and real-world problems using linear equations with one variable with one solution, infinitely many solutions, or no solutions. Interpret solutions in the original context.
	PA.A.4.2	Represent, write, solve, and graph problems leading to linear inequalities with one variable in the form $px + q > r$ and $px + q < r$, where p, q, and r are rational numbers.
	PA.A.4.3	Represent real-world situations using equations and inequalities involving one variable.

ITEM SPECIFICATIONS

Emphasis:

- Demonstrate an ability to represent and solve mathematical and real-world problems using linear equations and interpret solutions in the original context.
- Demonstrate an ability to represent and solve mathematical and real-world problems using linear inequalities.
- Represent real-world situations using equations and inequalities.

Stimulus Attributes:

- Test items may include algebraic equations, strict and non-strict inequalities, graphs, number lines, tables, and situations.

Format:

- Illustrate linear equations
- Write linear equations
- Solve linear equations
- Solve linear inequalities graphically
- Solve linear inequalities algebraically
- Identify the appropriate situation that corresponds to an algebraic equation
- Identify the appropriate algebraic equation that represents a situation described graphically or verbally
- Identify the appropriate situation that corresponds to an algebraic inequality
- Identify the appropriate algebraic inequality that represents a situation described graphically or verbally

Content Limits:

- Limit inequalities to rational coefficients
- Limit real-world and mathematical contexts to age appropriate situations
- Limit to linear equations
- Limit inequalities to one variable

OAS STRAND—ALGEBRAIC REASONING & ALGEBRA (A): STANDARD PA.A.4

ITEM SPECIFICATIONS

Primary Process Standards:

- Develop Strategies for Problem Solving
- Develop the Ability to Communicate Mathematically
- Develop Mathematical Reasoning
- Develop a Deep and Flexible Conceptual Understanding
- Develop the Ability to Make Conjectures, Model, and Generalize

Distractor Domain:

- Computational errors
- Common algebraic misconceptions
- Incorrectly filled or blank points on number lines
- Misdirected arrows on number lines or wrong shading of graphs

12 **A wall in Cynthia's attic is a triangle with an area of 6 square meters and a base of 4 meters. What is the height, in meters, of the triangular wall?**

A 3 meters

B 4 meters

C 12 meters

D 24 meters

Standard: PA.A.4.1 Illustrate, write, and solve mathematical and real-world problems using linear equations with one variable with one solution, infinitely many solutions, or no solutions. Interpret solutions in the original context.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to use the formula for the area of a triangle to solve for the height when given the area and base.

Distractor Rationale:

A. **Correct. The student demonstrated the ability to write and solve real world problems using linear equations with one variable with one solution.**

B. The student thought the base and height were the same.

C. The student computed $\frac{1}{2} \times 6 \times 4$, confusing the area and the height.

D. The student computed 6×4.

13 **The product of 24 and *n* is greater than –96. Which inequality represents the possible values for *n*?**

A $n > -4$

B $n > -120$

C $n < -4$

D $n < -120$

Standard: PA.A.4.2 Represent, write, solve, and graph problems leading to linear inequalities with one variable in the form $px + q > r$ and $px + q < r$, where p, q, and r are rational numbers.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to translate a verbal description into an inequality when simplifying is necessary.

Distractor Rationale:

A. **Correct. The student demonstrated an ability to write a linear inequality with one variable.**

B. The student confused sum and product.

C. The student reversed the inequality sign.

D. The student confused sum and product and reversed the inequality sign.

14 **Tom has read 11 pages of a 215-page book. He will read 6 pages each day until he finishes the book. Which equation can be used to find the number of days, *d*, it will take Tom to finish reading the book?**

A $6+ 11d = 215$

B $11+ 6d = 215$

C $17d = 215$

D $6d = 215$

Standard: PA.A.4.3 Represent real-world situations using equations and inequalities involving one variable.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to represent a real-world situation expressed in words using an equation.

Distractor Rationale:

A. The student confused the slope and the y-intercept.

B. **Correct. The student demonstrated an ability to write a linear equation for a real-world problem with one variable.**

C. The student did $11d + 6d$.

D. The student ignored the 11 pages already read.

OAS STRAND—GEOMETRY & MEASUREMENT (GM): STANDARD PA.GM.1

OAS STANDARD

PA.GM.1 Solve problems involving right triangles using the Pythagorean Theorem.

OAS OBJECTIVES

PA.GM.1.1 Informally justify the Pythagorean Theorem using measurements, diagrams, or dynamic software and use the Pythagorean Theorem to solve problems in two and three dimensions involving right triangles.

