Comments on OK Academic Standards: Mathematics

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In addition to some wordsmithing contained in the Standards document itself, this document discusses some areas that I had more thorough questions or concerns about, as well as other general comments.

First, I would like to introduce myself to the writing team. I will spare you a condensed version of a resume– that's not what I want to discuss. What I would like to make clear is that I am viewing the drafts of these standards through several different lenses, including, but probably not limited to:

> As someone who holds a doctorate in mathematics;
>
> As someone who has taught pre-service and in-service teachers for 12 years;
>
> As someone who has coached teachers, co-planned and co-taught lessons, worked with administrators and with students, in a variety of K-12 classrooms over those years; and
>
> As someone who has written about the mathematics of PK-12 in various formats and consulted with various entities around PK-12 math.

So please know that when reviewing the OK Mathematics Standards, I am trying to balance each of these perspectives.

In addition, when participating in a group that is discussing something hotly contested like a standards document, I want to assure you that I am *not* the type who folds his arms, digs in his heels, and challenges everyone and anyone that might offer a different perspective. One of my strengths is that I prefer to have a rigorous discussion around the math, and to let the mathematics and what's best for students have agency in the discussion. I do not have an agenda.

All that said, please take the comments that follow in the spirit of simply attempting to help create the best possible standards for Oklahoma students and their teachers. Several of my comments are more food-for-thought than outright revisions, but there are a few areas that I feel are critical and need to be addressed. And please forgive me if I sound pedantic at times– I am most likely just trying to cover all the bases as I have not met any of you yet and therefore do not know my audience well.

I am looking forward to the opportunity to discuss the standards soon!

Sincerely,

CY

Grades PK-4

I think the PK-4 standards do an excellent job of laying the foundation for later grade levels. There are many standards in PK-2 that explore numbers and their representations, operations with numbers, and geometric shapes with a focus on composing and decomposing, important for understanding fractions later. In Grades 3-4, multiplication and division are introduced, as well as fractions and decimals, and it is there that I find a particular area of concern with respect to area and the area model for multiplication, which I will focus on in a moment.

Equivalence vs. Equality

In Grades 1-4 we see the following standards:

> **1.N.1.9** Demonstrate equivalence and equality (e.g., using balance scales, various manipulatives).
>
> **2.N.1.7** Recognize the difference between equivalence and equality (e.g. use balance scales to demonstrate that $2+4$ is equivalent to $3+3$.)
>
> **2.N.1.8** Demonstrate non-equivalence (e.g., balance scales, various manipulatives).
>
> **3.N.1.5** Recognize non-equivalence (e.g., $7+1>2+3$, $6+3$ is not equivalent to 4).
>
> **4.N.1.7** Determine the unknown addend or factor in equivalent and non-equivalent expressions (e.g., $5+6=4+\square$, $3\times 8<3\times\square$).

I have rarely seen equivalence and equality so explicitly laid out and included in content standards. Doing some simple research online, I don't see a definitive distinction that would be particularly relevant to elementary school students, so someone might need to explain to me what the intent of these standards is. Now, I may concede in the end that these standards are all necessary for the goals of the standards, but I will simply point out some potential inconsistencies that I could see arising as they are currently stated.

First, an obvious place that comes to mind in which I see a clear distinction between equivalence and equality in elementary school mathematics is probably in the area of fraction equivalence. This is a context where more advanced mathematics might agree, for example, when one defines a rational number as a single object (a fraction) that represents an entire group of objects (all the fractions that are "equivalent" to it). In this context, we define fractions a/b and c/d to be **equivalent** if and only if $ad=bc$. Then, we might say that while $1/2$ and $3/6$ represent the same number on the number line, they are merely equivalent, as **equal** implies that they are the *exact same object*. Calling these objects equal clearly could be confusing for students, as $1/2$ and $3/6$ do not look exactly the same! So while there is something the same (equal) about them, we don't call them equal.

