Dr. Larry Gray's suggestions (July 2015)

Summary: Dr. Gray reviewed grades 6 - Algebra II of the OAS Mathematics Standards. The writing team co-chairs spoke to Dr. Gray via conference call for further clarification and his complete review of the standards draft. The following represents a summary of that conversation. He was overall complimentary of the draft he reviewed given the short timeline of the writing team. He also commented that he could see evidence of the team's reliance on the Minnesota Standards particularly in the middle grades. The majority of his comments were focused on 7th grade and Algebra II. He indicated that it was apparent that we were focusing on proportional reasoning in 7th grade but needed to strengthen that focus. In his estimation, the MN Standards had a strong focus on proportional reasoning in 7th grade and we could follow them more closely to capture that emphasis. His concerns about Algebra II were related to benchmarks that were too broad. He encouraged us to consider including fewer benchmarks and setting tight limitations on topics like polynomial equations. Further we discussed the focus on two-variables in Algebra I as well as the focus in the 2nd draft on linear equations only through Algebra I. We discussed how we could informally introduce non-linear functions in Algebra I to serve as counter examples for students to help clarify their understanding of linear equations. The review was thorough and thoughtful. His final comment was related to the coding (numbering) of our standards and benchmarks. Although not imperative, he hoped that we could align these across the grade levels if time allowed for ease of use for teachers. Many of Dr. Gray's comments are below in red.

2nd draft	Dr. Gray's suggestion	Response
6.N.3.2 Using the meaning of fractions, meanings of whole number multiplication and division, and inverse relationships to model multiplication and division of fractions and decimals using a variety of representations (e.g., fraction strips, area models, number lines, Cuisenaire rods).	6.N.3.2 is really a recommendation about pedagogy rather than a standard or benchmark.	Removed this benchmark
6.D.1.1 For a given set of data, explain and defend which measure of central tendency (mean, median, and mode) would provide the most descriptive information.	**6.D.1.1** For a given set of data, explain and defend which measure of central tendency (mean, median, and mode) would provide the most descriptive information. Is this word needed?	**6.D.1.1** Explain and justify which measure of central tendency (mean, median, and mode) would provide the most descriptive information for a given set of data.
7.A. 1.1 Create and use rules, tables, spreadsheets and graphs to describe patterns of change and make predictions and generalizations about real-world and mathematical problems; translate between any two of these representations.	**7.A.1.1** Create and use rules, tables, spreadsheets and graphs to describe patterns of change and make predictions and generalizations about real-world and mathematical problems; translate between any two of these representations. Except for the inclusion of the "spreadsheets", this is identical to the 6th grade benchmark.	**7.A.2.1** Represent proportional relationships with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another. Determine the unit rate (constant of proportionality or slope) given any of these representations.

7.A.4.1 Use properties of algebra to generate equivalent numerical and algebraic expressions containing rational numbers, grouping symbols and whole number exponents. Properties of algebra include associative, commutative and distributive laws.	**7.A.4.2** Use properties of algebra to generate equivalent numerical and algebraic expressions containing rational numbers, grouping symbols and whole number exponents. Properties of algebra include associative, commutative and distributive laws. Probably would be good to include "positive and negative" here.	7.A.5.1 Use properties of algebra to generate equivalent numerical and algebraic expressions containing positive and negative rational numbers, grouping symbols and whole number exponents. Properties of algebra include associative, commutative and distributive laws.
7.A.1 Apply mathematical actions and processes to create and use patterns, tables, graphs and rules to solve real-world and mathematical problems. **7.A.1.1** Create and use rules, tables, spreadsheets and graphs to describe patterns of change and make predictions and generalizations about real-world and mathematical problems; translate between any two of these representations. **7.A.3 Apply mathematical actions and processes to use ratios to solve real-world and mathematical problems.** **7.A.3.1** Apply the relationship between ratios, equivalent fractions and percents to solve problems in various contexts, including those involving mixtures and concentrations. **7.A.3.2** Use scaling up, scale factor, and unit rate reasoning to solve ratio and rate problems	This seems like a very cursory nod to working with proportional relationships in the Algebra section, given that it is supposed to be one of your main themes in 7th grade. Minnesota has 5 further benchmarks about proportional relationships in the Algebra section. What message do you want to convey to teachers? I realize that 7.A.3 goes in this direction, but only ratios are mentioned there, and this doesn't make it clear that a proportional relationship is really a 2-variable concept. In general, 7th grade has the same issue as 6th grade involving the avoidance of 2 variables, and this is causing some weaknesses.	**7.A.1 Apply mathematical actions and processes to understand the concept of proportionality in real-world and mathematical situations, and distinguish between proportional and other relationships.** **7.A.1.1** Understand that a relationship between two variables, *x* and *y,* is proportional if it can be expressed in the form $\frac{y}{x}=k$ or $y=kx$. Distinguish proportional relationships from other relationships, including inversely proportional relationships ($xy=k$ or $y=\frac{k}{x}$). **7.A.1.2** Understand that the graph of a proportional relationship is a line through the origin whose slope is the unit rate (constant of proportionality). Know how to use graphing technology to examine what happens to a line when the unit rate is changed. **7.A.2 Apply mathematical actions and processes to recognize proportional relationships in real-world and mathematical situations; represent these and other relationships with tables, verbal descriptions, symbols and graphs; solve problems involving proportional relationships and explain results in the original context.** **7.A.2.1** Represent proportional relationships with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another. Determine the unit rate (constant of proportionality or slope) given any of these