PA.GM.1.2 Use the Pythagorean Theorem to find the distance between any two points in a coordinate plane.

ITEM SPECIFICATIONS

Emphasis:
- Demonstrate the ability to use the Pythagorean Theorem to solve problems involving right triangles.
- Demonstrate the ability to use the Pythagorean Theorem to find the distance between two points in a coordinate plane.

Stimulus Attributes:
- Plane geometric figures, word problems, dynamic software, and three-dimensional figures.

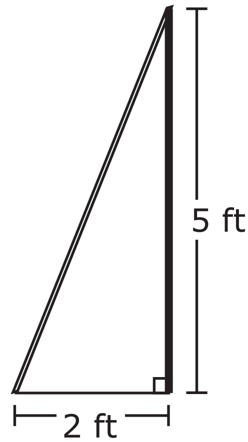
Format:
- Identify situations in which the use of the Pythagorean Theorem is appropriate in finding missing lengths of the sides of geometric figures and line segments given in various contexts through age appropriate word problems
- Justify the Pythagorean Theorem

Content Limits:
- Limit geometric figures to right triangles, rectangles, or combined forms which include triangles, rectangles, circles, or half-circles
- Limit final answer choices to whole numbers (after rounding) and radicals (simplification not needed, except for perfect squares)
- Limit real-life and mathematical contexts to age appropriate situations

Primary Process Standards:
- Develop Strategies for Problem Solving
- Develop Mathematical Reasoning
- Develop a Deep and Flexible Conceptual Understanding
- Develop the Ability to Make Conjectures, Model, and Generalize

Distractor Domain:
- Computational errors
- Incorrect procedures

15 **A wire is attached to the top of a 5-foot tall pole. The other end of the wire is secured to the ground 2 feet from the base of the pole.**



What is the length, in feet, of the wire?

A $\sqrt{7}$ feet

B $\sqrt{14}$ feet

C $\sqrt{21}$ feet

D $\sqrt{29}$ feet

Standard: PA.GM.1.1 Informally justify the Pythagorean Theorem using measurements, diagrams, or dynamic software and use the Pythagorean Theorem to solve problems in two and three dimensions involving right triangles.

Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to complete a simple procedure, using the Pythagorean Theorem to solve for the hypotenuse of a triangle.

Distractor Rationale:

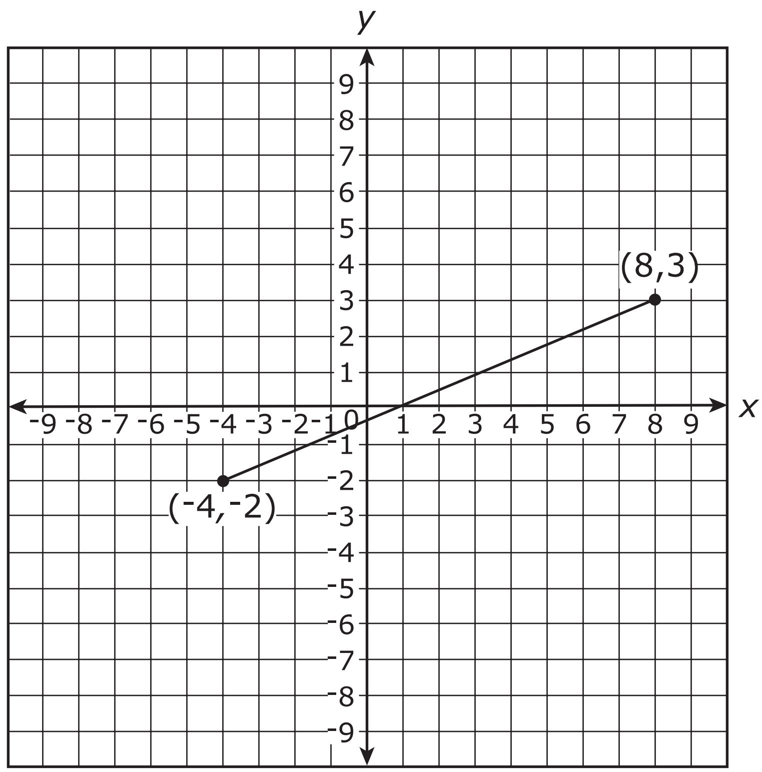
A. The student thought $a + b = c^2$.