So, I'm completely onboard that there is a difference between equivalence and equality, but I'm not sure I see the distinction that is attempted to be made in the examples given in the standards cited above. The best I can ascertain is the idea that the expression $2+4$ is equivalent (but not equal) to $3+3$, because while they both represent the same sum of 6, they are not exactly the same (i.e. one has a 2 and 4 being added, the other has two 3s being added). I would love it if someone could clarify this for me. Unfortunately, I no longer have a copy of "Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School," by Carpenter, Loef-Franke, and Levi to consult!

The reason I think this is important to bring up at all is that there may be a minor inconsistency with the usage of these concepts in later standards. Take for instance the standard below from Grade 2:

> **2.N.2.4** Use strategies and algorithms based on knowledge of place value and equality to add and subtract two-digit numbers (e.g. mental strategies, standard algorithm, decomposition, expanded notation, partial sums, differences).

I can imagine a student coming up with an efficient mental strategy of combining tens and ones and then combining the results to quickly add in her head, something like

$$53 + 28 = (50 + 3) + (20 + 8) = (50 + 20) + (3 + 8) = 70 + 11 = 81.$$

I would assume that this is the sort of strategy that is being called for in this standard. However, unless I'm reading the standards cited above incorrectly, it would seem this student is using *equivalent* representations. So is this standard calling for knowledge of equality or equivalence?

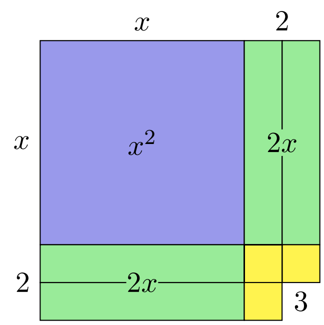
In addition, the transitive property of equality says that since $2+4=6$, and $6=3+3$, we should have $2+4=3+3$. So are these equal or equivalent?

Development of Area and Multiplication in Grades 3-4 (and 5)

My major concern with the Grades 3-4 (and by extension 5) standards is the development of the concept of area and its use in student understanding of multiplication. One of the most important representations for multiplication is the area model, which is first mentioned in standard **3.N.2.1**,* well before the standard requiring student understanding of the area of a rectangle as the product of its side lengths (standard **5.GM.2.1**†). I feel this is a critical misstep and should be rectified by developing the concept of area alongside the concept of the area model for multiplication.

Forgive me if I sound way too pedantic– it's not my intention. But I feel very strongly about how the area model is developed and want to lay out a complete argument for it.

First, consider just one example of the importance of the area model for students' mathematical development. The power of the area model is in its abstraction to algebra, for instance, in completing the square. The process of completing the square is the process of writing a quadratic polynomial $ax^2 + bx + c$ as the square of a binomial plus or minus a correcting term, crucial for deriving the quadratic formula. For an example, take the quadratic $x^2 + 4x + 3$, which cannot be factored as a perfect square. The distributive property shows that $(x + 2)^2 = x^2 + 4x + 4$, so that $x^2 + 4x + 3$ is 1 unit off from being a perfect square. Now, the name for the process of completing the square comes from the idea that when we try to represent $x^2 + 4x + 3$ as the area of a square, we cannot:



When we add one unit to $x^2 + 4x + 3$, we quite literally add another unit square to the picture above and "complete the square." This is just one example of the usefulness of the area model for multiplication in later grades.

Ask many teachers and students what multiplication means, i.e., how we interpret the product $a \times b$, and they very often respond that this can be interpreted as a groups with b objects in each group. This is the repeated addition model for multiplication, which makes the most sense when a and b are whole numbers:

$$a \times b = \underbrace{b + b + \cdots + b}_{a \text{ addends}}$$

* **3.N.2.1** Represent multiplication facts by using a variety of approaches, such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line and skip counting.

