South Central Comprehensive Center (SC3) Feedback on
Oklahoma Academic Standards for Mathematics
Draft Standards Released on July 1, 2015

Comments are noted as either "(Standards)," meaning they directly relate to something that should be considered or addressed prior to State Board adoption, or "(Instruction Guide)," meaning they are something that could be considered as instructional resources for teachers when implementing the standards, but do not need immediate attention.

General Comments

- We would like to acknowledge the apparent thoughtfulness and seriousness with which the writing committee members approached their task. The hard work is evident in the standards themselves, as well as in the other components of the document. All comments provided are only intended to assist with further refinement of what is already a strong set of standards for the state of Oklahoma.

- (Standards) Please provide clarity on the terminology of "standards" and how to read the relationships between various statements on each page.
  - Each page includes a statement at the top that appears to be a standard, and it also includes one or more additional statements under the heading "Mathematical Standard." We understand the writing committee has used language of "standard" for the words at the top of the page (e.g., 5.A.1) and "sub-standard" for the list below (e.g., 5.A.1.1). This should be explained in the introduction to the document. Consider revising the heading "Mathematical Standard" for clarity.
  - Some of the standards (e.g., 3.GM.1; A1.F.3) are not fully fleshed out within the sub-standards underneath, and some of the sub-standards (e.g., 4.GM.3.5; 5.A.1.2) are not clearly aligned to the standard above. These will be noted in the specific content standard comments below.

- (Instruction Guide) The general layout of the document is helpful for seeing relationships. We believe teachers will find great value in seeing sample problems or classroom activities immediately associated with the standards to be mastered by students in that grade level.
  - That being said, it is important to distinguish for teachers, administrators, and the general public the difference between "the standards" and "instructional resources" or "instructional guidance." Perhaps the document should be divided into two sections – one that is only the standards, and one that includes the instructional guidance.
  - Instructional guidance is something that can be further defined and fleshed out throughout the implementation process and not needed prior to adoption of the standards. In fact, instructional guidance may prove to be a hindrance to adoption of the standards in some instances.
  - Components of the instructional guidance could include some of the comments detailed below relating to mathematical actions and processes, relationships

between and among standards, and visual representations to assist in curriculum development and concept connections.

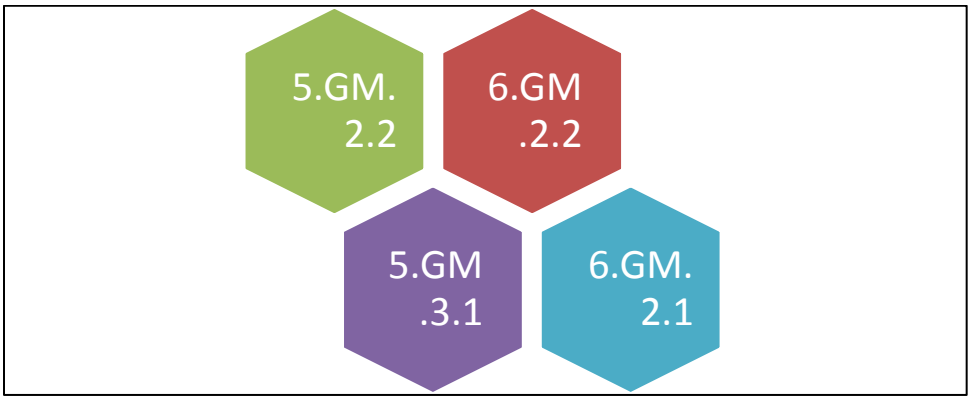
- (Instruction Guide) Inclusion of the Mathematical Actions and Processes throughout the document is to be commended! However, it is unclear why the exact same language is listed on each page with no variation based on the standard or grade level. The section of Mathematical Actions and Processes on each page could be significantly more beneficial if the language was customized to the standard.
  - There are at least three options for this:
    1. Eliminate the Mathematical Actions and Processes that are not closely aligned to the Mathematical Standard;
    2. Customize the language of each Mathematical Action and Process to the grade level, although this may not provide the clarity teachers would ultimately need within a given grade or course; or
    3. Customize the language of each Mathematical Action and Process to the Mathematical Standard.
       - For example, on 4.N.1, instead of "Develop Mathematical Reasoning," the statement could read "Explain mathematical reasoning of strategies and algorithms based on knowledge of place value, equality, and properties of operations."
       - Another example, on 7.GM.2, instead of "Develop the Ability to Make Conjectures, Model, and Generalize," the statement could read "Make conjectures, model, and generalize the relationship between the diameter and circumference of any circle based on exploration with a limited number of circles."
       - Many of the standards that have the word "develop" in them (e.g., 7.GM.3.1) would align nicely with "Develop a Deep and Flexible Conceptual Understanding." (See later note about instructional language.)
  - Again, this is a piece that could be fleshed out over the months and years ahead and does not have the immediacy of writing the actual standards.