		representations. **7.A.2.2** Solve multi-step problems involving proportional relationships in numerous contexts. **7.A.2.3** Use knowledge of proportions to assess the reasonableness of solutions. **7.A.2.4** Represent real-world or mathematical situations using equations and inequalities involving variables and positive and negative rational numbers. **7.A.4 Apply mathematical actions and processes to use ratios to solve real-world and mathematical problems.** **7.A.4.1** Use reasoning about multiplication and division to solve ratio and rate problems. **7.A.4.2** Use proportional reasoning to solve problems involving ratios in various contexts. **7.A.4.3** Use knowledge of proportions to assess the reasonableness of solutions.
7.A.4.3 Use properties of algebra to generate equivalent numerical and algebraic expressions containing rational numbers, grouping symbols and whole number exponents. Properties of algebra include associative, commutative and distributive laws.	**7.A.4.4** Use properties of algebra to generate equivalent numerical and algebraic expressions containing rational numbers, grouping symbols and whole number exponents. Properties of algebra include associative, commutative and distributive laws. Probably would be good to include "positive and negative" here.	**7.A.5.1** Use properties of algebra to generate equivalent numerical and algebraic expressions containing positive and negative rational numbers, grouping symbols and whole number exponents. Properties of algebra include associative, commutative and distributive laws.
7.GM.3.2 Use various tools and strategies to measure the volume and surface area of rectangular prisms. Use cubic units to label volume measurements.	**7.GM.3.2** Use various tools and strategies to measure the volume and surface area of rectangular prisms. Use cubic units to label volume measurements. What about area measurements?	**7.GM.3.1** Using a variety of tools and strategies, develop the concept that surface area of a rectangular prism can be found by wrapping the figure with same-sized square units without gaps or overlap. Use appropriate measurements such as cm^2. **7.GM.3.2** Using a variety of tools and strategies, develop the concept that the volume rectangular prisms can be found by counting the total number of same-sized cubic units that fill a shape without

		gaps or overlaps. Use appropriate measurements such as cm^3.
PA.A.1.1 Identify that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable. Use functional notation, such as $f(x)$, to represent such relationships.	**PA.A.1.1** Identify that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable. Use functional notation, such as $f(x)$, to represent such relationships. (Strange wording?)	**PA.A.1.1** Recognize that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable. Use functional notation, such as $f(x)$, to represent such relationships.
PA.A.3.1 Evaluate algebraic expressions using a variety of methods including modeling and substitution.	**PA.A.3.1** Evaluate algebraic expressions using a variety of methods including modeling and substitution. (What does this mean?)	**PA.A.3.1** Evaluate algebraic expressions using a variety of methods.
A1.N.1.1 Simplify radicals with or without variables. **A1.N.1.2** Add, subtract, and multiply radicals with or without variables, and divide radicals without variables.	**A1.N.1.1** Simplify radicals with or without variables. (We avoided this word in Minnesota, with only one exception in the entire K-12 standards. In this particular benchmark, I don't think it is good.) (With variables? Is this supported elsewhere?) **A1.N.1.2** Add, subtract, and multiply radicals with or without variables, and divide radicals without variables. (I don't know what is expected here.)	**A1.N.1.1** Write square roots and cube roots of monomial expressions in simplest radical form. **A1.N.1.2** Add, subtract, multiply, and simplify square roots of monomial algebraic expressions and divide square roots of whole numbers.
A1.A.1.1 Solve equations symbolically and graphically. (this is a broad statement about equations to help group objectives together and make less laundry-list like) **A1.A.1.1a** Solve real-world problems by using first-degree equations (i.e. using monomial or binomial expressions as angle measures with vertical, complementary, supplementary angles, geometric formulas, science, or statistics).	**A1.A.1.1** Solve equations symbolically and graphically. (This is a sudden change of format ...) **A1.A.1.1a** Solve real-world problems by using first-degree equations (shouldn't you just say "linear"?) (i.e. using monomial or binomial expressions as (this phrase could be omitted) angle measures with vertical, complementary, supplementary angles, geometric formulas, science, or statistics).	**A1.A.1 Apply mathematical actions and processes to represent and solve mathematical and real world problems using linear equations (including absolute value equations) and systems of equations; interpret solutions in the original context.** **A1.A.1.1** Use knowledge of solving multi-step equations to represent and solve mathematical and real-world problems (e.g., angle measures, geometric formulas, science, or statistics) and interpret the solutions in the original context.