† **5.GM.2.1** Develop and use formulas to determine the area of rectangles. Justify why the length and width are multiplied to find the area of a rectangle by breaking the rectangle into one unit by one unit squares and viewing these as grouped into rows and columns.

Of course, this is only *one* way to interpret multiplication, and it has its advantages and limitations. (For comparison, this is the first presentation of multiplication in the Common Core Standards.)

In contrast, the area model for multiplication involves interpreting the product $a \times b$ as representing the area of a rectangle of dimensions a units and b units. This can be related to the repeated addition model, but is much more flexible since, for example, we can interpret a product like

$$\frac{3}{5} \times \frac{2}{7}$$

as being the area of a rectangle of the fractional dimensions given. Note that the repeated addition model does not apply in a natural way here (for instance, there is nothing to add as we do not have a whole group of either factor).

Now, the development of the area model for multiplication is obviously connected to the concept of area, in particular, the area of a rectangle. So how might one develop this model?

Because Grade 3 students are doing operations only with whole numbers at this point, it is natural to first view multiplication with a repeated addition approach, as described above. In my eyes, the models mentioned in the standard itself (equal-sized groups, arrays, equal jumps on a number line, and skip counting) can all be considered examples of this model. Specifically, the amount being repeatedly added consists of

- the size of the group in the equal-sized groups model;
- the amount in each row (or column) in the array model;
- the size of the jump on the number line model; and
- the size of the "skip" when skip counting.

There is a natural transition from the equal-groups model to the array model, which can be quite simply recognized as taking the objects in the equal-sized groups and arranging them into rows. For instance, if we represent 4×5 as "4 groups of 5 balls," we would have

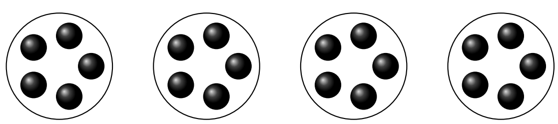


Figure 1: Circling equal-sized groups of 5 balls.

Then we would count up the balls: $5 + 5 + 5 + 5 = 20$. However, we then move towards arranging the balls in a more organized way that may make it easier to keep track of how many groups there are and how many are in each group, such as in the array below:

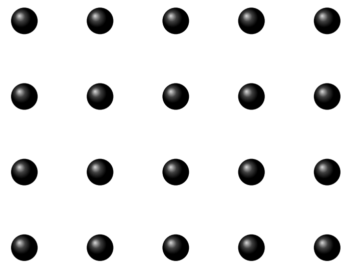


Figure 2: Arranging the balls into an array of 4 rows of 5 balls.

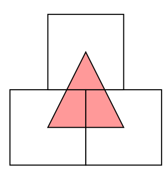
To me, this is the key picture that allows us to transition to understanding the relationship between area and multiplication.

But first, we need to define the area of a 2-D object. Assuming students have an understanding of "flat, closed" shapes, we can define the area as the *number of unit squares that completely* **fill** *a closed, planar shape without gaps or overlaps.* This is a common layperson's definition.

Please note that the current Grade 4 standard that defines area does so incorrectly. The standard states,

> **4.GM.2.3** Find the area of 2-D figures by counting the total number of same-size square units that cover the shape without gaps or overlaps.

Unless I'm missing something, with this definition the triangle below has area 3 square units, since it can be covered with 3 unit squares without gaps or overlaps:



It is possible that I'm misunderstanding the intended meaning of "overlaps" in the standard, but typically the non-overlap is among the units themselves, and not the unit and the shape. On the other hand, a quick internet search reveals that a definition involving "covering" does indeed come up (too frequently if you ask me). But if we are to use the word "cover" here, then I believe the standard would need to refer to the total number of unit squares *that the shape covers*, not the other way around. (But even that definition is somewhat imprecise.)