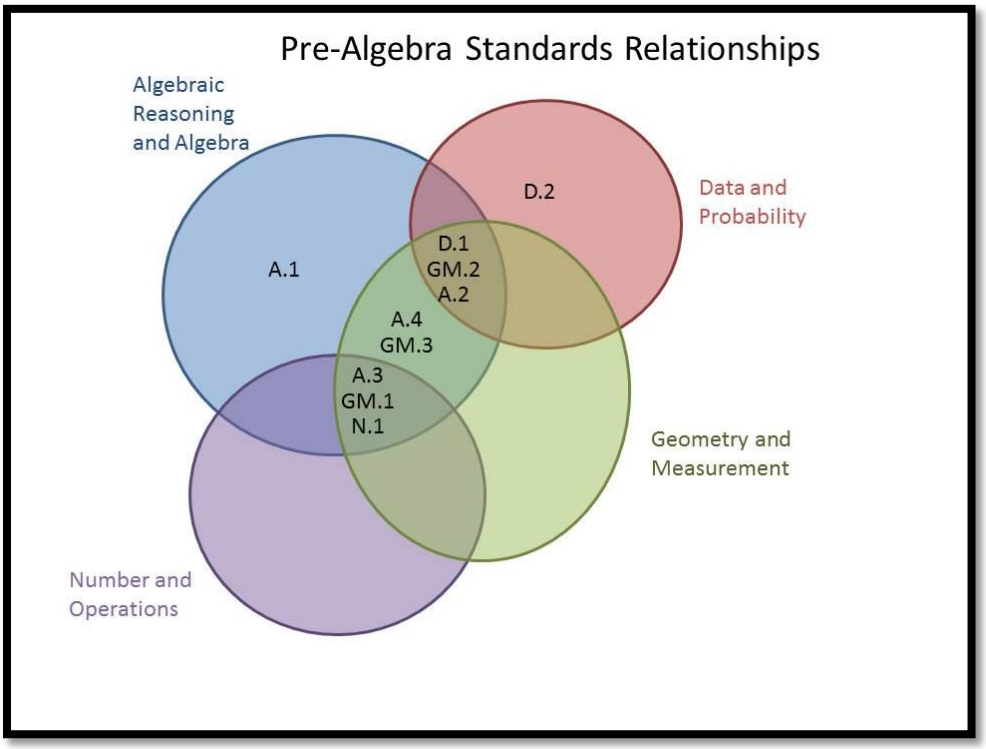
- (Instruction Guide) Consider finding an appropriate way to show relationships between and among standards within the same grade level, as well as extensions of previously mastered standards.
  - Same-grade-level connection examples include 1.GM.3.1 and 2.GM.3.1 as currently written could connect time to fraction standards (1.N.3 and 2.N.3, respectively); 3.N.2.5 could connect division to fraction standards (3.N.3); and 6.GM.2.2 could connect to solving algebraic equations (6.A.3).
  - In some places, without the connection between standards, the mathematics seem irrelevant. For example, A2.N.3 (computation on matrices) connects to A2.A.1.6 (solving systems of equations). Without the connection, what is the purpose of matrices? Also, A2.A.2 (factoring/equivalent expressions) connects to A2.F.1 (modeling situations and finding solutions). Without the connection, there is no real-world situation for equivalent expressions.
  - Extension on prior knowledge examples include 5.GM.3.1 (classifying angles) is built upon in 6.GM.2.1 (angle relationships), and A1.F.3 (computations on functions) is built upon in A2.F.3 (composition of functions and inverses).

- Perhaps the layout of the page could include a section for Related Standards.