B. The student thought $c^2 = 2a + 2b$.

C. Balance distractor

D. Correct. The student demonstrated an ability to use the Pythagorean Theorem to solve a problem involving a right triangle.

16



What is the length of the segment connecting the points that have the coordinates (–4, –2) and (8, 3)?

A 12

B 13

C 14

D 15

Standard: PA.GM.1.2 Use the Pythagorean Theorem to find the distance between any two points in a coordinate plane.

Depth-of-Knowledge: 2

This item is a DOK 2 because the student must first realize that the Pythagorean Theorem is necessary and then utilize it to find the distance between two points on the coordinate plane.

Distractor Rationale:

A. The student found the length of the horizontal leg.

B. Correct. The student demonstrated an ability to use the Pythagorean Theorem to the find the distance between two points on the coordinate plane.

C. The student found the difference of the two legs and then multiplied by 2.

D. The student added the lengths of the two legs.

OAS STRAND—GEOMETRY & MEASUREMENT (GM): STANDARD PA.GM.2

OAS STANDARD

PA.GM.2 Calculate surface area and volume of three-dimensional figures.

OAS OBJECTIVES

PA.GM.2.1 Calculate the surface area of a rectangular prism using decomposition or nets. Use appropriate measurements such as cm^2.

PA.GM.2.2 Calculate the surface area of a cylinder, in terms of π and using approximations for π, using decomposition or nets. Use appropriate measurements such as cm^2.

PA.GM.2.3 Develop and use the formulas $V = lwh$ and $V = Bh$ to determine the volume of rectangular prisms. Justify why base area (B) and height (h) are multiplied to find the volume of a rectangular prism. Use appropriate measurements such as cm^3.

PA.GM.2.4 Develop and use the formulas $V = \pi r^2 h$ and $V = Bh$ to determine the volume of right cylinders, in terms of π and using approximations for π. Justify why base area (B) and height (h) are multiplied to find the volume of a right cylinder. Use appropriate measurements such as cm^3.

ITEM SPECIFICATIONS

Emphasis:
- Determine the surface area of rectangular prisms and cylinders.
- Determine the volume of rectangular prisms and cylinders.

Stimulus Attributes:
- Test items may include rectangular prisms, cylinders, and other geometric solids.

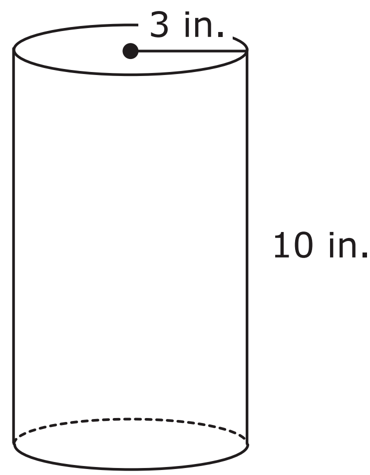
Format:
- Determine the surface area of right rectangular prisms using decomposition or nets
- Determine the surface area of a cylinder, using decomposition or nets
- Use formulas for determining volume of right rectangular prisms and right cylinders

Content Limits:
- Limit solids to rectangular prisms and cylinders

Primary Process Standards:
- Develop Strategies for Problem Solving
- Develop the Ability to Communicate Mathematically
- Develop Mathematical Reasoning
- Develop a Deep and Flexible Conceptual Understanding
- Develop the Ability to Make Conjectures, Model, and Generalize

Distractor Domain:
- Computational errors
- Incorrect procedures
- Interchange volume and surface area

17 **Jason decorated the outside of this cylinder.**



What is the surface area of the cylinder, in square inches?

A 72π square inches

B 78π square inches

C 192π square inches

D 260π square inches

Standard: PA.GM.2.2 Calculate the surface area of a cylinder, in terms of π and using approximations for π, using decomposition or nets. Use appropriate measurements such as cm^2.

Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to complete a simple procedure, finding the surface area of a cylinder.

Distractor Rationale:

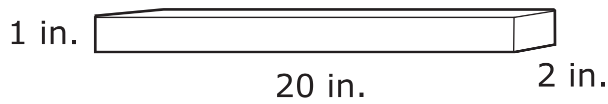
A. The student computed 3^2 as 6 instead of 9.

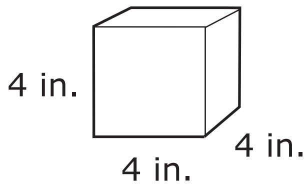
B. Correct. The student demonstrated an ability to calculate the surface area of a cylinder in terms of π.

