## OKLAHOMA SCHOOL TESTING PROGRAM

TEST BLUEPRINT AND ITEM SPECIFICATIONS

# GRADE 8 MATHEMATICS 

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## Grade 8 Mathematics Test

## Purpose

A robust assessment system is predicated upon the knowledge that no one assessment is able to provide answers to all questions affecting instructional decisions. An assessment system utilizes different types of assessment to gather multiple pieces of evidence to provide timely, relevant, actionable, and reliable information about what students know and can do relative to a set of standards.

## Assessments According to the Oklahoma ESSA Plan

According to page 48 of the Oklahoma ESSA Consolidated State Plan, Oklahoma recognizes that a robust assessment system is tied closely to students' learning and teachers' instructional practices by valuing and promoting local, classroom-based formative assessments that help make student learning visible. At the same time, that system should provide a strong summative assessment program that fits as a component within a multifaceted state, district, and school accountability system.


The OSDE supports an assessment system by working with Oklahoma educators and stakeholders to:

- Ensure that state and federally required annual summative assessments delivered through the Oklahoma School Testing Program (OSTP) are effective and meaningful to families, districts, educators, and members of the community;
- Develop instructional resources to support local formative and interim assessments through the curriculum frameworks projects and assessment guidance toolkit; and
- Build and deliver professional learning through face-to-face and web-based resources to support local assessment needs and interpretation of state assessment data.
Annual assessments delivered through the OSTP are aligned to the Oklahoma Academic Standards and can therefore provide point-in-time data for programmatic and curricular decisions by supporting criterion-referenced interpretations at appropriate levels and grain size (e.g., grade, student group, teacher, building/district administrator, state). Standards-based formative and interim assessments conducted at the local level can provide additional information and evidence
of learning at a smaller grain size to inform instructional decisions made at the student and classroom level.

While state summative assessments are only one measure of what students know and can demonstrate, having Oklahoma students take OSTP assessments:
$\checkmark$ Helps students, their families, and the public know how students have grown over time and how they are performing relative to the standards, their peers in Oklahoma, and the nation;
$\checkmark$ Enables teachers to see how their students are performing against grade-level expectations communicated through the Performance Level Descriptors (PLDs) to support evaluation and enhancement of curriculum and programs for the next school year;
$\checkmark$ Provides a standardized and reliable measure for school/district leaders, the state, policymakers, and the public to determine how well a system is meeting the goals of helping every child grow along a continuum to prepare them for careers, college, and life; and
$\checkmark$ Provides comparable information and data to inform continuous improvement of a system and appropriately support federal and state accountability decisions.

## Test Structure, Format, and Scoring

The Grade 8 Mathematics test will consist of 50 operational items and 10 field-test items, written at a reading level about two grade levels below a Grade 8 audience. The total 60 items will be divided into two test sections.

Each item is scored as correct or incorrect. Only the 50 operational items contribute to a student's scaled score on the test. Correct and incorrect field-test items do not contribute to a student's score.

The student's raw score is converted to a scaled score using the number correct scoring method.

## Test Alignment with Oklahoma Academic Standards (OAS)

## Criteria for Aligning the Test with the Oklahoma Academic Standards Content Strands and Standards

## 1. Categorical Concurrence

The test is constructed so that there are at least six items measuring each OAS strand. The number of items, six, is based on estimating the number of items that could produce a reasonably reliable estimate of a student's mastery of the content measured.

## 2. Range-of-Knowledge Correspondence

The test is constructed so that every standard for each OAS strand has at least one corresponding assessment item.

## 3. Source of Challenge

Each test item is constructed in such a way that the major cognitive demand comes directly from the targeted OAS strand or standard being assessed, not from specialized knowledge or cultural background that the test-taker may bring to the testing situation.

## OKLAHOMA SCHOOL TESTING PROGRAM TEST BLUEPRINT MATHEMATICS GRADE 8

This blueprint describes the content and structure of an assessment and defines the ideal range of test items by standard of the Oklahoma Academic Standards (OAS).
IDEAL
PERCENTAGE
OFITEMS
$\mathbf{1 6 - 2 0 \%}$
44-48\%

18-22\% GEOMETRY AND MEASUREMENT

14-18\%
STRANDS AND
STANDARDS
NUMBER AND OPERATIONS
, PA.N. 1 Real Number Operations
44-48\% ALGEBRAIC REASONING AND ALGEBRA
, PA.A. 1 Linear and Non-Linear Functions
, PA.A. 2 Linear Function Representations and Problem Solving
, PA.A. 3 Algebraic Expressions
, PA.A. 4 Equations and Inequalities
, PA.GM. 1 Pythagorean Theorem
, PA.GM. 2 Surface Area and Volume
DATA AND PROBABILITY
, PA.D. 1 Data Analysis and Scatter Plots
, PA.D. 2 Probability

100\%

Standards will be assessed using a combination of multiple choice items; some are linked with a common stimulus and some are technology-enhanced items.
Reporting category names are taken from the Strands and Standards named in the OAS-Mathematics.
(Please note this blueprint does not include items that may be field-tested.)

## Depth-of-Knowledge Assessed by Test Items

The Grade 8 test will approximately reflect the following "depth-of-knowledge (DOK)" distribution of items:

| Depth-of-Knowledge | Percent of Items |
| :--- | :---: |
| Level 1-Recall | $10-20 \%$ |
| Level 2-Skills/Concept | $65-75 \%$ |
| Level 3-Strategic Thinking | $15-25 \%$ |

DOK Ranges are based on the DOK of the OAS. The standards increase grade-level expectations and rigor, and set expectations for students to be college- and career-ready.

- Level 1 (Recall and Reproduction) requires the student to recall facts, terms, definitions, or simple procedures, perform simple algorithms or apply formulas. One-step, well-defined, or straight algorithmic procedures should be included at this level.
- Level 2 (Skills and Concepts) requires the student to make some decisions as to how to approach the problem or activity. Level 2 activities include making observations and collecting data; classifying, comparing, and organizing data; and organizing and displaying data in tables, charts, and graphs.
- Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking. Level 3 activities include making conjectures, drawing conclusions from observations, citing evidence and developing a logical argument for concepts, explaining phenomena in terms of concepts, and using concepts to solve nonroutine problems.
Note: These descriptions are adapted from Review Background Information and Instructions, Standards and Assessment Alignment Analysis, CCSSO TILSA Alignment Study, May 21-24, 2001, Version 2.0. For an extended description of each depth-of-knowledge level, see the web site at sde.ok.gov/sites/default/files/documents/files/Math\ WebbAlign DOK Summary Table.pdf.


## Universal Design for Learning (UDL) Considerations

Universal Design for Learning (UDL), as applied to assessments, is a framework that provides flexibility in the way information is presented and in the ways students demonstrate knowledge and skills. This reduces barriers while maintaining high expectations for all students, including students with disabilities and students who are limited English proficient. In the Oklahoma Grade 8 tests, items and instructions have been designed to provide maximum readability, comprehensibility, and legibility for all students. This includes such design aspects as reducing the language load in content areas other than Language Arts, increasing the font size, displaying fewer items per page, and boxing the items to assist visual focus.

## Test Administration Details

## Online Administration

Test questions will be presented one at a time.
Answers may be selected by using the mouse to click on the radio button to the left of the answer choice.