Geometry and Measurement 6.GM.2 Apply mathematical actions and processes to understand and use relationships between and among angles in geometric figures.
Related Standards: 6.A.3, …. Extension of Standards: 5.GM.2, 5.GM.3, …

- Alternatively, a visual or two at the beginning of each grade level could show how standards might be taught in conjunction with one another, and how standards build on prior knowledge. (The following rough sketches are only that and could be designed in the future in more elaborate and meaningful ways.)





- (Standards) There are a few places throughout the standards where the language seems to be more instructional (curriculum- or classroom activity-based) rather than rooted in outcomes or student expectations. Some instances are minor (e.g., 2.A.2.2 and 2.N.1.5) and are perhaps intended as limitations or clarifications of the standards. Others are more obtrusive (e.g., G.3.1) and could be interpreted as the state dictating teacher practice.
  - The words "develop" (2.GM.1), "discover" (G.3.1), "explore," and "investigate" are all words that seem to indicate classroom activities rather than student knowledge and skills by the end of the grade or course.
  - It seems the student expectation is to know and be able to apply a theorem, property, or relationship, and the best way to acquire that knowledge is through discovery, investigation, or exploration.
  - The clarifier of "introduction… but not mastery" (2.A.2.2, 3.A.2.2, and 4.A.2.1) is also bleeding into the curriculum/classroom activity arena rather than student outcomes. Perhaps this should be in notes in the Instruction Guide rather than in the standards.

- (Standards) Throughout the standards, when "strategies and algorithms" or "efficient and generalizable procedures… including standard algorithms" are expected, should students be able to explain the strategy/procedure/algorithm? This is not stated. The standards only expect students to use them. Perhaps this is an example of how Mathematical Actions and Processes could be differentiated throughout the document. Unless a student can explain them, how do we know they have the conceptual understanding of the strategy? They may have only memorized it, or even used a different strategy unbeknownst to anyone reviewing their work.

- (Standards) There are a few places where the same knowledge/skill is expected year after year with only incremental changes from the prior year(s). Rather than teaching the concept to a level of depth that did not require additional repetition the following year, some standards documents have unnecessarily and unproductively made minor (and sometimes arguably inconsequential) variations in subsequent years. For example, if a student understands fractional relationships, measuring to 1/4-inch, 1/8-inch, and 1/16-inch are essentially the same process. Rather than repeating it in subsequent years with only this small level of increased rigor, perhaps it is better to expect depth of understanding in the most appropriate year and not require it be introduced or "re-mastered" in other years.
  - This is most often noticed in the Geometry and Measurement strand (e.g., 1.GM.3.1, 2.GM.3.1, and 3.GM.3.1 about time; 4.GM.3.5 and 5.GM.3.2 about length); but it is also apparent in other sections (e.g., 2.N.1 and 3.N.1 about place value).

- (Standards) The sixth grade content seems light. In the next section, we suggest a few extensions to help the content go deeper.

- (Instruction Guide) On the order of the strands within a grade level, is there a reason that sometimes Number and Operations comes before Algebraic Reasoning and Algebra (i.e., Algebra 1, Algebra 2, and Appendix C) when the other times Algebraic Reasoning and Algebra comes before Number and Operations (i.e., PK-7 and Pre-Algebra)? The inconsistency seems to imply meaning that may not actually exist.

- (Instruction Guide) Throughout the document, there is inconsistency in the font style for variables. It appears some were created in Equation Editor, which might automatically italicize. Others are not italicized.

- (Standards/Instruction Guide) Please provide a glossary of specific mathematical terms that do not have widely used common definitions, for example, "dot plot." Consider including suggested mathematical vocabulary for each grade level. While this is most closely related to instructional resources, having this tool available - or knowing that it will be available – could change the language of the standards so as not to have to deal with this directly in some of the standards and sub-standards. (For example, rather than state the vocabulary is not expected in the standard or sub-standard itself, this could be an Instruction Guide note.)

- (Instruction Guide) Consider adding an explanation of which formulas should be memorized and which formulas should always be available as references.

- (Instruction Guide) Consider adding suggested use of tools such as computers, graphing entities, and calculators as an instructional resource rather than including this in the standards themselves.

