Note on Definition of Length

This had occurred to me earlier, but I suppose I hadn't fully decided whether to bring it up until after our conversation yesterday. We had been discussing an appropriate and accurate definition of **area**, and in that discussion the definition of **length** came up, specifically, the part of the definition that says "from one end of the object to another." Here is the standard in Grade 1:

> **1.GM.2.2** Illustrate that the length of an object is the number of same-size units of length that, when laid end-to-end with no gaps or overlaps, reach from one end of the object to the other.

I had originally thought there might be some issue with this definition, since for example, the following would illustrate the "length" of a circle, since it is a measure "from one end of the object to another."
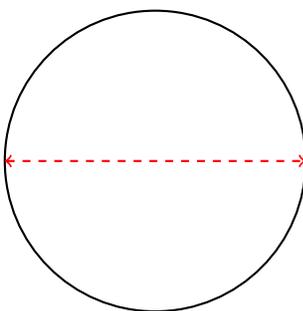

Figure 1: Does the length of the red line indicate the "length" of the circle?

This came up for me again and I decided to revisit it after we talked about area as "filling a shape" and that it was simply implied, similar to how length being end-to-end was implied. Also, the discussion about finding the perimeter of the key on a basketball court, and estimating its length using line segments made me rethink things. To illustrate further, by the definition in the standard above, the length of the S-curve below might be found by measuring the length of the red line, since it reaches from one end of the object to the other.
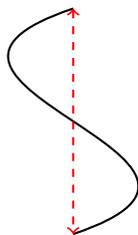

Figure 2: Does the length of the red line indicate the "length" of this curve?

Now, I also understand that students could measure the length of such a curve using a piece of string that they then straighten out, but I wonder if that is much more challenging than we give students credit for.

For example, do young children know that if a bent piece of wire is straightened, then there is an intrinsic quality of that piece of wire that stays the same, namely, its length? I

recall research (Van Hiele?) about how young children will think a shorter container holds less than a taller, because they cannot yet understand that the circumference of the container contributes to its volume as well. And I am also aware that there is a developmental stage involving understanding conservation of measure (for example area). So is there a similar developmental stage with "conservation" of length in the piece of string context?

I think the upshot is that a revision of standard **1.GM.2.2** might be needed that explicitly states that length is a measure of **line segments** (but perhaps not in that exact language–not sure). Once length of segments is established, students can discuss finding lengths of straight **sides** of objects, like finding lengths of **sides of polygons**. By the way, this could help remove ambiguity around the "length" of an object, as mentioned for example in the next standard (and possibly in others):

> **1.GM.2.3** Measure the same object/distance with units of two different lengths and describe how and why the measurements differ.

When stated this way, it isn't clear what the length of an object is; for example a student might argue that the length of the diagonal of a rectangle is the rectangle's "length," especially since it is the "longest" measure within the rectangle.
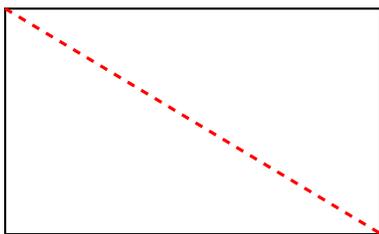
Figure 3: Is this the "length" of this object?

Furthermore, I think such a change would lead *very nicely* into a standard about approximating lengths of curves by measuring lengths of line segments that better and better approximate the curve.
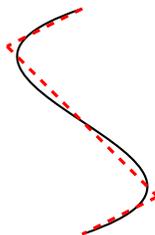
Figure 4: When we know how to measure lengths of line segments, we can *estimate* the lengths of curves.

And this may get us closer to students developing an understanding of circumference in a more natural way, as desired.

Just some more food for thought.
Sincerely, CY