Draft Oklahoma Academic Standards for Mathematics
Review and Comments

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for
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Overview and General Conclusions

The *Draft Oklahoma Standards for Mathematics* (July 1, 2015), full document and vertical progressions, were reviewed against the following criteria: rigor, content versus pedagogy, grade level appropriateness, measurability, and plain language.

The Common Core State Standards (CCSS) for Mathematics, the Virginia Mathematics Standards of Learning, experiences in public policy setting and professional judgment served as a frame of reference in reviewing the draft Oklahoma (OK) standards.

Overall, the OK **standards statements** appear to be content specific, rigorous, measurable, and written in plain language. The draft OK mathematics standards should prepare students for successful transition to college and a career.

The *Draft Oklahoma Academic Standards for Mathematics* include most of the content found in the Common Core State Standards (CCSS) and the Virginia Standards of Learning with some differences in scope and sequence. The OK standards prescribe content for pre-kindergarten students, which is consistent with Oklahoma's long-standing investment in early childhood education. The draft OK Pre-K standards appear to be age appropriate, and the content is consistent with Virginia's Foundation Blocks for Early Learning. The Virginia standards have been endorsed by early childhood professionals at all levels.

The OK **standards statements** focus primarily on academic content expected of all students. The OK standards statements are less complex

than the CCSS and are written in plainer language for the benefit of parents and the public. It appears OK made a deliberate attempt to write academic content standards, limiting the inclusion of teaching strategies within the standards statements.

It should be noted, however, the phrase—*apply mathematical actions and processes*—introduces each set of standards and may trigger questions on what is actually expected of students. The mathematical actions and processes are consistent with the process standards recommended by the National Council of Teachers of Mathematics (NCTM). However, many of the OK standards do not require all of the prescribed mathematical actions and processes for student mastery. OK may want to revisit whether this universal lead-in phrase does what is intended. A crisp and clear general statement of the standard followed by a listing of specific content and skills intended may cause less confusion to teachers, parents, and the public. The mathematical actions and processes could be described within the introduction to each set of grade-level standards.

The draft standards document says the organization of the final document will include sample problems and activities for teachers. This type of information in a policy document may initiate criticism that OK is prescribing strategies and activities that some may view as inappropriate. While this type of information is valuable to classroom teachers and curriculum specialists, parents and the public may not understand how the suggested strategies and activities are to be used. The draft OK standards are specific enough they may not need additional explanation for teachers to begin implementing. OK may want to consider creating a curriculum framework and strategies document that is separate from the standards document. Sample problems, strategies, and activities should be updated periodically and not require adoption by a policy board.

Based on the alignment of the OK standards with the CCSS and other state standards (e.g., Virginia) having been validated for college and career readiness, OK students who successfully complete the grade level and course standards should successfully transition to college and a career. To further validate this claim, the OK standards are being evaluated by the Oklahoma Department of Education, State Regents for Higher Education, the State Board of Career and Technology Education, and the Oklahoma Department of Commerce. Certification by state higher education and

workforce entities should provide OK with solid footing on which to validate college and career readiness.

More specifically, the following observations, comments, and suggestions are noted on the organization and structure of the standards, introductory statements, and grade-level and course standards.

Specific Comments and Suggestions

Introductory Statements and Structure

The organization of the draft Oklahoma mathematics standards into strands appears consistent with the broad topics recommended by the NCTM and other state and national standards used in this review. The Algebraic Reasoning and Algebra Strand might be further delineated to help parents of very young students better understand the intended content. An alternative heading sometimes used to describe this strand is Patterns, Functions, and Algebra.

The sequencing of the draft OK standards by strand, as opposed to a developmental or linear progression, may make it more difficult for teachers to translate the standards into a teaching scope and sequence. Ordering the standards numerically by strand often separates content that should be taught sequentially.

The Vertical Progressions document should be very helpful to teachers and curriculum specialists. This type of document could be maintained even if the standards were reorganized and renumbered.

As worded, the OK mathematical actions and processes could be interpreted as “developmental” outcomes versus terminal outcomes expected of mathematically literate students. It states “…mathematically literate students will develop…” which is weaker than saying, for example, “mathematically students will communicate mathematically or model mathematics or demonstrate procedural fluency.”

Also problematic is that these actions and processes are repeated verbatim in the tables for each standard and not customized to reflect age or grade-level maturity.