(Standards) Grade Level- or Standard-Specific Conceptual Comments

Progression

- 1.GM.3.1, 2.GM.3.1, and 3.GM.3.1: What is the conceptual rationale for separating half hour (1st grade), quarter hour (2nd grade), and minute (3rd grade), particularly in reference to digital clocks? When looking at a clock reading 3:17, we would not say 3:30. The estimation/rounding skill of saying that it is just past 3:15 may be valuable, but that is not expressed in this standard. This seems to be an ideal place to use depth rather than breadth by doing all of digital in 1st grade, then connecting digital to analog at the hour, half-hour, and quarter-hour in 2nd grade (fraction concepts) and to analog at the minute in 3rd grade (application of fractions).
- 2.N.1 and 3.N.1: Some research indicates that doing the same processes with additional digits year after year negatively impacts the understanding of place value. What is the rationale for these two standards not occurring in the same learning process? If there is a conceptual understanding change between the two grade levels, it is not clear from the standards/sub-standards.

Related Standards

- 2.N.2: Why is this standard focused on application of only one- and two-digit addition and subtraction when 2.N.1 requires conceptual understanding of 3-digit numbers? The process (conceptually and through the standard algorithm) is the same with two digits and with three digits. It might be more reasonable to connect the two standards so both can be taught at once with depth of conceptual connection.
- 3.N.2.7: Why are students asked to use standard algorithms for multi-digit multiplication without first demonstrating fluency of single digit facts and fact families? It seems they would need to use fluency of single digit facts to efficiently use standard algorithms, or almost any other strategy for that matter.

Standard/Sub-Standard Relationship

- 3.A.2 and 4.A.2: It appears addition and subtraction should be added to the standard language since they are included in the sub-standard language.
- 3.GM.1: Why use the broad term "shapes" if only "lines" are meant? If students are expected to describe and create a variety of shapes, even though lines are the only shapes specifically mentioned below, then the broader term should stay in the standard and additional sub-standard(s) should be included.

Grade 4

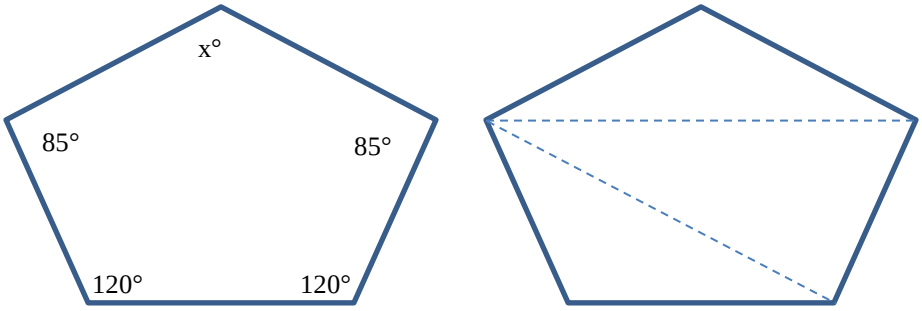
- 4.N.3: Is this standard intended to only be about the value of coins, not including bills? Also, is it intended to focus on the value of the coins and not on the monetary transactions? The sub-standard appears to indicate differently.
  - Consider revising the standard to read: "Apply mathematical actions and processes to solve monetary transactions."
  - 4.N.3.1 could remain the same.
  - Consider adding 4.N.3.2 to read: "Determine which combination of bills and coins could be used to equal a given monetary value."
  - Would it then also be appropriate to add 4.N.3.3, which connects the two by applying to real-world situations?
- 4.GM.3: Two of the sub-standards (4.GM.3.5 and 4.GM.3.6) do not appear to align with this standard. Consider adding 4.GM.4 as a new standard for these two.
- 4.D.1.1: Is "dot plot" used here different than "line plot" used in earlier grades? If not, consider using line plot throughout for consistency.