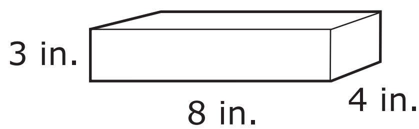
C. The student used the diameter, 6, instead of the radius, 3.

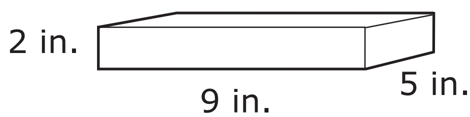
D. Balance distractor

18 **Caitlin wants to pack her craft supplies in the box with the greatest volume. Which box has the greatest volume?**

A 

B 

C 

D 

Standard: PA.GM.2.3 Develop and use the formulas $V = lwh$ and $V = Bh$ to determine the volume of rectangular prisms. Justify why base area (B) and height (h) are multiplied to find the volume of a rectangular prism. Use appropriate measurements such as cm^3.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to use the formula for the volume of rectangular prisms to find the prism with the greatest volume.

Distractor Rationale:

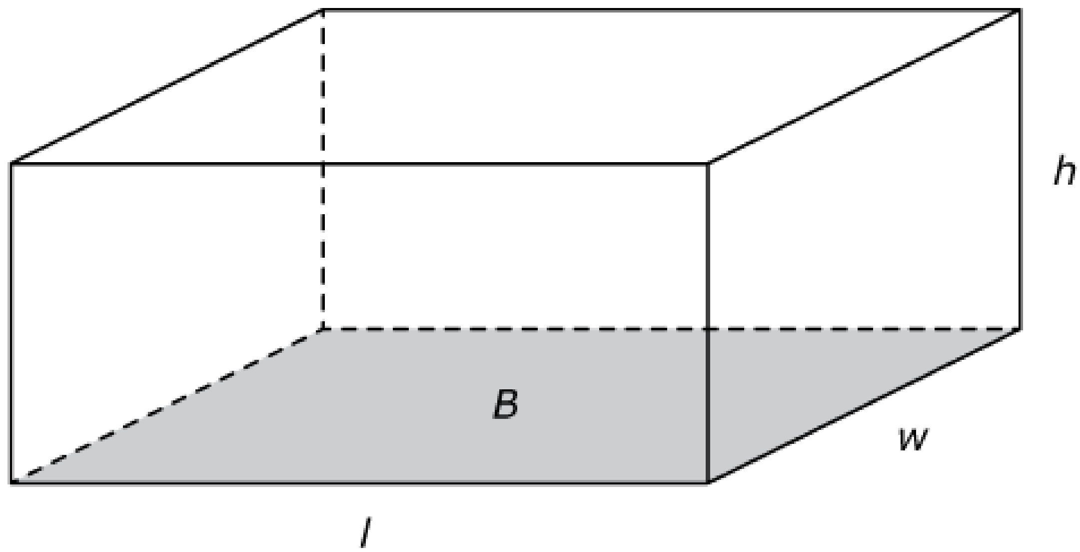
A. The student chose the prism with the greatest single side length.

B. The student thought a cube would have a greater volume than a rectangular prism.

C. Correct. The student demonstrated an ability to determine the volume of rectangular prisms.

D. The student chose the prism with the greatest width.

19 **A right rectangular prism has a base area of 24 cm^2. Its volume, in cubic centimeters, is not a whole number.**



Select the measures that could be two of the dimensions of this prism.
To select a measure, click the measure. To deselect a measure, click it again.

length = 6 cm width = 4 cm	length = 6.1 cm height = 2 cm	width = 3.9 cm height = 8.4 cm
length = 7.5 cm width = 3.5 cm	length = 12 cm height = 15.1 cm	width = 10.5 cm height = 9 cm

Standard: PA.GM.2.3 Develop and use the formulas $V = lwh$ and $V = Bh$ to determine the volume of rectangular prisms. Justify why base area (B) and height (h) are multiplied to find the volume of a rectangular prism. Use appropriate measurements such as cm^3.

Depth-of-Knowledge: 3

This item is a DOK 3 because it requires the student to use the formula for the volume of a right rectangular prism to determine possible measures for the length and width, when the exact volume is not given, but only that it is not a whole number.

Sample Distractor Rationales:

Correct

length = 6 cm width = 4 cm		width = 3.9 cm height = 8.4 cm
	length = 12 cm height = 15.1 cm	

Incorrect

length = 6 cm width = 4 cm		

The student identified only the dimensions that show an area of 24 cm^2.

	length = 6.1 cm height = 2 cm	width = 3.9 cm height = 8.4 cm
length = 7.5 cm width = 3.5 cm	length = 12 cm height = 15.1 cm	width = 10.5 cm height = 9 cm

The student selected all dimensions that included at least one non-whole number.