Navigation buttons appear at the bottom of the page for each question. For longer items, a scroll bar will appear on the right-hand side of the window to allow scrolling through the answer choices.

Tools (including a scientific calculator on the Grade 8 Mathematics assessment) appear at the bottom of the screen/page to aid in answering questions.

Students will be able to use scratch paper for all online assessments. This paper must be collected and destroyed by the test administrator immediately following the test. The test administrator must not look at what the student has written on the scratch paper.

## Paper Administration

Paper/pencil testing is used only as a testing accommodation. In the paper/pencil test booklet, any technology-enhanced items that appear in the online test form will be replaced by equivalent multiple-choice items that target the same constructs.

Students will be able to use scratch paper or blank grid paper for the paper Grade 8 Math test. This paper must be collected and destroyed by the test administrator immediately following the test. The test administrator must not look at what the student has written on the scratch paper.

## Estimated Testing Time

This section appears in all of the test specification documents and is provided to give the reader a general sense of the overall testing program at this particular grade level.

Each Grade 8 test is meant to be administered in two sessions within one day with a break given between sessions or on consecutive instructional days. Estimated time for scheduling purposes is given in the table below.

Grade 8 Mathematics Estimated Online Testing Time

| Distributing login information | Approximately 5 minutes |
| :--- | :--- |
| Test instructions/tutorial and reviewing sample items | Approximately 15 minutes |
| Total: | Approximately 20 minutes |
| Administering Section 1 of the G8 Mathematics Online Test | Approximately 40 minutes |
| Administering Section 2 of the G8 Mathematics Online Test | Approximately 40 minutes |
| Total testing time (Suggested Maximum Time: $\mathbf{2 0 0}$ minutes) | Approximately $\mathbf{1 0 0}$ minutes |

## Introduction

The test will consist of a combination of multiple-choice and technology-enhanced items.
Most stems are positively worded—avoiding the use of the word "not." If a negative is required, it is emphasized (e.g., if a bag has the same number of red, blue, and black marbles, what is the probability that a marble randomly selected from the bag is not red?).

## Multiple-Choice Item Specifications

- All items must clearly indicate what is expected in a response and direct students to focus on their responses.
- Each multiple-choice item has a stem (question, statement, and/or graphic component) and four answer options-the correct answer and three distractors. Distractors will be developed based on the types of errors students are most likely to make.
- Multiple-choice item stems ask a question or pose a clear problem so that students will know what to do before looking at the answer choices. Students should not need to read all answer choices before knowing what is expected. A stem will seldom include an incomplete sentence.


## Technology-Enhanced Item Specifications

- Technology-Enhanced Items (TEIs) should be used to more authentically address some aspects of the OAS performance expectations and/or provide more opportunity for students to construct rather than select their response.
- For each TEI, the interaction type used is that which is the most appropriate and enhancing to the construct to be measured.
- Each TEI is structured to contain the question (content) first followed by directions for how to complete the interaction in that item. Consistent style and language are used in these directions (e.g., "Drag the pictures," "Click the object," etc.).

In summary, Grade 8 test items assess whether students understand algebraic concepts and procedures, whether they can communicate their understandings effectively in mathematical terms, and whether they can approach problems and develop viable solutions.

## Stimulus Materials

Stimulus materials are the tables, charts, graphs, passages, and illustrations students must use in order to respond to items. The following characteristics are necessary for stimulus materials:

A stimulus that gives information must precede a question or a set of questions.
When students are given information to evaluate, they should know the question and the purpose of the information.
Passages, graphics, tables, etc., provide sufficient information for assessment of multiple objectives.

Stimulus materials for a set of items may be a combination of multiple stimuli.
Information in stimulus materials is based on situations students would encounter in or beyond school.

For conceptual items, stimulus materials are necessary but not conceptually sufficient for student response.

There is a balance of graphic and textual stimulus materials within a test form. Approximately 50 percent of the items will have appropriate pictorial or graphical representations. Graphs, tables, or figures are clearly associated with their intended items. The stimulus and question will appear on the screen at the same time.

## General Considerations-Oklahoma School Testing Program

Items deal with issues and details that are of consequence in the stimulus and central to students' understanding and interpretation of the stimulus.
Test items are varied and address all OAS standards listed in the Test Blueprint.
To the greatest extent possible, no item or response choice clues the answer to any other item.
All items reviewed and approved by the Oklahoma Item Review Committee are assigned an OAS strand, standard, and/or objective. The Test Blueprints and score reports reflect the degree to which each OAS strand and standard is represented on the test.
Test items are tied closely and particularly to the stimuli from which they derive, so that the impact of outside (prior) knowledge, while never wholly avoidable, is minimized.
Each multiple-choice item contains a question and four answer options, only one of which is correct. Correct answers will be approximately equally distributed among answer options.
Distractors adopt the language and sense of the material in the stimuli so that students must think their way to the correct answer rather than simply identify incorrect responses by virtue of a distractor's obviously inappropriate nature.
Distractors should always be plausible (but, of course, incorrect) in the context of the stimulus. Students should not be able to rule out a wrong answer or identify a correct response solely because it looks different from the other answer choices.
Order of presentation of item types is dictated by logic (chronologically, spatially, etc.).
Items are worded precisely and clearly. The better focused an item, the more reliable and fair it is certain to be, and the more likely all students will understand it in the same way.
The range of items measuring an OAS standard consisting of more than one skill will provide a balanced representation of those skills.
Items should be focused on what all students should know and be able to do as they complete their Grade 8 coursework.
The responses "Both of the above," "All of the above," "None of the above," and "Neither of the above" will not be used.
The material presented is balanced, culturally diverse, well written, and of interest to Grade 8 test level students. The stimuli and items are fairly presented in order to gain a true picture of students' skills.
Across all forms, a balance of gender and active/passive roles by gender is maintained.
Forms attempt to represent the ethnic diversity of Oklahoma students.
Approved calculators and the provided formula sheet may be used on the Grade 8 Mathematics test. No other resource materials may be used by students during the test. The calculator policy and formula sheet can be found at sde.ok.gov/documents/ostp-accommodation-manuals-companion-documents.

Accommodations, designated features embedded in the online testing platform, and paper-based test formats are available for students with an indicated need per their IEP or 504 Plan.
The stimuli avoid subject matter that might prompt emotional distress on the part of the students.
Permission to use stimuli from copyrighted material is obtained as necessary by testing vendor.

## Considerations Specific to the Grade 8 Mathematics Test

It is necessary to create test items that are reliable, fair, and targeted to the Oklahoma Academic Standards listed on the following pages. There are some general considerations and procedures for effective item development.

These considerations include, but are not limited to, the following:
Each test form contains items assessing all content standards.
Test items that assess each standard are not limited to one particular type of response format.
Test questions attempt to focus on content that is authentic and that Grade 8 level students can relate to and understand.
Test items are worded precisely and clearly. The better focused an item, the more reliable and fair it is likely to be, and the more likely all students will understand what is required of them.
All items are reviewed to eliminate language that shows bias or that would otherwise likely disadvantage a particular group of students. That is, items do not display unfair representations of gender, race, ethnicity, disability, culture, or religion; nor do items contain elements that are offensive to any such groups.
Items are written so that calculations are kept to a minimum, and numbers are selected to minimize the time spent on computations.
All test items and answer choices have appropriate labels and units.
Most graphs are placed on a gray grid, with the $x$ - and $y$-axes labeled and marked.