Grade 5

- 5.A.1: Introduction of the coordinate system in 5.A.1.2 is very important. However, it seems to be an afterthought, since it is not included in the standard language, but only in the sub-standard language.
  - Consider inserting a new sub-standard, or even standard, about understanding the coordinate system for displaying algebraic relationships. Perhaps it should include identification of the origin, axes, and coordinates.
  - It might be beneficial for students to understand there are four quadrants to the coordinate plane even if they are only going to graph in Quadrant 1 for the time being.
- 5.N.1.3: Why is this limited to whole numbers? This might be an appropriate place to show sustained knowledge of prior-grade mastery in decimals. There are also strong connections here to 5.N.1.2 and 5.N.3.3 if fractions and/or decimals are included.
- 5.GM.2.2: Consider adding "or combinations of triangles and parallelograms" to the end. This digs deeper into the conceptual understanding of how shapes are composed and may be a faster calculation process than subdividing the parallelograms into triangles.
- 5.GM.3.2: The only extension over 4.GM.3.5 is 1/4-inch to 1/16-inch. Why split this over two years? What is the developmental rationale? Since length was introduced to the nearest whole or 1/2-unit in 3.GM.2.1, perhaps students can make the progression to 1/4, 1/8, and 1/16 in the same learning process, either in 4th grade or 5th grade.
  - If 1/16 is too complex for 4th grade and needs to be included in 5th grade, consider rewording the sub-standard to read: "Choose an appropriate instrument… centimeter or 1/16-inch in order to solve problems that involve length." This would make a much stronger connection to the perimeter (prior year) and area (current year) standards.
- 5.D.1: Should mode also be included in the standard language?

Grade 6

- 6.N.2.1: The last sentence discusses comparing quantities using subtraction. It is unclear what is meant. Is this referencing <, >, etc.? If so, this should be explained. If not, this should also be explained.
- 6.GM.1: The list of sub-standards appears to be incomplete. There is no mention of perimeter, real-world problems, or two-dimensional figures other than quadrilaterals.
- 6.GM.2.2: To address the concern that 6th Grade is light on content, here are two suggestions:
  - Consider adding the phrase "Solve equations to…" at the beginning of the sentence to make a connection to 6.A.3.
  - Consider adding the following sentence to the end, which would extend knowledge: "Determine missing angle measures in quadrilaterals and other polygons that can be subdivided into triangles." There is no need to know or use the formula for the sum of the angles of n-sided polygons at this stage, nor is it necessary for these to be complex missing angle measure problems.
    - Example:



Since there are 3 triangles, the total must be 3 x 180= 540.
540-(120+120+85+85)=x

- 6.GM.3: Consider adding estimation of conversions between measurement systems to determine reasonableness. This would likely require a new 6.GM.3.2 that reads: "Use benchmark equivalencies between measurement systems to determine reasonableness of conversions of weight, capacity, and length." Alternatively, the language could be only about comparing measures rather than converting.
  - Example 1: Julio knows 1 liter is a little larger than 1 quart. He determined there must be about 3 liters in one gallon. Explain whether Julio's argument makes sense.
  - Example 2: Since 1 yard is a little shorter than 1 meter, about how many inches are in 5 meters? (a) 150 inches, (b) 200 inches, or (c) 500 inches
- 6.D.2.2: Consider replacing "know that" in the last phrase with "understand why."
- 6.D.2.3: Consider adding a new sub-standard (see notes on sample problems for 6.D.2.2) that reads: "Conduct experiments for situations in which the probabilities are unknown to make inferences about the sample space."

Grade 7

- 7.GM.2.3: Consider adding a new sub-standard that reads: "Solve equations to find a missing diameter or radius when circumference is given." (7.A.5 could then be listed as a related standard.)
- 7.D.1: Should the standard include "collect?" If so, the statement would read: "Apply mathematical actions and processes to collect, display, and interpret data in a variety of ways, including circle graphs and histograms."

Pre-Algebra

- PA.A.4: Should the standard language limit the equations to be solved and graphed to linear? If so, the next to last sentence would read: "Solve and graph linear equations and inequalities…"
    - Part of PA.A.4.3 appears to be duplicative of PA.A.4.1. They both say to "solve multi-step [linear] equations in one variable" (although PA.A.4.3 does not have linear in the phrase).
    - PA.A.4.6 appears to be duplicative of PA.A.4.4. They both require solving and graphing linear inequalities with one variable.
- PA.A.4.7: This sub-standard seems out of place in PA.A.4. Should it be with PA.A.3 or perhaps PA.N.1? Everything else in this standard is linear.

Algebra 1

- A1.A.3: Is "mathematical change" the same as "rate of change?" If so, that may be more communicative.
- A1.F.3: Is inverse appropriate at this level of understanding of functions? It is not mentioned in the sub-standards, and the explanation of inverses at Algebra 2 requires an understanding of composite functions. Should inverse be left completely for Algebra 2?
- A1.D.1: The sophistication of these standards is nice!

Geometry

- G.3.2 and G.3.3: Are these intentionally separated? It appears that if angles were added to G.3.3, there would be no need for G.3.2. If they are intentionally separated, why are perimeter and area left out of G.3.2?
- G.4.2: The meaning of "them" is ambiguous.