20 **A cylindrical container has a diameter of 6 centimeters and a height that is $\frac{5}{3}$ times the diameter of the cylinder. What is the volume of the container, in cubic centimeters?**

A 60π cubic centimeters

B 90π cubic centimeters

C 120π cubic centimeters

D 360π cubic centimeters

Standard: PA.GM.2.4 Develop and use the formulas $V = \pi r^2 h$ and $V = Bh$ to determine the volume of right cylinders, in terms of π and using approximations for π. Justify why base area (B) and height (h) are multiplied to find the volume of a right cylinder. Use appropriate measurements such as cm^3.

Depth-of-Knowledge: 3

This item is a DOK 3 because it requires the student to use the formula for the volume of a cylinder when the height is given as a fraction of the diameter.

Distractor Rationale:

A. The student thought 3^2 is 6.

B. Correct. The student demonstrated an ability to use the formula $V = \pi r^2 h$ to determine the volume of a right cylinder.

C. The student used 6 for the radius and thought 6^2 is 12.

D. The student used 6 for the radius.

OAS STRAND—DATA & PROBABILITY (D): STANDARD PA.D.1

OAS STANDARD

PA.D.1 Display and interpret data in a variety of ways, including using scatterplots and approximate lines of best fit. Use line of best fit and average rate of change to make predictions and draw conclusions about data.

OAS OBJECTIVES

PA.D.1.1 Describe the impact that inserting or deleting a data point has on the mean and the median of a data set. Know how to create data displays using a spreadsheet and use a calculator to examine this impact.

PA.D.1.2 Explain how outliers affect measures of central tendency.

PA.D.1.3 Collect, display, and interpret data using scatterplots. Use the shape of the scatterplot to informally estimate a line of best fit, make statements about average rate of change, and make predictions about values not in the original data set. Use appropriate titles, labels, and units.

ITEM SPECIFICATIONS

Emphasis:

- Demonstrate an ability to create data displays.
- Understand the impact that inserting or deleting a data point has on the mean and the median of a data set.
- Understand how outliers affect measures of central tendency.
- Estimate a line of best fit for a set of data displayed in a scatterplot and use the line to make predictions.

Stimulus Attributes:

- Test items may include graphs, scatter plots, and data tables.

Format:

- Determine the outcome on the mean and mean when a data point is inserted or deleted from a data set
- Determine the affect of outliers on the measures of central tendency
- Identify characteristics of a scatter plot
- Determine the best-fit line of a set of data
- Interpret the results of data using linear models

Content Limits:

- For lines of best fit, limit data displayed to strong positive or negative correlations
- With the exception of scatterplots, limit data sets to 20 data points
- Limit data sets to numerical data

Primary Process Standards:

- Develop Strategies for Problem Solving
- Develop Mathematical Reasoning
- Develop a Deep and Flexible Conceptual Understanding
- Develop the Ability to Make Conjectures, Model, and Generalize

Distractor Domain:

- Computational errors
- Common algebraic misconceptions

21 **Brandon used an indoor rock-climbing wall seven times. His climbing times, in minutes, are shown in this list.**

35, 16, 17, 18, 13, 13, 14

Why is the median the most useful measure of central tendency for these times?

A The median is not affected by an outlier.

B The median is equal to the range of the data.

C The median is the time that occurs most often.

D The median is a larger value than the mean of the data.

Standard: PA.D.1.2 Explain how outliers affect measures of central tendency.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to identify why the median is the best measure for a given set of data.

Distractor Rationale:

A. **Correct. The student demonstrated an ability to explain how outliers affect measures of central tendency.**

B. The student confused median and range.

C. The student defined median, but this did not answer the question.

D. The student chose an explanation that is not true for this data set, nor did it answer the question.

22 **These data show the ages of students in a community play.**

12, 9, 11, 9, 15, 11, 9, 11, 12

Which data point could be removed from the set without changing the mean or the median of the data?

A 9

B 11

C 12

D 15

Standard: PA.D.1.1 Describe the impact that inserting or deleting a data point has on the mean and the median of a data set. Know how to create data displays using a spreadsheet and use a calculator to examine this impact.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to have a deep understanding of mean and median in order to determine which data point could be removed from a set of data without changing the mean or median.