Once we have defined the area of a 2-D object as the number of pre-determined unit squares that fill a shape without gaps or overlap, then we can make the transition from the array model to the area model for multiplication. We might simply take the picture in Figure 2 and surround each ball with a unit square, noting how the method of counting horizontal rows of unit squares is completely analogous to counting the balls like we did earlier:

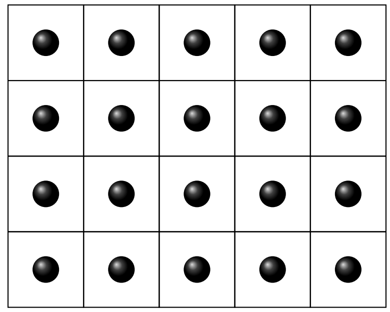


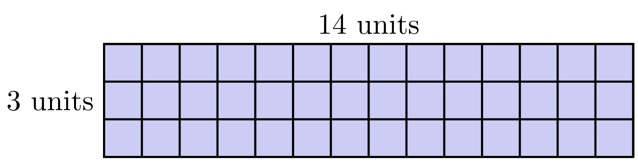
Figure 3: We transition from seeing objects arranged in arrays to viewing unit squares comprising rectangles. The method of counting and finding the total number of unit squares (i.e., the product) remains virtually the same.

Now, as mentioned earlier, there *is* a standard that deals with finding areas of rectangles using multiplication, by realizing we can count unit squares in rows (or columns) in a similar way to counting objects in an array– but this standard is in Grade 5:

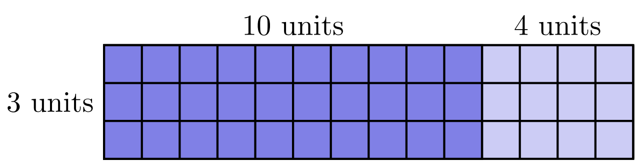
> **5.GM.2.1** Develop and use formulas to determine the area of rectangles. Justify why length and width are multiplied to find the area of a rectangle by breaking the rectangle into one unit by one unit squares and viewing these as grouped into rows and columns.

I feel strongly that this standard needs to be developed in tandem with the area model for multiplication. It is just too imporant of a concept.

I can provide a number of references that illustrate the use of the area model for multiplication in developing multiplication facts and properties, such as the distributive property of multiplication over addition. Here, base-ten blocks are often a tool, as is graph paper. Students might draw a rectangle with height 3 and base 14 on graph paper, understanding that to find the product 3×14, they must determine the area of this rectangle:



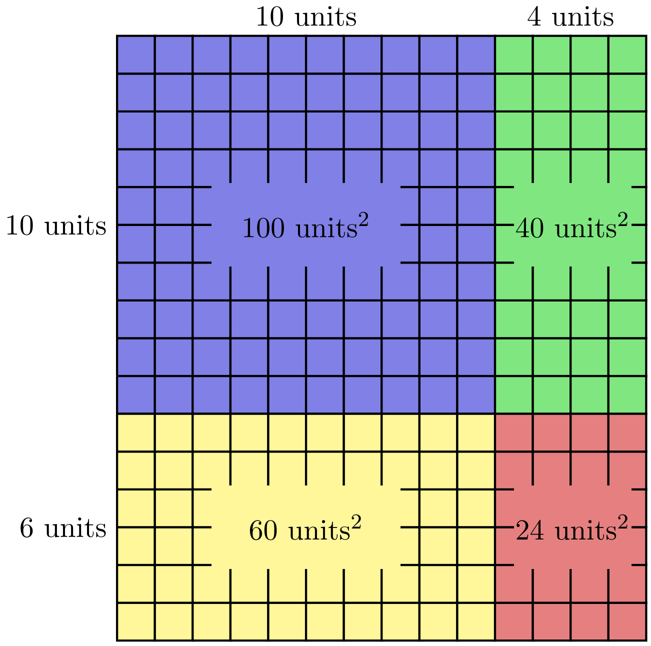
They learn that they can see the rectangle as being subdivided into a 3×10 rectangle and a 3×4 rectangle:



Students have decomposed shapes in earlier grades, through which they begin to understand that area is additive. So they see that to find the area of the original rectangle, they can find the areas of the smaller rectangles, which involves using simpler facts, and which they find by multiplying. In other words, they see that

$$3 \times 14 = 3 \times 10 + 3 \times 4.$$

Of course, the distributive property, combined with seeing rectangles subdivided into smaller rectangles, leads naturally to generalizable algorithms like the partial products algorithm:



$$\begin{array}{rrl} & 16 & \\ \times & 14 & \\ \hline & 24 & (4 \times 6) \\ & 40 & (4 \times 10) \\ & 60 & (10 \times 6) \\ & 100 & (10 \times 10) \\ \hline & 224 & \end{array}$$

I hope this illustrates the importance of developing the area model for multiplication and the concept of area together.

Development of Decimals in Grade 4 (and Grade 5)

The following standard in Grade 4 introduces the concepts of tenths and hundredths and models for representing them, and the succeeding standard refers to students writing decimals:

> **4.N.2.5** Represent tenths and hundredths with concrete models, making connections between fractions and decimals.
>
> **4.N.2.6** Represent, read and write decimals up to at least the hundredths place in a variety of contexts including money.

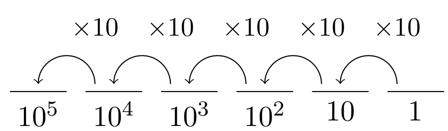
These are appropriate standards that emphasize the connection between fractions and decimals, and contexts in which they are used.

However, there is a standard in Grade 5 that does not seem to fit with these standards, nor does it seem to contribute anything new.

> **5.N.2.1** Represent decimal fractions (e.g. 1/10, 1/100) using a variety of models (e.g. 10 by 10 grids, rational number wheel, base-ten blocks, meter stick) and make connections between fractions and decimals (e.g. the visual for 1/10 is the same as for 0.1).

If this standard exists as a review or continuation of the ideas started in Grade 4, then that is fine with me. However, I have a problem with the statement "the visual for 1/10 is the same as for 0.1." I'll discuss that below in the Grade 5 section. Some thoughts on decimals and place value follow.

Students in elementary school are learning about the amazing base-10 number system, which allows for efficient representations of numbers as well as efficient algorithms for performing operations on them. They begin by learning numbers up to 10, including how to read, write, and represent the digits 0 through 9. They eventually learn that "10" not only means ten individual objects, but that it also can be thought to represent "1 group of *ten* and 0 units." This concept lays the foundation for understanding place value: that the location of a digit indicates its value, as in "a 6 in the hundreds place represents 6 groups of 100." In particular, they learn that there is a multiplication pattern (of grouping by tens) when moving from right to left in the number system:



When students encounter fractions, they naturally study tenths and hundredths as examples of fractions, but which will *eventually* turn out to be powers of 10:

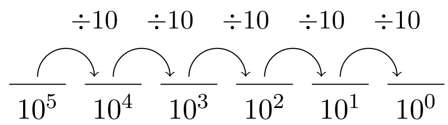
$$\frac{1}{10} = 10^{-1} \quad \text{and} \quad \frac{1}{100} = 10^{-2}.$$

However, what they should see first is that

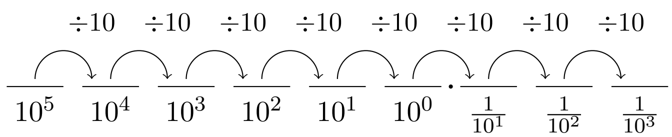
$$1 \div 10 = \frac{1}{10} \quad \text{and} \quad \frac{1}{10} \div 10 = \frac{1}{100}.$$

So how are these fractions related to the base-10 representation of a number and thus to decimals?