All items developed using these specifications are reviewed annually by Oklahoma educators and approved by the Oklahoma State Department of Education. The distribution of newly developed items is based on difficulty, cognitive ability, percentage of art/graphics, and grade-level appropriateness as determined by an annual Item Development Plan approved by the Oklahoma State Department of Education.

## Overview of Item Specifications

For each OAS strand, item specifications are organized under the following headings:

- OAS Strand
- OAS Standard
- OAS Objectives
- Item Specifications
a. Emphasis
b. Stimulus Attributes
c. Format
d. Content Limits
e. Primary Process Standard(s)
f. Distractor Domain
g. Sample Test Items

The headings "OAS Strand" and "OAS Standard" state the OAS strand followed by the OAS standard being measured in the mathematics section of the Oklahoma Academic Standards document.

For each standard, the information under the heading "Item Specifications" highlights important points about a test item's emphasis, format, content limits, and distractor domain. Sample test items are provided with each strand to illustrate these specifications. Although it is sometimes possible to score single items for more than one concept, all items in these tests are written to address a single objective as the primary concept.

Note: With the exception of content limits, the Item Specifications offer suggestions of what might be included and do not provide an exhaustive list of what can be included. For this reason, Item Specifications are only meant to be a supplemental resource for classroom instruction.

In addition, the sample test items are not intended to be definitive in nature or construction-the stimuli and the test items that follow them may differ from test form to test form, as may their presentations. Sample test items are not intended to predict a student's performance on the actual test, but rather to allow students to familiarize themselves with the item types and formats that they may see on the test.

## OAS STRAND-NUMBER \& OPERATIONS (N): STANDARD PA.N. 1

|  | PA.N. 1 | Read, write, compare, classify, and represent real numbers and use them to solve problems in various contexts. |
| :---: | :---: | :---: |
|  | PA.N.1.1 | Develop and apply the properties of integer exponents, including $a^{0}=1$ (with $a \neq 0$ ), to generate equivalent numerical and algebraic expressions. |
|  | PA.N.1.2 | Express and compare approximations of very large and very small numbers using scientific notation. |
|  | PA.N.1.3 | Multiply and divide numbers expressed in scientific notation and express the answer in scientific notation. |
|  | PA.N.1.4 | Compare and order real numbers; locate real numbers on a number line. Identify the square roots of perfect squares to 400 or, if it is not a perfect square root, locate it as an irrational number between two consecutive positive integers. |
| Emphasis: |  |  |
| ITEM SPECIFICATIONS | - Demonstrate an understanding of the properties of integer exponents and generate equivalent expressions that involve integer exponents. <br> - Demonstrate an understanding of scientific notation. <br> - Demonstrate an ability to multiply and divide numbers expressed in scientific notation. <br> - Demonstrate an understanding of the rational number system and the irrational number system. <br> - Demonstrate an understanding of perfect squares and square roots, and integers and approximate square roots. <br> - Demonstrate an ability to locate real numbers on a number line. <br> Stimulus Attributes: <br> - Test items may include calculator displays, tables, graphs, charts, maps, scale drawings, data sets, other diagrams, number lines, two-dimensional geometric figures, coordinate graphs, 10 -by- 10 grids, and counting manipulatives. |  |
|  |  |  |
|  | - Use the rules of exponents in mathematical and real-life contexts, including $a^{0}=1$ (with $a \neq 0$ ) <br> - Demonstrate an understanding of the magnitude of very large and very small numbers and the role of scientific and exponential notation in the representation of these numbers <br> - Use the rules of exponents in mathematical and real-life contexts to generate equivalent numerical and algebraic expressions <br> - Demonstrate the concepts of positive and negative exponents using patterns <br> - Demonstrate an understanding of scientific notation <br> - Compare, order, and translate among representations of rational numbers <br> - Demonstrate an understanding of the rational and irrational number system |  |

## OAS STRAND-NUMBER \& OPERATIONS (N): STANDARD PA.N. 1

## Format (continued):

- Use numbers to explore, estimate, and identify square roots and perfect squares in mathematical, geometric, and real-world contexts
- Use graphic representations including arrays and models of multiples to explore, estimate, and identify square roots and perfect squares in mathematical, geometric, and real-world contexts
- Use a number line to locate real numbers

Content Limits:

- Limit mathematical and real-life contexts to age-appropriate situations
- Limit to no more than two operations on exponential or scientific numbers
- Limit to integer exponents with the same base
- Limit computations involving only fractions to halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths
- Limit graphic representations to common two-dimensional geometric figures
- Limit square roots to up to the square root of 400

Primary Process Standards:

- Develop Strategies for Problem Solving
- Develop the Ability to Communicate Mathematically
- Develop Mathematical Reasoning
- Develop a Deep and Flexible Conceptual Understanding
- Develop the Ability to Make Conjectures, Model, and Generalize


## Distractor Domain:

- Computational errors
- Incorrect use of rules or properties
- Comparison errors
- Common errors
- Rounding errors
- Use of incorrect equivalencies
- Incorrect interpretation of data display

1 Which of the following is equivalent to the expression below?

$$
\frac{4^{8}}{4^{2}}
$$

A $4^{4}$
B $4^{6}$
C $4^{10}$
D $4^{16}$
Standard: PA.N.1.1 Develop and apply the properties of integer exponents, including $a^{0}=1$ (with $a \neq 0$ ), to generate equivalent numerical and algebraic expressions.

## Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to perform a simple one-step procedure, dividing with exponents.

## Distractor Rationale:

A. The student divided the exponents.
B. Correct. The student demonstrated an ability to apply the properties of integer exponents.
C. The student added the exponents.
D. The student multiplied the exponents.

2 The weight of a ruby-throated hummingbird was 3,100 milligrams.
How is this weight written in scientific notation?
A $3.1 \times 10^{-3}$ milligrams
B $3.1 \times 10^{-2}$ milligrams
C $3.1 \times 10^{2}$ milligrams
D $3.1 \times 10^{3}$ milligrams

Standard: PA.N.1.2 Express and compare approximations of very large and very small numbers using scientific notation.

## Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to perform a simple procedure, writing a given number in scientific notation.

## Distractor Rationale:

A. The student confused -3 and 3 .
B. The student counted zeroes and confused -2 and 2 .
C. The student counted zeroes.
D. Correct. The student demonstrated an ability to express numbers using scientific notation.

3 A space shuttle travels at $2.6 \times 10^{\mathbf{4}}$ feet per second. An hour is $3.6 \times 10^{\mathbf{3}}$ seconds. This expression can be used to find the number of feet the space shuttle travels in an hour.

## $\left(2.6 \times 10^{4}\right)\left(3.6 \times 10^{\mathbf{3}}\right)$

## How many feet does the shuttle travel in an hour?

A $6.2 \times 10^{1}$ feet
B $6.2 \times 10^{12}$ feet
C $9.36 \times 10^{7}$ feet
D $9.36 \times 10^{12}$ feet

Standard: PA.N.1.3 Multiply and divide numbers expressed in scientific notation and express the answer in scientific notation.

## Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to multiply numbers expressed in scientific notation.