Distractor Rationale:

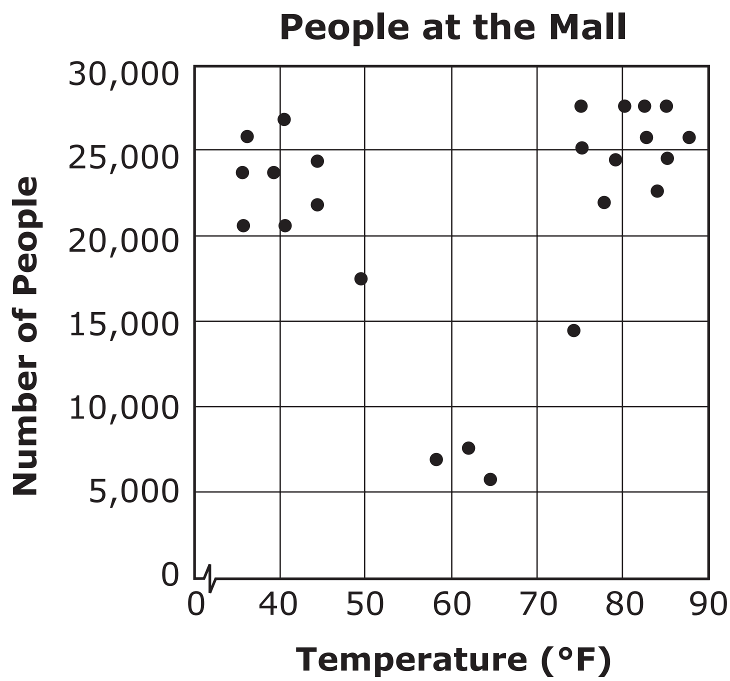
A. The student saw that this will not change the median.

B. Correct. The student demonstrated an ability to describe the impact that deleting a data point has on the mean and the median of a data set.

C. The student saw that this will not change the median.

D. The student saw that this will not change the median.

23 **This scatter plot shows the number of people at a mall each day and the average temperature for the day.**



Based on the scatter plot, which statement is true?

A The number of people at the mall always increases as the temperature rises.

B The number of people at the mall always decreases as the temperature rises.

C Fewer people are at the mall when the temperature is between 70 °F and 90 °F.

D Fewer people are at the mall when the temperature is between 50 °F and 70 °F.

Standard: PA.D.1.3 Collect, display and interpret data using scatterplots. Use the shape of the scatterplot to informally estimate a line of best fit, make statements about average rate of change, and make predictions about values not in the original data set. Use appropriate titles, labels and units.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to analyze the scatter plot and then determine which statement about the data is true.

Distractor Rationale:

A. The student does not know how to read the data displayed on the scatterplot.

B. The student saw that the data decreased from the left to the middle.

C. The student does not know how to read the data displayed on the scatterplot or confused fewer and more.

D. Correct. The student demonstrated an ability to interpret data displayed on a scatterplot.

OAS STRAND—DATA & PROBABILITY (D): STANDARD PA.D.2

OAS STANDARD

PA.D.2	Calculate experimental probabilities and reason about probabilities to solve real-world and mathematical problems.

OAS OBJECTIVES

PA.D.2.1	Calculate experimental probabilities and represent them as percents, fractions, and decimals between 0 and 1 inclusive. Use experimental probabilities to make predictions when actual probabilities are unknown.
PA.D.2.2	Determine how samples are chosen (random, limited, or biased) to draw and support conclusions about generalizing a sample to a population.
PA.D.2.3	Compare and contrast dependent and independent events.

ITEM SPECIFICATIONS

Emphasis:

- Demonstrate the ability to determine and predict experimental and actual probabilities in specified mathematical and real-world contexts.
- Explain how samples are chosen to make conclusions about a population.
- Demonstrate an ability to compare and contrast dependent and independent events.

Stimulus Attributes:

- Test items may include illustrations of coordinate graphs, number lines, tables, graphs, and charts, such as frequency charts, line, bar, or picture graphs, Venn diagrams, stem-and-leaf plots, box-and-whisker plots, scatter plots, histograms, circle graphs, data sets, spinners, and other diagrams.