Algebra 2

- A2.N.2: Should this standard have a sub-standard about understanding the purpose and use of complex numbers?

(Instruction Guide) Sample Problems or Classroom Activities Comments

- The last question in the sample problem for 3.A.1.3 might be more effective as "What patterns do you notice?" (open-ended) as opposed to the current language "Do you notice any patterns?" (closed, and could be answered correctly with "NO" or "YES").
- Consider a classroom activity that connects 3.N.2.5 (division as equal sharing and forming groups) with 3.N.3.1 and 3.N.3.2 (read, write, and construct fractions).
- In the example for 6.A.1.3, "Use y=2x…" should begin a new example; however, it appears to be a continuation of the prior example.
- The example for 6.D.2.2 is when the sample space (and therefore the probabilities) is unknown. 6.D.2.2 is about when the probabilities are known. See conceptual comments for how this could be remedied with a new sub-standard.
- In the example for A2.N.2.2, why are there values in the blue box if the task is to have a value of $4-i$ before entering the blue box?

(Standards/Instruction Guide) Grammar Comments

- On Page 3 and throughout, there is inconsistency on the use of a comma before "and" in a list of three or more items.

- On Page 9 and each grade level introductory page, there is an "and" missing in the list in the last paragraph. The statement should read: "crucial to a student's understanding, appreciation of, and disposition for the subject."
- On Page 13 and throughout, punctuation should be inside quotation marks at the end of a sentence or phrase.
- In PK.GM.2.2, the verb should be compare, not compares.
- In 2.A.2.2, 3.A.2.2, 4.A.2.1, and perhaps in other places not noted, there is a clarifying phrase in parentheses that this standard is an introduction without mastery of vocabulary. This phrase is a sentence fragment. Consider whether an asterisk or superscript and note would be more appropriate. Is this language that should be in the "standard" or only in the "instruction guide?"
    - There is a deeper concern that regular use of introductions as standards bleeds into the curriculum/classroom activity arena rather than student outcomes. Be sure the student knowledge and skills at this level are clearly articulated.
- In 2.N.1.5, 3.N.1.4, and perhaps in other places not noted, there is a sentence fragment as the last sentence. Again, consider the use of a note here.
- In 2.GM.1, wording should read: "Apply mathematical actions and processes to analyze attributes of two- and three-dimensional figures and develop generalizations about their properties." There is an extra word ("standards") and a missing word (perhaps "and").
- There is a missing period in 2.D.1 (it currently says 2.D1) and an extra period at the end of the sentence. The period is also missing in each of the sub-standard labels (e.g., 2.D.1.1, not 2.D1.1).
- There is a missing period in 3.D.1 and 4.D.1 and in each of the sub-standard labels on those pages.
- In 4.A.2, there is a missing "and" between multiplication and division. See note on conceptual comments about adding addition and subtraction.
- 4.GM.2 is missing a verb. Perhaps "describe" is the intended verb, but it is unclear.
- 5.N.3.2 and 6.N.3.2 should begin with "Use" instead of "Using."
- 5.D.1 has an extra period at the end of the sentence.
- In 6.GM.2 and 6.GM.2.1, should the word "between" be "among?" Or is it "between and among?"
- PA.A.2.2 and PA.A.2.5 appear to have a couple of extra spaces. Also, PA.A.2.2 may need different punctuation in the parentheses to separate the two examples (a semi-colon, perhaps).
- PA.N.1.4: "Or" should be replaced by "and." Consider rewording the statement so there is not a conditional phrase. The second sentence could read as follows: "Identify the square root of a perfect square to 169, and locate the real number square root of a non-perfect square between two consecutive positive integers." (There may be better wording for this if it is divided into two sentences.)
- PA.GM.1.1: Consider separating into two sentences. "Informally justify… computer software. Use the Pythagorean Theorem… triangles."
- PA.GM.3.1 and PA.GM.3.2: For consistency, list V=Bh first in both sub-standards.
- A1.A.1.1a has a parenthetical comment that begins with "i.e." It appears that "e.g.," is what intended. Also the list that follows is unclear. Some semi-colons may help.
- A1.A.3.3: In the example, the first "base" should be "based."
- G.2 through G.5 are missing "es" on the end of processes in the standard verbiage.