Oklahoma might consider restating the actions and processes into topical headings followed by descriptions. For example:

- Mathematical Procedural Fluency (not sure this is needed, but ok if needed to make a point computational fluency important)
    - [description]
- Mathematical Problem Solving
    - [description]
- Mathematical Reasoning
    - [description]
- Mathematical Disposition
    - [description]
- Mathematical Modeling
    - [description]
- Mathematical Communication
    - [description]

A cleaner policy document could be created by restricting the contents to include background information, grade-level introductions, standards, and corresponding skills and concepts. The mathematical actions and processes could be part of each introduction to the grade-level standards. For example on page 3, substitute "Following each of the standards are Sample Problems or Classroom Activities" with "Following each statement of the standard are specific skills, concepts, and procedures that amplify the standard and define the content knowledge, skills, and understandings that are expected of students."

On page 3, consider substituting "developmentally appropriate" with age appropriate or grade appropriate to avoid sensitivities to vocabulary. Then, be consistent throughout the document.

On pages 3-4, the phrase …"regardless of their personal characteristics, backgrounds, or physical challenges" could be sensitive to some groups. Consider striking this phrase and simplify the statement to say, "All students must have…." to avoid questions of what else should be considered and having to elaborate on what is meant by these descriptors.

All means all.

Be careful not to suggest in the explanation that “reasonable and appropriate” are new federal terms.

On page 4 under Guiding Principal 3, it says, “…requires teachers who have a deep knowledge of mathematics as a discipline.” While the statement is appropriate, the narrative that follows does not address teachers, only students.

It is good to see Technology addressed in the Guiding Principles. The appropriate use of technology in teaching and learning mathematics is important and should not be left to chance. Also, setting out expectations for use of technology in a policy document lays the groundwork for securing resources to support the standards. Having standards that require students to use certain technologies will support policies allowing these tools to be used on state assessments.

On page 7, consider using a stronger verb than “pursue” when describing conceptual and procedural strands. “Demonstrate” is stronger.

The layout and headings in the charts may confuse the reader as to what constitutes the standard. The table describes the right-hand column as the Mathematical Standard. However, the lead-in sentence at the top of the chart appears to be the broad statement of standard and the right-hand column is the standard amplified. It will be important to make clear both comprise the actual standard, especially when assessments are developed.

The phrase “apply mathematical actions and processes” at the beginning of each standard is vague. This lead-in phrase could weaken the student expectation in cases where there are specific procedures students should execute.

The listing of actions and processes in the left-hand column gives little guidance to teachers on what is expected of students since they are the same for each standard throughout Prek-12.

<u>Pre-Kindergarten</u>

The draft Oklahoma mathematics standards for Pre-Kindergarten, Kindergarten, and First Grade appear to be comprehensive and age appropriate. The standards should prepare students for successful entry into kindergarten and lay a strong mathematics foundation in the early childhood years.

In the Pre-K standards, OK should consider adding a data and statistics standard that is age appropriate. This strand is absent. Children are naturally inquisitive and develop opinions at an early age. They could begin to collect data (information) about questions of interests and then talk about what they found.

In PK.N.3.1, consider including "fewer and same" in this standard to start distinguishing fewer from less when finite objects are used.

Consider strengthening the measurement standards by adding an expectation that students will use, informally, measuring tools and know their names and what they measure (e.g. clocks, calendars, thermometers, rulers, measuring cups, and scales).

Kindergarten

In the Kindergarten standards, students are not asked to write the corresponding numeral representing the number of objects counted in a set.

Kindergarten includes a standard on recognizing coins. Consider extending this expectation to determine the value of a collection of pennies or nickels.

The concept of fraction is delayed until Grade 1 (1.N.3.1). Some states begin conceptual development of fractions earlier.

First Grade

In Grade 1, no explicit standard was found requiring students to recall basic facts with sums less than 18. In standard 1.N.2.2, addition and subtraction is associated with applying basic fact strategies. This might be interpreted as an alternative algorithm or imply a teaching method.

The concept of equality is not introduced until Grade 2 except for one mention of equivalence found in Grade 1 (1.N.1.8 … exploring equivalence through the use of balance scales).

Second Grade

In Grade 2, fluency with basic facts is required, but there may be confusion over what "demonstrate fluency" means versus "recall."
2.N.2.2 Demonstrate fluency with basic addition facts and related subtraction facts up to 20.

A concern may be raised that in 2.N.2.4 use of strategies are dictated and may be more important than the student outcome of adding and subtracting two-digit numbers. The standard algorithm is not in the list of strategies. Use of the general term "algorithm" could mean some alternatives not popular with the public today.
2.N.2.4 Use mental strategies and algorithms based on knowledge of place value and equality to add and subtract two-digit numbers. Strategies may include decomposition, expanded notation, and partial sums and differences.

Using the symbols less than and greater than is delayed until third grade.