## Distractor Rationale:

A. The student computed $2.6+3.6$ to get the base number and then computed $4-1$ to get the exponent.
B. The student computed $2.6+3.6$ to get the base number and then computed $4 \times 3$ to get the exponent.
C. Correct. The student demonstrated an ability to multiply numbers expressed in scientific notation.
D. The student computed the base number correctly, but then multiplied instead of added to find the exponent.

## OAS STRAND—ALGEBRAIC REASONING \& ALGEBRA (A): STANDARD PA.A. 1

PA.A. $1 \quad$| Explain the concept of function in mathematical situations and distinguish between |
| :---: |
| the concepts of linear and nonlinear functions. |

PA.A.1.1 $\quad$| Recognize that a function is a relationship between an independent variable and a |
| :--- |
| dependent variable in which the value of the independent variable determines the |

value of the dependent variable.

## OAS STRAND-ALGEBRAIC REASONING \& ALGEBRA (A): STANDARD PA.A. 1

## Primary Process Standards:

## ITEM SPECIFICATIONS

- Develop Strategies for Problem Solving
- Develop the Ability to Communicate Mathematically
- Develop Mathematical Reasoning
- Develop a Deep and Flexible Conceptual Understanding
- Develop the Ability to Make Conjectures, Model, and Generalize


## Distractor Domain:

- Computational errors
- Interchange range and domain
- Common algebraic misconceptions
- Conceptual errors

4 A smoothie shop makes $\$ 3.00$ for every smoothie sold after subtracting the costs of ingredients and packaging. They have additional costs of $\$ 150.00$ per day. Which linear model expresses how much the shop makes, $y$, for selling $x$ smoothies in a day?

A $y=3 x$
B $y=3 x-150$
C $y=150 x-3$
D $y=3 x+150$
Standard: PA.A.1.2 Use linear functions to represent and model mathematical situations.

## Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to translate a verbal description into an equation.

## Distractor Rationale:

A. The student ignored the fixed costs.
B. Correct. The student demonstrated an ability to write a linear function to represent a real-world problem.
C. The student reversed the slope and $y$-intercept.
D. The student added the fixed costs instead of subtracting.

## 5 A taxi company charges a base fee of $\$ 2.00$ plus $\$ 0.50$ per mile. Which of

 these represents the cost of a taxi ride, $y$, for the distance of $x$ miles?A $y=2.5 x$
B $y=50 x+2$
C $y=0.50 x+2$
D $y=2 x+0.50$

Standard: PA.A.1.2 Use linear functions to represent and model mathematical situations.

## Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to translate a verbal description into an equation.

## Distractor Rationale:

A. The student added the $\$ 2$ and the $\$ 0.50$.
B. The student confused 50 and 0.50 .
C. Correct. The student demonstrated an ability to use a linear function to represent a real-world situation.
D. The student swapped the slope and the $y$-intercept.

## OAS STRAND-ALGEBRAIC REASONING \& ALGEBRA (A): STANDARD PA.A. 2

PA.A.2 $\quad$| Identify and justify linear functions using mathematical models and situations; solve |
| :--- |
| problems involving linear functions and interpret results in the original context. |

PA.A.2.1 $\quad$| Represent linear functions with tables, verbal descriptions, symbols, and graphs; |
| :--- |
| translate from one representation to another. |

PA.A.2.2 $\quad$ Identify, describe, and analyze linear relationships between two variables.

## OAS STRAND-ALGEBRAIC REASONING \& ALGEBRA (A): STANDARD PA.A. 2

## Content Limits:

- Limit real-world and mathematical contexts to age appropriate situations
- Limit to linear equations
- Limit linear equations to rational coefficients
- Limit equations to slope-intercept form
- Limit change of slope to opposites, integers, and common fractions, or simple identification of steepness, or closer to vertical or horizontal (without calculation)
- Limit multistep processes to no more than two steps for each component stage

Primary Process Standards:

- Develop Strategies for Problem Solving
- Develop the Ability to Communicate Mathematically
- Develop Mathematical Reasoning
- Develop a Deep and Flexible Conceptual Understanding
- Develop the Ability to Make Conjectures, Model, and Generalize

Distractor Domain:

- Computational errors
- Interchange range and domain
- Common algebraic misconceptions
- Common errors
- Incorrect procedures
- Inappropriate operations with variables
- Use of inappropriate formulas

| $x$ | $\boldsymbol{y}$ |
| :---: | :---: |
| -7 | -12 |
| -5 | -8 |
| -3 | -4 |
| -1 | 0 |
| 1 | 4 |

## Which statement best describes the values of $\boldsymbol{x}$ and $\boldsymbol{y}$ in the table?

A As the value of $x$ increases by 2 , the value of $y$ increases by 4 .
B As the value of $x$ increases by 2, the value of $y$ decreases by 4 .
C As the value of $x$ increases by 4 , the value of $y$ increases by 2 .
D As the value of $x$ increases by 4 , the value of $y$ decreases by 2 .

Standard: PA.A.2.2 Represent linear functions with tables, verbal descriptions, symbols, and graphs; translate from one representation to another.

## Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to analyze the data presented in the table and then describe the relationship between $x$ and $y$.

## Distractor Rationale:

A. Correct. The student demonstrated an ability to represent a linear function with a verbal description.
B. The student identified the correct relationship for $x$ only.
C. The student reversed $x$ and $y$.
D. The student reversed $x$ and $y$ and thought $y$ was decreasing.

7 When planning a trip to the local amusement park, Leah drew a graph to show her possible costs.


## Based on the graph, what is the cost per ride?

A $\$ 1.00$
B $\$ 2.00$
C $\$ 2.60$
D $\$ 6.00$

Standard: PA.A.2.3 Identify graphical properties of linear functions including slope and intercepts. Know that the slope equals the rate of change, and that the $y$-intercept is zero when the function represents a proportional relationship.

## Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to analyze the graph to find the cost per ride.
Distractor Rationale:
A. The student saw that the values on the $x$-axis increase by 1 but did not recognize the scale of the $y$-axis is 2 .
B. Correct. The student demonstrated an ability to identify graphical properties of a linear function.
C. Balance distractor
D. The student saw that the first data point has a cost of $\$ 6$.

Match the equation in the left column to the description of the slope and $y$-intercept of its graph in the right column. To connect an equation to a description, click an equation in the left column and then a description in the right column, and a line will automatically be drawn between them. To remove a connection, hold the pointer over the line until it turns red, and then click it. Each equation in the left column matches to exactly two descriptions in the right column.

|  | slope $=-\frac{2}{3}$ |
| :--- | :--- |
| $3 x-2 y=8$ | slope $=\frac{2}{3}$ |
| $2 x+3 y=12$ | slope $=\frac{3}{2}$ |
| $3 y+6=2 x$ | $y$-intercept $=-4$ |
| -intercept $=-2$ |  |
| $y$-intercept $=4$ |  |

Standard: PA.A.2.3 Identify graphical properties of linear functions including slope and intercepts. Know that the slope equals the rate of change, and that the $y$-intercept is zero when the function represents a proportional relationship.

## Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to identify the slope and $y$-intercept of equations presented in slopeintercept form.

## Sample Distractor Rationales:

## Correct



Incorrect


The student confused $-2 / 3$ and $2 / 3$.

9 The graph of the equation $y=-3 x-5$ is shown below.


## Which best represents the graph of the equation $y=-3 x-5$ when the slope is changed to 1 and the $y$-intercept remains the same?