Students should also discover a division pattern in the base-10 system when moving from left to right:



This becomes the key to representing tenths and hundredths in the base-10 number system: we extend this division-by-10 pattern to the right of the ones place, including a decimal point to separate place values representing amounts less than one:



This is the key concept that connects the so-called "decimal fractions" to decimals, and it should be made explicit to students.

Concerning the Grade 4 standards, I would strongly advocate for a clarifying statement along the lines of "Understand that the two places to the right of the units place in a number represent tenths and hundredths, respectively," added to standard **4.N.2.5** above. (Perhaps that is already the intent of the standard, but a later standard in Grade 5 gives me pause–see below.)

Grades 5-7 & PA

I only have a few comments on Grades 5-8, as overall I think that the major ideas are developed very well: continued work with number sense; ratio and proportion, which moves into functions; the continued development of algebraic reasoning; etc.

Area, the Area Model for Multiplication, and Fractions and Operations

One major concern I have with this grade band is the development of the concept of area, as I mentioned earlier in the Grades 3-4 comments. Please see my discussion of the area model for multiplication. Just for clarity, I will again state that I think the standard below:

> **5.GM.2.1** Develop and use formulas to determine the area of rectangles. Justify why length and width are multiplied to find the area of a rectangle by breaking the rectangle into one unit by one unit squares and viewing these as grouped into rows and columns.

appears too late in the progression, given that it is the key to fully understanding the very important area model for multiplication.

Development of Decimal Understanding in Grades 4 and 5

As noted previously, there is a standard in Grade 5 that seems not to fit with the decimal standards started in Grade 4 and which doesn't seem to contribute much new.

> **5.N.2.1** Represent decimal fractions (e.g. 1/10, 1/100) using a variety of models (e.g. 10 by 10 grids, rational number wheel, base-ten blocks, meter stick) and make connections between fractions and decimals (e.g. the visual for 1/10 is the same as for 0.1).

What I am most concerned with is the parenthetical example in this standard. In particular, seeing as 0.1 has no intrinsic meaning unless it is clear we are working with the base-10 number system, I don't see how there is a visual for 0.1 at all; there seems to be an implication that a visual (concrete model) for 0.1 can be constructed without referencing the fact that 0.1 represents 1/10. Someone may need to explain the intent of this standard to me.

An example that illustrates why I see some difficulty here might be how to explain to a confused student that 0.2 does not represent 1/2, 0.3 does not represent 1/3, 0.4 does not represent 1/4, etc. How might we explain this? These may seem logical, especially since his teacher tells him that 0.10 represents 1/10! We would most likely explain that 0.1 only has meaning since in our base-10 number system, the first place to the right of the units place represents a new denomination of units, called *tenths*, and so a "1" there represents 1/10. Then, we can go on and note that since 1/2 is equivalent to $5/10 = 5(1/10)$, we can represent 1/2 with 0.5, etc. We can say then that $0.3 = 3/10$, and 3/10 is not a fraction equivalent to 1/3.

The point is, unless I'm mistaken, there is no way to have a visual that represents 0.1 without explicitly making use of the fact that 0.1 represents 1/10. I would much rather see a statement like, "Students understand that the place directly to the right of the ones place represents tenths, and so 0.1 is the same as 1/10," or something to that effect. See the discussion in my Grades 3-4 comments as well.

Ratio, Rate, and Definitions in Grades 6-7

Something I appreciate after having worked with the Common Core standards for quite a few years is the attentiveness to clear definitions of certain terms that have seemingly been forever part of the K-12 mathematics lexicon, but that have never really been clearly defined.