Third Grade

In Grade 3, the underlined statements below may weaken the expectation that students add and subtract whole numbers using the standard algorithm. (This language is used throughout the grade levels.) Also, the phrase "using arithmetic" is vague.
3.N.2. Apply mathematical actions and processes to add and subtract multi-digit whole numbers; represent multiplication and division in various ways; solve real-world and mathematical problems using arithmetic.
3.N.2.2 Add and subtract multi-digit numbers, using efficient and generalizable procedures and strategies based on knowledge of place value, which may include standard algorithms.

It is not clear whether students are expected to know multiplication and division facts by Grade 3. The fluency standard appears in Grade 4 (4.N.1.1 Demonstrate fluency with multiplication and division facts up to 12 x 12.) Consider moving 4.N.1.1 to Grade 3 and rewording it to say "recall of basic multiplication and corresponding division facts through 12's table".

While important for conceptual development, Standards 3.N.2.4 and 3.N.2.5 do not go far enough to prepare students for success on 3.N.2.7.

3.N.2.4 Represent multiplication facts by using a variety of approaches, such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line and skip counting.

3.N.2.5 Represent division facts by using a variety of approaches, such as repeated subtraction, equal sharing and forming equal groups.

3.N.2.7 Use strategies and algorithms based on knowledge of place value, equality and properties of addition and multiplication to multiply a two-digit number by a one-digit number. Strategies may include mental strategies, partial products, the standard algorithm, and the commutative, associative, and distributive properties.

Use of the standard algorithm for multiplication is not expected until Grade 4 based on wording of the above third-grade standards and the following fourth-grade standard.

4.N.1.3 Multiply 3-digit by 1-digit or a 2-digit by 2-digit whole numbers, using efficient and generalizable procedures and strategies, based on knowledge of place value, including standard algorithms.

Fourth Grade

Standard 4.N.1.6 requires students to divide by single-digit whole numbers, using a list of strategies that does not include the standard algorithm.

The Grade 4 standards on fractions and decimals may be weak. Standard 4.N.2 and the specific expectations underneath it restrict work with fractions to like denominators. There are no operations with decimals required. Standards dealing with common multiples and factors are delayed until Grade 6.

Conversion of measurements within the same measurement system is not included in the Grade 4 standards. This review did not find conversion within measurement systems explicit until the Grade 6 standards (6.GM.3).

The Grade 4 Geometry standards do not include parallel and perpendicular lines or identification of points, lines, line segments, rays, angles, endpoints and vertices.

The Grade 4 standards introduce the concept of an unknown in an equation, which begins to lay a solid foundation for algebraic reasoning.

The draft standards do not appear to include an explicit development of the concept of equivalence in early grades. One could argue the development is implicit. Clearly, the Oklahoma standards place an emphasis on algebra for all students.

Some of the terminology used in several standards throughout the grades may be considered vague. For example, “arithmetic problems” as used in 4.A.2.2 and use of the terminology “real world and mathematical problems” in 4.A.1 may need clarification. How does one distinguish between problem types?

Probability is not introduced until the Grade 6 standards.

Grade 5

Fluency in multiplying multi-digit whole numbers using the standard algorithm only is not explicit in the OK standards for Grade 4 or 5.

The move from working with like to unlike fractions is not explicit in the OK standards. The Grade 4 standards include working with like denominators and Grade 6 includes using common multiples and greatest common factor in the context of operations with fractions. It must be assumed work with unlike fractions is intended in Standard 5.N.3.3. Otherwise, this requirement is delayed until Grade 6 Standard 6.N.1.5.

5.N.3.3 Add and subtract fractions and decimals, using efficient and generalizable procedures, including standard algorithms in order to solve real world and mathematical problems including those involving money, measurement, geometry, and data.

6.N.1.5….Use common factors and common multiples to calculate with fractions and find equivalent fractions.

There is a good treatment of mean, median and mode in Grade 5 (5.D.1).

The Grade 5 algebra standards require students to evaluate expressions and solve equations when values for variables are given. These are rigorous expectations for all students.

Volume does not appear to be developed until Grade 7 in Standard 7.GM.3.1. Classifying two-dimensional figures in a hierarchy based on properties is not explicit in Grade 5 but may be implicit in the Grade 4

Standard 4.GM.1.

Grade 6

Clarification may be needed on the use of terminology in OK Standard 6.N.3 that refers to ”requiring arithmetic”.

6.N.1.4 references prime numbers. Where is composite numbers introduced?

Expectations for operations with fractions and decimals culminate in Grade 6, which is appropriate. However, as noted in the Grade 4 and 5 comments, OK may want to revisit whether enough has been done in the earlier grades to prepare students for success.