A

B

C

D


Standard: PA.A.2.4 Predict the effect on the graph of a linear function when the slope or y-intercept changes. Use appropriate tools to examine these effects.

Depth-of-Knowledge: 2
This item is a DOK 2 because it requires the student to use an understanding of graphs and equations to find a graph that matches an equation with a different slope, but same $y$-intercept.
Distractor Rationale:
A. The student changed the $y$-intercept to 1 .
B. The student changed both the slope and the $y$-intercept to 1 .
C. The student changed the slope to -1 .
D. Correct. The student demonstrated an ability to predict the effect on the graph when the slope changes.

10 A thermometer showed a temperature of 40 degrees Celsius. This equation can be used to find the temperature in degrees Fahrenheit, $F$, given the temperature in degrees Celsius, $C$.
$9 C=5(F-32)$
What was the temperature, in degrees Fahrenheit?
A $40^{\circ} \mathrm{F}$
B $72^{\circ} \mathrm{F}$
C $104^{\circ} \mathrm{F}$
D $360^{\circ} \mathrm{F}$

Standard: PA.A.2.5 Solve problems involving linear functions and interpret results in the original context.
Depth-of-Knowledge: 2
This item is a DOK 2 because it requires the student to solve for a variable in an equation with more than one step.

## Distractor Rationale:

A. The student identified the temperature in degrees Celsius.
B. The student computed $40+32$.
C. Correct. The student demonstrated an ability to solve a problem using a linear function.
D. The student reversed $C$ and $F$ and then multiplied by 9 instead of dividing by 9 .

## OAS STRAND-ALGEBRAIC REASONING \& ALGEBRA (A): STANDARD PA.A. 3

|  | PA.A. 3 Generate equivalent numerical and algebraic expressions and use algebraic properties to evaluate expressions. |
| :---: | :---: |
|  | PA.A.3.1 Use substitution to simplify and evaluate algebraic expressions. <br> PA.A.3.2 Justify steps in generating equivalent expressions by combining like terms and using order of operations (to include grouping symbols). Identify the properties used, including the properties of operations (associative, commutative, and distributive). |
|  | Emphasis: <br> - Simplify and evaluate algebraic expressions using substitution. <br> - Justify steps in generating equivalent expressions by identifying the properties used. <br> Stimulus Attributes: <br> - Test items may include illustrations of the following: tables, graphs, charts, data sets, equivalency statements, and algebraic expressions. <br> Format: <br> - Use variables as unknowns <br> - Substitute numerical values for variables in algebraic expressions <br> - Use the rules for order of operations with rational numbers to find the values of numerical and algebraic expressions <br> - Items may include parentheses and other grouping symbols <br> - Identify applications of the properties of operations |
|  | Content Limits: <br> - Properties of operations to include associative, commutative, and distributive laws <br> - Limit exponents to integers <br> Primary Process Standards: <br> - Develop Accurate and Appropriate Procedural Fluency <br> - Develop Mathematical Reasoning <br> - Develop the Ability to Communicate Mathematically <br> Distractor Domain: <br> - Common errors <br> - Incorrect procedures <br> - Computational errors <br> - Incorrect use of rules or properties <br> - Order of operations errors |

11 The total amount of money, in dollars, Sandy earns for working $\boldsymbol{h}$ hours is represented by this expression.

## 15h

How much money does Sandy earn for working $\mathbf{3 5}$ hours?
A $\$ 20$
B $\$ 50$
C $\$ 525$
D $\$ 1535$

Standard: PA.A.3.1 Use substitution to simplify and evaluate algebraic expressions.

## Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to complete a simple procedure, substituting 35 for $h$ in a single step expression.

## Distractor Rationale:

A. The student subtracted instead of multiplying.
B. The student added instead of multiplying.
C. Correct. The student demonstrated an ability to use substitution to evaluate an algebraic expression.
D. The student substituted 35 for $h$ and then read left to right, not understanding the implied multiplication.

12

$$
\begin{aligned}
& \text { Complete these equations. Select the numbers } \\
& \text { you want to choose and drag and drop them into } \\
& \text { the boxes. To drag a number, click and hold it, and } \\
& \text { then drag to the desired box. To change a number, } \\
& \text { click and hold it, then drag it back to the desired } \\
& \text { box. You may use each number once, more than } \\
& \text { once, or not at all. } \\
& \text { If } x=3 \text {, then } 5 x+2= \\
& \text { If } x=1 \text {, then } 2(x-4)= \\
& \text { If } x=-2 \text {, then }-3 x^{2}+4 x+15= \\
& \text { 19 }
\end{aligned}
$$

Standard: PA.A.3.1 Use substitution to simplify and evaluate algebraic expressions.
Depth-of-Knowledge: 1
This item is a DOK 1 because it requires the student to complete a simple procedure, substituting values for $x$ expressions.

## Sample Distractor Rationales:

Correct
If $x=3$, then $5 x+2=17$.
If $x=1$, then $2(x-4)=-6$.
If $x=-2$, then $-3 x^{2}+4 x+15=-5$.
Incorrect
If $x=3$, then $5 x+2=17$.
If $x=1$, then $2(x-4)=-6$.
If $x=-2$, then $-3 x^{2}+4 x+15=19$.
The student thought $(-2)^{2}$ was -4 .
If $x=3$, then $5 x+2=17$.
If $x=1$, then $2(x-4)=6$.
If $x=-2$, then $-3 x^{2}+4 x+15=19$.
The student computed (1-4) as (4-1) for the second equation.

## OAS STRAND-ALGEBRAIC REASONING \& ALGEBRA (A): STANDARD PA.A. 4

| PA.A. $4 \quad$Represent and solve problems using mathematical models and situations with <br> equations and inequalities involving linear expressions. |
| :---: | :---: | :---: |

## OAS STRAND-ALGEBRAIC REASONING \& ALGEBRA (A): STANDARD PA.A. 4

Primary Process Standards:

- Develop Strategies for Problem Solving
- Develop the Ability to Communicate Mathematically
- Develop Mathematical Reasoning
- Develop a Deep and Flexible Conceptual Understanding
- Develop the Ability to Make Conjectures, Model, and Generalize

Distractor Domain:

- Computational errors
- Common algebraic misconceptions
- Incorrectly filled or blank points on number lines
- Misdirected arrows on number lines or wrong shading of graphs


## 13 The product of 24 and $\boldsymbol{n}$ is greater than -96. Which inequality represents the possible values for $n$ ?

A $n>-4$
B $n>-120$
C $n<-4$
D $n<-120$

Standard: PA.A.4.2 Represent, write, solve, and graph problems leading to linear inequalities with one variable in the form $p x+q>r$ and $p x+q<r$, where $p, q$, and $r$ are rational numbers.

## Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to translate a verbal description into an inequality when simplifying is necessary.
Distractor Rationale:
A. Correct. The student demonstrated an ability to write a linear inequality with one variable.
B. The student confused sum and product.
C. The student reversed the inequality sign.
D. The student confused sum and product and reversed the inequality sign.

14 Tom has read 11 pages of a 215 -page book. He will read 6 pages each day until he finishes the book.

Which equation can be used to find the number of days, $\boldsymbol{d}$, it will take Tom to finish reading the book?

A $6+11 d=215$
B $11+6 d=215$
C $17 d=215$
D $6 d=215$

Standard: PA.A.4.3 Represent real-world situations using equations and inequalities involving one variable.

## Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to represent a real-world situation expressed in words using an equation.

## Distractor Rationale:

A. The student confused the slope and the $y$-intercept.
B. Correct. The student demonstrated an ability to write a linear equation for a real-world problem with one variable.
C. The student did $11 d+6 d$.
D. The student ignored the 11 pages already read.

## OAS STRAND-GEOMETRY \& MEASUREMENT (GM): STANDARD PA.GM. 1



15 A wire is attached to the top of a 5-foot tall pole. The other end of the wire is secured to the ground $\mathbf{2}$ feet from the base of the pole.


## What is the length, in feet, of the wire?

A $\sqrt{7}$ feet
B $\sqrt{14}$ feet
C $\sqrt{21}$ feet
D $\sqrt{29}$ feet

Standard: PA.GM.1.1 Justify the Pythagorean theorem using measurements, diagrams, or dynamic software to solve problems in two dimensions involving right triangles.

## Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to complete a simple procedure, using the Pythagorean Theorem to solve for the hypotenuse of a triangle.

## Distractor Rationale:

A. The student thought $a+b=c^{2}$.
B. The student thought $c^{2}=2 a+2 b$.
C. Balance distractor
D. Correct. The student demonstrated an ability to use the Pythagorean Theorem to solve a problem involving a right triangle.


What is the length of the segment connecting the points that have the coordinates (-4, -2) and (8, 3)?

A 12
B 13
C 14
D 15

Standard: PA.GM.1.2 Use the Pythagorean Theorem to find the distance between any two points in a coordinate plane.
Depth-of-Knowledge: 2
This item is a DOK 2 because the student must first realize that the Pythagorean Theorem is necessary and then utilize it to find the distance between two points on the coordinate plane.

## Distractor Rationale:

A. The student found the length of the horizontal leg.
B. Correct. The student demonstrated an ability to use the Pythagorean Theorem to the find the distance between two points on the coordinate plane.
C. The student found the difference of the two legs and then multiplied by 2.
D. The student added the lengths of the two legs.

## OAS STRAND-GEOMETRY \& MEASUREMENT (GM): STANDARD PA.GM. 2

|  | PA.GM. 2 | Justify and use formulas to calculate surface area and volume of three-dimensional figures. |
| :---: | :---: | :---: |
|  |  |  |

PA.GM.2.1 Calculate the surface area of a rectangular prism using decomposition or nets. Use appropriate measurements such as $\mathrm{cm}^{2}$.

PA.GM.2.2 Calculate the surface area of a cylinder, in terms of $\pi$ and using approximations for $\pi$, using decomposition or nets. Use appropriate units (e.g., $\mathrm{cm}^{2}$ ).

PA.GM.2.3 Justify why base area $(B)$ and height $(h)$ in the formula $V=B h$ are multiplied to find the volume of a rectangular prism. Use appropriate units (e.g., $\mathrm{cm}^{3}$ ).
PA.GM.2.4 Develop and use the formulas $V=(\pi \mathrm{r})^{2} h$ and $V=B h$ to determine the volume of right cylinders, in terms of $\pi$ and using approximations for pi $(\pi)$. Justify why base area $(B)$ and height ( $h$ ) are multiplied to find the volume of a right cylinder. Use appropriate units (e.g., $\mathrm{cm}^{3}$ ).

## Emphasis:

- Determine the surface area of rectangular prisms and cylinders.
- Understand volume of rectangular prisms is determined by the area of the base and the height.
- Determine the volume of cylinders.


## Stimulus Attributes:

- Test items may include rectangular prisms, cylinders, and other geometric solids.


## Format:

- Determine the surface area of right rectangular prisms using decomposition or nets
- Determine the surface area of a cylinder, using decomposition or nets
- Use formulas for determining volume of right rectangular prisms and right cylinders

Content Limits:

- Limit solids to rectangular prisms and cylinders

Primary Process Standards:

- Develop Strategies for Problem Solving
- Develop the Ability to Communicate Mathematically
- Develop Mathematical Reasoning
- Develop a Deep and Flexible Conceptual Understanding
- Develop the Ability to Make Conjectures, Model, and Generalize


## Distractor Domain:

- Computational errors
- Incorrect procedures
- Interchange volume and surface area

17 Jason decorated the outside of this cylinder.


What is the surface area of the cylinder, in square inches?
A $72 \pi$ square inches
B $78 \pi$ square inches
C $192 \pi$ square inches
D $260 \pi$ square inches

Standard: PA.GM.2.2 Calculate the surface area of a cylinder, in terms of $\pi$ and using approximations for $\pi$, using decomposition or nets. Use appropriate units (e.g., $\mathrm{cm}^{2}$ ).

## Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to complete a simple procedure, finding the surface area of a cylinder.

## Distractor Rationale:

A. The student computed $3^{2}$ as 6 instead of 9 .
B. Correct. The student demonstrated an ability to calculate the surface area of a cylinder in terms of $\pi$.
C. The student used the diameter, 6 , instead of the radius, 3 .
D. Balance distractor

18 A right rectangular prism has a base area of $\mathbf{2 4} \mathbf{~ c m}^{\mathbf{2}}$. Its volume, in cubic centimeters, is not a whole number.


Select the measures that could be two of the dimensions of this prism. To select a measure, click the measure. To deselect a measure, click it again.

| length $=6 \mathrm{~cm}$ <br> width $=4 \mathrm{~cm}$ | length $=6.1 \mathrm{~cm}$ <br> height $=2 \mathrm{~cm}$ | width $=3.9 \mathrm{~cm}$ |
| :--- | :--- | :--- |
| height $=8.4 \mathrm{~cm}$ |  |  |
| length $=7.5 \mathrm{~cm}$ | length $=12 \mathrm{~cm}$ | width $=10.5 \mathrm{~cm}$ |
| width $=3.5 \mathrm{~cm}$ | height $=15.1 \mathrm{~cm}$ | height $=9 \mathrm{~cm}$ |

Standard: PA.GM.2.3 Justify why base area $(B)$ and height $(h)$ in the formula $V=B h$ are multiplied to find the volume of a rectangular prism. Use appropriate units (e.g., $\mathrm{cm}^{3}$ ).
Depth-of-Knowledge: 3
This item is a DOK 3 because it requires the student to use the formula for the volume of a right rectangular prism to determine possible measures for the length and width, when the exact volume is not given, but only that it is not a whole number.

## Sample Distractor Rationales:

Correct

| length $=6 \mathrm{~cm}$ <br> width $=4 \mathrm{~cm}$ |  | width $=3.9 \mathrm{~cm}$ <br> height $=8.4 \mathrm{~cm}$ |
| :--- | :--- | :--- |
|  | length $=12 \mathrm{~cm}$ <br> height $=15.1 \mathrm{~cm}$ |  |

Incorrect

| length $=6 \mathrm{~cm}$ <br> width $=4 \mathrm{~cm}$ |  |  |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |

The student identified only the dimensions that show an area of $24 \mathrm{~cm}^{2}$.

|  | length $=6.1 \mathrm{~cm}$ | width $=3.9 \mathrm{~cm}$ |
| :--- | :--- | :--- |
|  | height $=2 \mathrm{~cm}$ | height $=8.4 \mathrm{~cm}$ |
| length $=7.5 \mathrm{~cm}$ | length $=12 \mathrm{~cm}$ | width $=10.5 \mathrm{~cm}$ |
| width $=3.5 \mathrm{~cm}$ | height $=15.1 \mathrm{~cm}$ | height $=9 \mathrm{~cm}$ |

The student selected all dimensions that included at least one non-whole number.