A particular area in which the Common Core authors felt clarity was needed was in defining ratio and proportion. I myself have been in numerous discussions with some of my curriculum-writing colleagues over the years with how to define a **ratio** in a precise way that is also accessible to teachers and students. It isn't easy.[‡] I personally do not think the CCSS authors define ratio terribly precisely, but they *do* explicitly define the rate associated with a ratio, and the unit rate. Thus, they end up with: the common notion of the **ratio** $a: b$ representing "a UNITS for every b *units*"; the **associated rate**, a/b UNITS per *unit*; and the **unit rate**, which is simply the number a/b. (Historically, what the CCSS calls the unit rate has been called the **value** of the ratio.) Agree with them or not, their intention in employing these definitions is evident when considering how and why we set up a proportion to solve certain ratio problems, for example: we are setting the unit rates (which are numbers without units) equal to each other since equivalent ratios must have equal unit rates.

In addition, the CCSS define a fraction in a very precise way (initially as a part-whole model in Grade 3), and the connection between ratios and fractions is explicitly made. Thus, when we have a lemonade consisting of 3 parts of lemon to 2 parts of sugar, the number 3/2 in this case represents 3/2 *lemons per sugar*, and we arrive at this quantity by trying to divide 3 lemons equally among 2 sugars: every sugar would be paired with $1\frac{1}{2}$ lemons. In contrast, take for example the previous California Standards (1997), which featured the following standard in Grade 6:

> **CA.6.NS.1.2** Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show relative sizes of two quantities, using appropriate notations (a/b, a to b, $a: b$.)

Now, the 1997 California standards in earlier grades only focused on a fraction as being part of a set or part of a whole, that is, there was no explicit connection made between a part-part interpretation and a fraction. Fraction notation was simply another way to represent a ratio, rather than a natural consequence of how we understand fractions and how a fraction is related to a ratio.

I bring all of this up because in Grades 6, 7, and PA ,the OK standards tread into similar waters with the introduction of ratio and unit rate. The writing team may want to carefully define a ratio, at least in student terms ("for every a UNITS there are b *units*?"). In addition, it seems that unit rate is explicitly defined only when a ratio has different units:

> **6.N.3.2** Determine the unit rate for ratios of quantities with different units.

While I have seen this definition of unit rate before, I don't believe it is a standard definition, and so the standards will want to explicitly define this term as well.

[‡]The definition I prefer is one involving equivalence classes of objects $[a: b]$, wherein two objects $a: b$ and $c: d$ are equivalent if there is a non-zero k such that $ka = c$ and $kb = d$ simultaneously– but this isn't very kid friendly.

Related to this is the definition of **similar** shapes. Note that there is a standard,

> **7.GM.4.1** Describe the properties of similarity, compare geometric figures for similarity, and determine scale factors.

that deals with similarity, but it is thus far unclear what it means for shapes to be similar, and so a definition should certainly be provided. For instance, are similar shapes those that can be matched together through a one-to-one correspondence such that corresponding lengths have a common ratio (or something to this effect)?

A popular curriculum in California is the College Preparatory Mathematics (CPM) curriculum. It is a good program overall, but I was dismayed to find that at some point in the newly revised Grade 7 textbook, "similar shapes" are defined as those having the "same shape but not necessarily the same size." This is a completely vague and inaccurate definition of similarity, and it is also presented earlier than the CCSS-M present an explicit definition of similarity (in terms of rigid motions and dilations). For instance, in this vague definition, any two triangles are similar, since while they might not have the same perimeter (different size), they are still triangles (same shape).

For another example, consider the common layperson's definition of **congruence**. Many people might say congruent shapes have the "same size and same shape." However, the rectangles below have the "same size" (they both have area 4 square units) and the "same shape" (they are both rectangles). But we know they are not congruent.

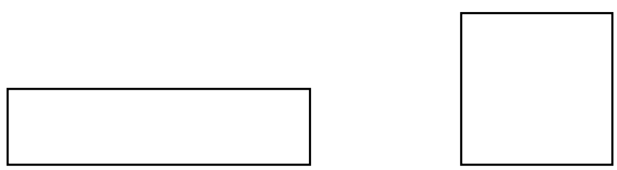


Figure 4: Do these rectangles not have the "same size and same shape?"