The OK standards include a good introduction to working with expressions and equations in 5.A.2 and 5.A.3. This work is formalized in Grade 6 Standards 6.A.1 and 6.A.2.

The Grade 6 standards dealing with integers might be made more explicit as Grade 7 requires the application of integers.

The treatment of ratio, proportion, and percent is strong. The standards include an explicit development of ratio leading to proportions.

Probability is introduced for the first time in Standard 6.D.2 and expanded in Grade 7.

The OK Grade 6 Standards do not include surface area and volume. These concepts are delayed to Grade 7.

Grade 7

The OK Grade 7 standards are rigorous and broad in scope. The introduction to the Grade 7 standards states that the standards should prepare students for pre-algebra. Students who successfully complete the Grade 7 standards should be successful in Pre-Algebra.

As a policy decision, OK may want to encourage more students to take Algebra I by the end of Grade 8. If so, some content shifts will be needed in

the standards to ensure all students have the opportunity to study a comprehensive middle school mathematics curriculum by the end of Grade 7.

OK delays work with integers until Grade 7 where there is an intense treatment. OK may want to revisit whether enough has been done in Grade 6 to develop conceptual understanding.

Standard 7.GM.3 refers to surface area and volume of three-dimensional figures. However, Standard 7.GM.3.2 restricts surface area and volume to rectangular prisms. There is no mention of cylinders in these standards.

Formulas for surface area and volume are not introduced in the OK standards until Grade 8 (Pre-Algebra) standards. Delaying the use of surface area and volume formulas until Grade 8 raises a timing concern for students who may skip the Pre-Algebra course and enroll directly in Algebra I.

Probability is expanded in Grade 7 but there is no reference to the concept of dependent or independent events.

Standard 7.A.5 requires students to solve equations symbolically. Consider replacing “symbolically” with “algebraically.”

Grade 8

The introduction to the OK Pre-Algebra (Grade 8) standards states that the standards prepare students for Algebra I. This review concurs with that goal. Students who complete the OK Pre-Algebra standards should be well prepared for success in a formal Algebra I course. The Pre-Algebra Standards complete a comprehensive middle school curriculum in numbers and operations, geometry and measurement, and data and probability. Additionally, this set of standards includes much of the content found in the beginning of a formal Algebra I course.

The concern expressed above in the Grade 7 standards is repeated for Pre-Algebra. As a policy decision, OK may want to encourage more students to take Algebra I by the end of Grade 8. If so, some content shifts will be needed in the standards to ensure all students have the opportunity

to study a comprehensive middle school mathematics curriculum by the end of Grade 7.

Scientific notation is delayed until Pre-Algebra. Consider introducing scientific notation in grade 7 using positive and negative integer exponents and reinforce in grade 8 as stated in PA.N.1. The same could be done with work with square roots.

Consider moving all or part of the surface area and volume standard PA.GM.3 to grade 7 and focus the Pre-Algebra standard on solving application problems.

The Pre-Algebra standards include more content on probability but does not address independent and dependent events or compound events. Introductory probability concepts are developed in Algebra I. If students take the Pre-Algebra course prior to enrolling in Algebra I, there is less concern about the amount of probability and statistics content required in Algebra I.

Algebra I

The OK Algebra I standards are typical of a formal Algebra I course combined with an emphasis on probability and statistics content. Students who take Pre-Algebra prior to enrolling in Algebra I will have studied the introductory concepts that are assumed mastered in Algebra I.

The sequencing of the Algebra I standards may flow better as a linear progression of content and skills versus being organized by strands.

The Algebra I standards require use of technology in creating and analyzing graphical representations of data. It is good to see the use of technology required as part of the content standards.

Algebra II

The OK Algebra II course standards are comprehensive and define a rigorous Algebra II course. Some teachers may question the amount of content in the course. If so, matrices might be delayed until the study of a pre-calculus course.

It is not clear whether all students must take a separate trigonometry course. The unit circle is not included in the Algebra II standards. If covered elsewhere, that is fine. There does not appear to be an option for students to combine Algebra II and Trigonometry courses.

The OK Standard A2.D.1.1 provides a slight treatment of normal curve but work with combinations, permutations, and z-scores are not explicit in the Algebra II standards.

A2.D.1.1 Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve) and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate.

Geometry

The OK Geometry standards are comprehensive and rigorous. Theoretical proofs are required but expectations are reasonable for all students.

The Geometry standards cover right-triangle trigonometry but not the unit circle. It is not clear whether all students must take a separate trigonometry course. The unit circle is not included in the Algebra II standards.