Format:

- Predict the probability of the outcome of a specified event or experiment in a mathematical or real-world context based on "or," "and," or "not" statements
- Express probabilities in various forms, including decimal, fraction, and percent
- Probability can be with or without replacement
- Determine how samples are chosen
- Compare and contrast dependent and independent events

Content Limits:

- Limit sample to no more than 20 pieces of data
- Limit real-world contexts to age-appropriate situations

Primary Process Standards:

- Develop Strategies for Problem Solving
- Develop Mathematical Reasoning
- Develop the Ability to Make Conjectures, Model, and Generalize

Distractor Domain:

- Common errors
- Incorrect procedures
- Computational errors
- Incorrect use of rules or properties
- Use of incorrect equivalencies

24 **Nancy has a bag of different colored golf balls. Each ball is white, yellow, or green.**

Nancy takes a ball from the bag without looking, records the color, and then returns the ball to the bag. She repeats this process until she has recorded the color of a ball 10 times. Her results are shown.

- **White: 7**
- **Yellow: 2**
- **Green: 1**

Nancy will take another golf ball from the bag without looking. Based on Nancy's data, what is the probability she will take a green golf ball from the bag?

A $\frac{1}{10}$

B $\frac{1}{9}$

C $\frac{1}{7}$

D $\frac{1}{3}$

Standard: PA.D.2.1 Calculate experimental probabilities and represent them as percents, fractions and decimals between 0 and 1 inclusive. Use experimental probabilities to make predictions when actual probabilities are unknown.

Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to complete a simple procedure, finding the experimental probability of a simple experiment.

Distractor Rationale:

A. **Correct. The student demonstrated an ability to calculate experimental probability and represent it as a fraction.**
B. The student compared green to non-green.
C. The student compared green to white.
D. The student used 3 for the denominator because there are 3 possible colors.

25 **School cafeteria workers conduct a survey on Monday to learn how many students will want to buy pizza on Friday.**

What sample should the cafeteria workers choose for the survey?

A every other student who buys lunch

B every teacher who brings a class to lunch

C every fourth student who enters the school

D every other student who brings a lunch from home

Standard: PA.D.2.2 Determine how samples are chosen (random, limited, biased) to draw and support conclusions about generalizing a sample to a population.

Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to use a term/definition to identify the best sample for a survey.

Distractor Rationale:

A. The student thought the sample should only include students who buy lunch.

B. The student selected a sample that is not representative of the population.

C. Correct. The student demonstrated an ability to determine the best sample for a survey.

D. The student selects a sample that is not representative of the population.

26

Drag each pair of events into the table to show whether the events are dependent or independent. To place a pair of events in the table, click and hold the pair, and then drag it to the desired space. To change the classification of a pair of events, click and hold it, and then drag it to the desired space.

Dependent	Independent

- one roll of a red number cube; one roll of a blue number cube
- pulling two cards, one at a time, from a deck of playing cards without replacement
- pulling one card from a deck of playing cards and rolling a number cube
- pulling one card from a deck of playing cards, replacing it, and pulling a second card

Standard: PA.D.2.3 Compare and contrast dependent and independent events.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to analyze pairs of events and determine which are dependent and which are independent.

Sample Distractor Rationales:

Correct

Dependent	Independent
pulling two cards, one at a time, from a deck of playing cards without replacement	one roll of a red number cube; one roll of a blue number cube
	pulling one card from a deck of playing cards and rolling a number cube
	pulling one card from a deck of playing cards, replacing it, and pulling a second card

Incorrect

Dependent	Independent
one roll of a red number cube; one roll of a blue number cube	pulling two cards, one at a time, from a deck of playing cards without replacement
pulling one card from a deck of playing cards and rolling a number cube	
pulling one card from a deck of playing cards, replacing it, and pulling a second card	

The student confused independent and dependent events.

Dependent	Independent
pulling two cards, one at a time, from a deck of playing cards without replacement	one roll of a red number cube; one roll of a blue number cube
pulling one card from a deck of playing cards, replacing it, and pulling a second card	pulling one card from a deck of playing cards and rolling a number cube

The student thought that pulling two cards, with and without replacement, is always a dependent event.

27

A student randomly selected one marble from a bucket of marbles 50 times with replacement. These are the results of his experiment.

- 5 green marbles
- 8 blue marbles
- 12 red marbles
- 25 purple marbles

Based on these results, what is the probability of selecting a green, blue, red, or purple marble?

Select one button in each row to indicate the expected probability for each color. To make a selection, click a button in one of the four columns, To remove a seleced button, click the button again.