## 19 A cylindrical container has a diameter of 6 centimeters and a height that

 is $\frac{5}{3}$ times the diameter of the cylinder. What is the volume of the container, in cubic centimeters?A $60 \pi$ cubic centimeters
B $90 \pi$ cubic centimeters
C $120 \pi$ cubic centimeters
D $360 \pi$ cubic centimeters

Standard: PA.GM.2.4 Develop and use the formulas $V=(\pi r)^{2} h$ and $V=B h$ to determine the volume of right cylinders, in terms of $\pi$ and using approximations for pi $(\pi)$. Justify why base area $(B)$ and height $(h)$ are multiplied to find the volume of a right cylinder. Use appropriate units (e.g., $\mathrm{cm}^{3}$ ).
Depth-of-Knowledge: 3
This item is a DOK 3 because it requires the student to use the formula for the volume of a cylinder when the height is given as a fraction of the diameter.

## Distractor Rationale:

A. The student thought $3^{2}$ is 6 .
B. Correct. The student demonstrated an ability to use the formula $V=\pi r^{2} h$ to determine the volume of a right cylinder.
C. The student used 6 for the radius and thought $6^{2}$ is 12 .
D. The student used 6 for the radius.

## OAS STRAND-DATA \& PROBABILITY (D): STANDARD PA.D. 1

| 是 | PA.D. 1 | Display and interpret data in a variety of ways, including using scatterplots and approximate lines of best fit. Use line of best fit and average rate of change to make predictions and draw conclusions about data. |
| :---: | :---: | :---: |
| 0 <br> 3 <br>  <br>  <br> 0 <br> 0 <br> 0 <br> 0 | $\begin{aligned} & \text { PA.D.1.1 } \\ & \text { PA.D.1.2 } \\ & \text { PA.D.1.3 } \end{aligned}$ | Describe the impact that inserting or deleting a data point has on the mean and the median of a data set. Create data displays using technology to examine this impact. <br> Explain how outliers affect measures of center and spread. <br> Collect, display, and interpret data using scatter plots. Use the shape of the scatter plot to find the informal line of best fit, make statements about the average rate of change, and make predictions about values not in the original data set. Use appropriate titles, labels and units. |
|  | Emphasis: <br> - Demonstrate an ability to create data displays. <br> - Understand the impact that inserting or deleting a data point has on the mean and the median of a data set. <br> - Understand how outliers affect measures of center and spread. <br> - Estimate a line of best fit for a set of data displayed in a scatterplot and use the line to make predictions. |  |
|  | Format: <br> - Determine the outcome on the mean and mean when a data point is inserted or deleted from a data set <br> - Determine the affect of outliers on the measures of central tendency <br> - Identify characteristics of a scatter plot <br> - Determine the best-fit line of a set of data <br> - Interpret the results of data using linear models |  |
|  | Content <br> - F <br> - W <br> - L | its: <br> lines of best fit, limit data displayed to strong positive or negative correlations h the exception of scatterplots, limit data sets to 20 data points it data sets to numerical data |
|  |  | cess Standards: <br> elop Strategies for Problem Solving <br> velop Mathematical Reasoning <br> velop a Deep and Flexible Conceptual Understanding <br> elop the Ability to Make Conjectures, Model, and Generalize |
|  |  | Domain: <br> mputational errors <br> mon algebraic misconceptions |

## 20 These data show the ages of students in a community play.

12, 9, 11, 9, 15, 11, 9, 11, 12

## Which data point could be removed from the set without changing the mean or the median of the data?

A 9
B 11
C 12
D 15

Standard: PA.D.1.1 Describe the impact that inserting or deleting a data point has on the mean and the median of a data set. Create data displays using technology to examine this impact.

## Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to have a deep understanding of mean and median in order to determine which data point could be removed from a set of data without changing the mean or median.

## Distractor Rationale:

A. The student saw that this will not change the median.
B. Correct. The student demonstrated an ability to describe the impact that deleting a data point has on the mean and the median of a data set.
C. The student saw that this will not change the median.
D. The student saw that this will not change the median.

## 21 Brandon used an indoor rock-climbing wall seven times. His climbing

 times, in minutes, are shown in this list.$$
35,16,17,18,13,13,14
$$

## Why is the median the most useful measure of central tendency for these times?

A The median is not affected by an outlier.
B The median is equal to the range of the data.
C The median is the time that occurs most often.
D The median is a larger value than the mean of the data.

Standard: PA.D.1.2 Explain how outliers affect measures of center and spread.
Depth-of-Knowledge: 2
This item is a DOK 2 because it requires the student to identify why the median is the best measure for a given set of data.

## Distractor Rationale:

A. Correct. The student demonstrated an ability to explain how outliers affect measures of central tendency.
B. The student confused median and range.
C. The student defined median, but this did not answer the question.
D. The student chose an explanation that is not true for this data set, nor did it answer the question.

22 This scatter plot shows the number of people at a mall each day and the average temperature for the day.


## Based on the scatter plot, which statement is true?

A The number of people at the mall always increases as the temperature rises.
B The number of people at the mall always decreases as the temperature rises.
C Fewer people are at the mall when the temperature is between $70^{\circ} \mathrm{F}$ and $90^{\circ} \mathrm{F}$.
D Fewer people are at the mall when the temperature is between $50^{\circ} \mathrm{F}$ and $70^{\circ} \mathrm{F}$.

Standard: PA.D.1.3 Collect, display, and interpret data using scatter plots. Use the shape of the scatter plot to find the informal line of best fit, make statements about the average rate of change, and make predictions about values not in the original data set. Use appropriate titles, labels and units.

## Depth-of-Knowledge: 2

This item is a DOK 2 because it requires the student to analyze the scatter plot and then determine which statement about the data is true.

## Distractor Rationale:

A. The student does not know how to read the data displayed on the scatterplot.
B. The student saw that the data decreased from the left to the middle.
C. The student does not know how to read the data displayed on the scatterplot or confused fewer and more.
D. Correct. The student demonstrated an ability to interpret data displayed on a scatterplot.