A1.F.2.2 Recognize the parent graph of the functions f(x)=k, f(x)=x, f(x)=abs(x), and predict the effects of transformations symbolically and graphically on the parent graph using various methods and tools which may include graphing calculators.	**A1.F.2.2** Recognize the parent graph of the functions f(x)=k, f(x)=x, f(x)=abs(x), and predict the effects of transformations symbolically and graphically on the parent graph using various methods and tools which may include graphing calculators. (Unnecessary technical term and notation.)	**A1.F.2.2** Recognize the graph of the functions f(x)=x and f(x)=|x| and predict the effects of transformations algebraically and graphically on the graph using various methods and tools which may include graphing calculators.
A1.D.1.3 Interpret graphs as being discrete or continuous based upon the context of the problem/situation.	**A1.D.1.3** Interpret graphs as being discrete or continuous based upon the context of the problem/situation. (You don't need a context to see that a graph is discrete.)	**A1.D.1.3** Interpret graphs as being discrete or continuous.
A2.N.1.2 Add, subtract, multiply, divide and simplify expressions containing rational exponents.	**A2.N.1.2** Add, subtract, multiply, divide and simplify expressions containing rational exponents. (Same issue as in A1 with radicals).	**A2.N.1.4** Add, subtract, multiply, divide and simplify radical expressions and expressions containing rational exponents.
A2.A.1.2 Solve rational equations, consider domain restrictions and extraneous solutions.	**A2.A.1.2** Solve rational equations, consider domain restrictions and extraneous solutions. (The wording is a little awkward.)	**A2.A.1.3** Solve rational equations with only one variable and limited to three or less denominators. Check for extraneous solutions.
A2.A.1.4 Solve polynomial equations using various methods and tools which may include factoring, polynomial division, and synthetic division.	**A2.A.1.4** Solve polynomial equations using various methods and tools which may include factoring, polynomial division, and synthetic division. (This is pretty open-ended. In Minnesota, we were much more restrictive.)	**A2.A.1.4** Solve polynomial equations with real roots using various methods and tools which may include factoring, polynomial division, synthetic division, graphing calculators or other appropriate technology.
A2.A.2.1 Factor polynomial and quadratics expressions involving common factors, trinomials, and differences of squares, using a variety of tools and strategies.	**A2.A.2.1** Factor polynomial and quadratics expressions involving common factors, trinomials, and differences of squares, using a variety of tools and strategies.	**A2.A.2.1** Factor polynomial expressions including but not limited to trinomials, difference of two squares, sum and difference of cubes, and factoring by grouping using a variety of tools and strategies.
A2.A.2.3 Connect various representations of a parabolic equation (vertex, factored, and standard forms) and manipulate to fit given context.	**A2.A.2.3** Connect various representations of a parabolic equation (vertex, factored, and standard forms) and manipulate to fit given context.	**A2.A.2.3** Recognize that a quadratic equation has different equivalent representations ($f(x) = ax^2 + bx + c$, $f(x) = a(x-h)^2 + k$, or in factored form) and identify the representation that is most appropriate for the situation (solving or graphing).
A2.F.1.5 Model a situation that can be described by an exponential or logarithmic function and use the model to answer questions about the situation.	**A2.F.1.5** Model a situation that can be described by an exponential or logarithmic function and use the model to answer questions about the situation. (Are there good situations for the logarithm at this level?)	Removed this benchmark

A2.F.1.7 Model a situation that can be described by a polynomial function and use the model to answer questions about the situation.	**A2.F.1.7** Model a situation that can be described by a polynomial function and use the model to answer questions about the situation. (Beyond quadratic?)	Removed this benchmark
A2.F.1.9 Model a situation that can be described by a rational function and use the model to answer questions about the situation. (Beyond simple ones like $1/x$ or $1/x^2$?)	**A2.F.1.9** Model a situation that can be described by a rational function and use the model to answer questions about the situation. (Beyond simple ones like $1/x$ or $1/x^2$?)	Removed this benchmark
A2.F.1.12 Read, interpret, and model piecewise graphs, including step functions.	**A2.F.1.12** Read, interpret, and model piecewise graphs, including step functions. (Beyond piecewise linear?)	**A2.F.1.8** Graph piecewise graphs with no more than three branches. Give a graph, analyze piecewise functions.
A2.D.1.2 Collect and use scatterplots to analyze patterns and describe linear, exponential or polynomial relationships between two variables. Using graphing technology, determine regression equation and correlation coefficients; use regression equations to make predictions and correlation coefficients to assess the reliability of those predictions.	**A2.D.1.2** Collect and use scatterplots to analyze patterns and describe linear, exponential or polynomial relationships (beyond quadratic?) between two variables. Using graphing technology, determine regression equation and correlation coefficients; use regression equations to make predictions and correlation coefficients to assess the reliability of those predictions.	**A2.D.1.2** Collect and use scatterplots to analyze patterns and describe linear, exponential or quadratic relationships between two variables. Using graphing technology, determine regression equation and correlation coefficients; use regression equations to make predictions and correlation coefficients to assess the reliability of those predictions.