Probability of Selecting

Color	**50%**	**0.16**	**24%**	$\frac{1}{10}$
green	○	○	○	○
blue	○	○	○	○
red	○	○	○	○
purple	○	○	○	○

Standard: PA.D.2.1 Calculate experimental probabilities and represent them as percents, fractions and decimals between 0 and 1 inclusive. Use experimental probabilities to make predictions when actual probabilities are unknown.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to determine the expected probabilities, represented as percents, decimals, and fractions, of selecting different colors in a probability experiment.

Sample Distractor Rationales:

Correct

Probability of Selecting

Color	**50%**	**0.16**	**24%**	$\frac{1}{10}$
green	○	○	○	●
blue	○	●	○	○
red	○	○	●	○
purple	●	○	○	○

Incorrect

Probability of Selecting

Color	50%	0.16	24%	$\frac{1}{10}$
green	●	○	○	○
blue	○	●	○	○
red	○	○	●	○
purple	●	○	○	○

The student focused on the 5 green marbles that were selected and thought this was the same as 50%.

Probability of Selecting

Color	50%	0.16	24%	$\frac{1}{10}$
green	●	○	○	○
blue	○	○	○	●
red	○	●	○	○
purple	○	○	●	○

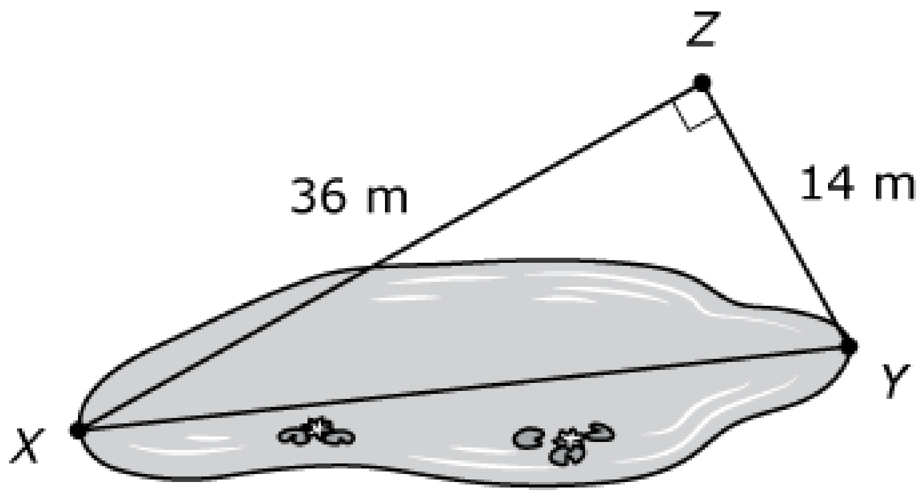
The student found the number that was close in some way to each result. 5 green means 50% because they both have 5s; 8 blue is closest to the 10 in 1/10; 12 red is close to 16 in 0.16; 25 purple is close to 24%.

Cluster Items

The following sample items are part of a cluster. The cluster is presented first and then the two items that follow require use of the cluster. The two items are from different standards.

Use this information to answer the following two questions.

Mathew wants to find the length of a pond. He picks three points and records the measurements, as shown in the diagram.



28 **Which measurement is closest to the length of the pond from point *X* to point *Y* in meters?**

A 10 meters

B 22 meters

C 39 meters

D 50 meters

Standard: PA.GM.1.1 Informally justify the Pythagorean Theorem using measurements, diagrams, or dynamic software and use the Pythagorean Theorem to solve problems in two and three dimensions involving right triangles.

Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to use the Pythagorean Theorem to determine the approximate value for the length of the hypotenuse of a triangle.

Distractor Rationale:

A. Balance distractor

B. The student computed 36 − 14.

C. Correct. The student demonstrated an ability to use the Pythaogorean Theorem to solve a problem.

D. The student computed 36 + 14.

29 **Mathew finds the deepest part of the pond to be $\sqrt{185}$ meters. Which measurement describes the depth of the pond?**

A between 13 and 14 meters

B between 14 and 15 meters

C between 92 and 93 meters

D between 93 and 94 meters

Standard: PA.N.1.5 Compare real numbers; locate real numbers on a number line. Identify the square root of a perfect square to 400 or, if it is not a perfect square root, locate it as an irrational number between two consecutive positive integers.

Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to complete a simple procedure, finding the square root of a number as between two consecutive whole numbers.

Distractor Rationale:

A. **Correct. The student demonstrated an ability to find the square root of a number as between two consecutive positive integers.**

B. Balance distractor

C. The student computed 185 ÷ 2.

D. The student computed 185 ÷ 2 incorrectly.

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