## OAS STRAND-DATA \& PROBABILITY (D): STANDARD PA.D. 2

\begin{tabular}{|c|c|c|}
\hline 是 \& PA.D. 2 \& Calculate experimental probabilities and reason about probabilities to model and solve problems. <br>
\hline  \& PA.D.2.1
PA.D.2.2

PA.D.2.3 \& | Calculate experimental probabilities and represent them as percents, fractions, and decimals between 0 and 1 . Use experimental probabilities to predict relative frequencies when actual probabilities are unknown. |
| :--- |
| Determine how samples are chosen (randomness) to draw and support conclusions about generalizing a sample to a population, including identifying limitations and biases. |
| Define, compare, and contrast the probabilities of dependent and independent events. | <br>

\hline \multirow{4}{*}{} \& \multicolumn{2}{|l|}{| Emphasis: |
| :--- |
| - Demonstrate the ability to determine and predict experimental and actual probabilities in specified mathematical and real-world contexts. |
| - Explain how samples are chosen to make conclusions about a population. |
| - Demonstrate an ability to compare and contrast dependent and independent events. |
| Stimulus Attributes: |
| - Test items may include illustrations of coordinate graphs, number lines, tables, graphs, and charts, such as frequency charts, line, bar, or picture graphs, Venn diagrams, stem-and-leaf plots, box-and-whisker plots, scatter plots, histograms, circle graphs, data sets, spinners, and other diagrams. |} <br>


\hline \& \multicolumn{2}{|l|}{| Format: |
| :--- |
| - Predict the probability of the outcome of a specified event or experiment in a mathematical or real-world context based on "or," "and," or "not" statements |
| - Express probabilities in various forms, including decimal, fraction, and percent |
| - Probability can be with or without replacement |
| - Determine how samples are chosen |
| - Compare and contrast dependent and independent events |} <br>


\hline \& \multicolumn{2}{|l|}{| Content Limits: |
| :--- |
| - Limit sample to no more than 20 pieces of data |
| - Limit real-world contexts to age-appropriate situations |} <br>


\hline \& \multicolumn{2}{|l|}{| Distractor Domain: |
| :--- |
| - Common errors |
| - Incorrect procedures |
| - Computational errors |
| - Incorrect use of rules or properties |
| - Use of incorrect equivalencies |} <br>

\hline
\end{tabular}

23 Nancy has a bag of different colored golf balls. Each ball is white, yellow, or green.

Nancy takes a ball from the bag without looking, records the color, and then returns the ball to the bag. She repeats this process until she has recorded the color of a ball $\mathbf{1 0}$ times. Her results are shown.

- White: 7
- Yellow: 2
- Green: 1

Nancy will take another golf ball from the bag without looking. Based on Nancy's data, what is the probability she will take a green golf ball from the bag?

A $\frac{1}{10}$
B $\frac{1}{9}$
C $\frac{1}{7}$
D $\frac{1}{3}$

Standard: PA.D.2.1 Calculate experimental probabilities and represent them as percents, fractions, and decimals between 0 and 1. Use experimental probabilities to predict relative frequencies when actual probabilities are unknown.

## Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to complete a simple procedure, finding the experimental probability of a simple experiment.

## Distractor Rationale:

A. Correct. The student demonstrated an ability to calculate experimental probability and represent it as a fraction.
B. The student compared green to non-green.
C. The student compared green to white.
D. The student used 3 for the denominator because there are 3 possible colors.

A student randomly selected one marble from a bucket of marbles 50 times with replacement. These are the results of his experiment.

- 5 green marbles
- 8 blue marbles
- 12 red marbles
- 25 purple marbles

Based on these results, what is the probability of selecting a green, blue, red, or purple marble?
Select one button in each row to indicate the expected probability for each color. To make a selection, click a button in one of the four columns, To remove a selected button, click the button again.

Probability of Selecting

| Color | 50\% | 0.16 | 24\% | $\frac{1}{10}$ |
| :---: | :---: | :---: | :---: | :---: |
| green | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| blue | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| red | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| purple | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |

Standard: PA.D.2.1 Calculate experimental probabilities and represent them as percents, fractions, and decimals between 0 and 1 . Use experimental probabilities to predict relative frequencies when actual probabilities are unknown. Depth-of-Knowledge: 2
This item is a DOK 2 because it requires the student to determine the expected probabilities, represented as percents, decimals, and fractions, of selecting different colors in a probability experiment.

## Sample Distractor Rationales:

Correct

Probability of Selecting

| Color | 50\% | 0.16 | 24\% | $\frac{1}{10}$ |
| :---: | :---: | :---: | :---: | :---: |
| green | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | - |
| blue | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| red | $\bigcirc$ | $\bigcirc$ | - | $\bigcirc$ |
| purple | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |

Incorrect

| Probability of selecting |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Celor | $\mathbf{5 0 \%}$ | 0.16 | $\mathbf{2 4 \%}$ | $\frac{1}{10}$ |  |
| green |  |  |  |  |  |
| blue |  |  | 0 | 0 |  |
| red |  |  |  | 0 |  |
| purple |  |  |  |  |  |

The student focused on the 5 green marbles that were selected and thought this was the same as $50 \%$.
Probability of selecting

| celor | $\mathbf{5 0 \%}$ | 0.16 | $\mathbf{2 4 \%}$ | $\frac{1}{10}$ |
| :---: | :---: | :---: | :---: | :---: |
| green |  | 0 | 0 |  |
| blue | 0 | 0 | 0 |  |
| red | 0 |  | 0 | 0 |
| purple |  |  |  | 0 |

The student found the number that was close in some way to each result. 5 green means $50 \%$ because they both have 5 s ; 8 blue is closest to the 10 in $1 / 10 ; 12$ red is close to 16 in $0.16 ; 25$ purple is close to $24 \%$.

## 25 School cafeteria workers conduct a survey on Monday to learn how many students will want to buy pizza on Friday. <br> What sample should the cafeteria workers choose for the survey?

A every other student who buys lunch
B every teacher who brings a class to lunch
C every fourth student who enters the school
D every other student who brings a lunch from home

Standard: PA.D.2.2 Determine how samples are chosen (randomness) to draw and support conclusions about generalizing a sample to a population, including identifying limitations and biases.

## Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to use a term/definition to identify the best sample for a survey.

## Distractor Rationale:

A. The student thought the sample should only include students who buy lunch.
B. The student selected a sample that is not representative of the population.
C. Correct. The student demonstrated an ability to determine the best sample for a survey.
D. The student selects a sample that is not representative of the population.

## Cluster Items

The following sample items are part of a cluster. The cluster is presented first and then the two items that follow require use of the cluster. The two items are from different standards.

## Use this information to answer the following questions.

Mathew wants to find the length of a pond. He picks three points and records the measurements, as shown in the diagram.


## 26 Which measurement is closest to the length of the pond from point $X$ to point $Y$ in meters?

A 10 meters
B 22 meters
C 39 meters
D 50 meters

Standard: PA.GM.1.1 Justify the Pythagorean theorem using measurements, diagrams, or dynamic software to solve problems in two dimensions involving right triangles.
Depth-of-Knowledge: 2
This item is a DOK 2 because it requires the student to use the Pythagorean Theorem to determine the approximate value for the length of the hypotenuse of a triangle.

## Distractor Rationale:

A. Balance distractor
B. The student computed 36-14.
C. Correct. The student demonstrated an ability to use the Pythaogorean Theorem to solve a problem.
D. The student computed $36+14$.

27 Mathew finds the deepest part of the pond to be $\sqrt{185}$ meters. Which measurement describes the depth of the pond?
A between 13 and 14 meters
B between 14 and 15 meters
C between 92 and 93 meters
D between 93 and 94 meters

Standard: PA.N.1.4 Compare and order real numbers; locate real numbers on a number line. Identify the square roots of perfect squares to 400 or, if it is not a perfect square root, locate it as an irrational number between two consecutive positive integers.

## Depth-of-Knowledge: 1

This item is a DOK 1 because it requires the student to complete a simple procedure, finding the square root of a number as between two consecutive whole numbers.

## Distractor Rationale:

A. Correct. The student demonstrated an ability to find the square root of a number as between two consecutive positive integers.
B. Balance distractor
C. The student computed $185 \div 2$.
D. The student computed $185 \div 2$ incorrectly.