This illustrates the need for a precise definition of **congruence**. Now, there is a standard in Oklahoma Grade 6 that takes a similar approach to the CCSS-M, but note that a clear definition of congruence may need to be given at some point.

> **6.GM.4.2** Recognize that translations, reflections, and rotations preserve congruency and use them to show that two figures are congruent.

This may need to be addressed in the Geometry course as well.

While these terms may be precisely defined in a forthcoming glossary, the real issue is whether students should understand these terms in the same language as what might appear there. If the glossary says that congruent shapes are those that can be mapped onto one another through a sequence of rigid transformations, then are students to understand congruence the same way? If so, this may need to be made explicit in the standards themselves. (Note that standard **6.GM.4.2** seems to indicate that translations, reflections, and rotations preserve congruency, but they are not being used to actually define congruence of shapes.)

Grades 9-12

I only have a few comments for Grades 9-12, as I really think the major topics found in the typical high school curriculum are covered, and well.

Algebra 1

I have a quick comment on standard **A1.A.3.6**, which introduces geometric sequences. Note the two very similar standards below:

> **A1.A.3.5** Recognize that arithmetic sequences are linear using equations, tables, graphs...
> **A1.A.3.6** Recognize that geometric sequences are exponential using equations, tables, graphs...

Earlier, in standard **PA.A.1.3**, a linear function is explicitly defined as one that can be expressed as $y = mx + b$ or one whose graph is a straight line. However, there is no comparable standard that defines an exponential function. So is this the first place that an exponential function is defined? If so, this may need to be explicitly stated with a similar standard as **PA.A.1.3**.

Geometry

First, the current font does not present "π" very well– please fix that!

This certainly seems like a somewhat traditional Geometry course, at least in the sense that it is heavily focused on proof and logical reasoning. I just want to offer a word of slight caution here, in that introducing such a heavily proofs-focused course without laying enough of a foundation for logical reasoning, justification, conjecture and experimentation, etc., in prior grades may present a challenge for most students (and by extension their teachers).

When I occasionally ask my students (often pre-service elementary teachers) to write a short essay describing a prior positive and a prior negative math experience from their lives, many of them cite their High School geometry course as the negative experience! And they often specifically connect it with being required to do proofs. So my word of caution is that the writing teams make sure that enough reasoning, justification, mathematical argument, etc., is included in prior grades so that when students get to Geometry there isn't such a shock from suddenly needing to not only write proofs, but to understand the logical arguments they are attempting to convey. I'm not advocating for lessening the proofs-focus of Geometry by any means; I am advocating for ensuring a consistent focus on reasoning and justification at every grade level.

Something else the writing team may want to consider is clearly defining the progression through Geometry that is taken here. The classical U.S. Geometry course followed Euclid's approach– stating postulates and axioms and using compass and straight-edge constructions to prove theorems about geometric relationships. In contrast, the CCSS has a different approach, assuming properties of transformations, verifying those properties experimentally, and then using these to define congruence and prove basic triangle congruence theorems, which are then used to prove other theorems. But based on the order in which the standards are presented in the OK Geometry course, it is a little unclear to me what the starting point is for the course– what are the axioms, postulates, and undefined terms that students will begin with and then use to construct arguments that justify further results? It may be helpful to make the intended sequence a little more explicit.

Algebra 2

I'm afraid I don't have many comments on Algebra 2, other than that there are thirty-four separate standards, and they are all fairly substantial. I'm not sure where to find more balance, but that may be something to consider. (While I normally wouldn't simply count standards, they are all fairly distinct, so thirty-four seems to be quite a few.)

Something that stands out to me is that it seems like traditional Algebra 1 courses culminated in the general solution to quadratic equations and a complete study of quadratic functions. But this is delayed until Algebra 2 in the OK standards. I'm not going to take a strong position for either case; I just point it out since Algebra 2, as written, is a very full course, and one place to lighten it up may be to shift some of the quadratic work down to Algebra 1.