



## Oklahoma Academic Standards for Mathematics

### Mathematical Glossary Terms, Tables & Illustrations

This glossary contains those terms found and defined from the following sources:

#### Glossary Sources

- (DPI) <http://dpi.wi.gov/standards/mathglos.html>  
 (H) <http://www.hbschool.com/glossary/math2/>  
 (M) <http://www.merriam-webster.com/>  
 (MW) <http://www.mathwords.com>  
 (MA) <http://www.doe.mass.edu/frameworks/current.html>  
 (NCTM) <http://www.nctm.org>  
 (PASS) <http://www.ok.gov./sde/sites/ok.gov.sde/files/C3%20PASS%20math.pdf>.

**AA similarity.** (Angle-angle similarity.) If two triangles have two pairs of corresponding angles that are congruent, then the triangles are similar. (MW)

**ASA congruence.** (Angle-side-angle congruence.) If two triangles have two corresponding angles and the side adjacent to both angles congruent, then two triangles have corresponding angles and sides that are congruent, the triangles themselves are congruent. (MW)

**Absolute value.** ~~A non-negative number equal in numerical value to a given real number.~~  
 (MW) The absolute value of a real number is its (non-negative) distance from 0 on a number line. Formally,

$$|k| = \begin{cases} k & \text{if } k \geq 0 \\ -k & \text{if } k < 0 \end{cases}$$

**Addend.** In the addition problem  $3+2+6 = 11$ , the addends are 3,2,and 6. (PASS)

**Addition and subtraction within 5, 10, 20, 100, or 1000.** Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0–5, 0–10, 0–20, or 0–100, respectively. *Example:  $8 + 2 = 10$  is an addition within 10,  $14 - 5 = 9$  is a subtraction within 20, and  $55 - 18 = 37$  is a subtraction within 100.* (MA)

**Additive inverses.** Two numbers whose sum is 0 are additive inverses of one another. *Example:  $3/4$  and  $-3/4$  are additive inverses of one another because  $3/4 + (-3/4) = (-3/4) + 3/4 = 0$ .* (MA)

**Algorithm.** A finite set of steps for completing a procedure, e.g., long division. (H)

**Analog.** Having to do with data represented by continuous variables, e.g., a clock with hour, minute, and second hands. (M)

**Analytic geometry.** The branch of mathematics that uses functions and relations to study geometric phenomena, e.g., the description of ellipses and other conic sections in the coordinate plane by quadratic equations.

- A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak/akstandards/standards/akstandards\\_elaandmath\\_080812.pdf](http://www.eed.state.ak/akstandards/standards/akstandards_elaandmath_080812.pdf). Accessed 12/1/2015.

**Comment [CY1]:** I am not a fan of the MathWords website myself for mathematical definitions. One of the defining characteristics (pun?) of mathematics is precision in defining new terms in terms of prior terms, all the way down to accepted "undefined" terms.

The MathWords website does not follow this approach. Many of the definitions are more loose, more in layperson terms, which can be good but is often bad. I anticipate correcting many of the MW terms moving forward.

**Comment [CY2]:** Be careful here. I checked the original reference (MW) and there was an image and a line "...as shown below." This is important here, as AA similarity refers to **two** corresponding angles being congruent determining similarity—the accompanying picture in original website illustrated this. (This happens with ASA below, and it may happen throughout so I'll look for it.) I've made a suggested change.



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**Argument of a complex number.** ~~The angle describing the direction of a complex number on the complex plane. The argument is measured in radians as an angle in standard position. For a complex number in polar form  $r(\cos \theta + i \sin \theta)$ , the argument is  $\theta$ .~~ The angle  $\theta$  when a complex number is expressed in polar form,  $r(\cos \theta + i \sin \theta)$ .

**Array.** (rectangular) an orderly arrangement of objects into a rectangular configuration (e.g., take six tiles and arrange two long and three wide to form a rectangle). (PASS)

**Associative property of addition.** See Table 3 in this Glossary.

**Associative property of multiplication.** See Table 3 in this Glossary.

**Assumption.** A fact or statement (as a proposition, axiom, postulate, or notion) taken for granted. (M)

**Attribute.** characteristics (e.g., size, shape, color, weight). (PASS)

**Benchmark fraction.** A common fraction against which other fractions can be measured, such as  $\frac{1}{2}$ . (MA)

**Binomial Theorem.** The theorem that gives the polynomial expansion of any whole-number power of a binomial. A method for distributing powers of binomials. For  $n \geq 0$ ,

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k.$$

**Comment [CY3]:** Given this is a HS-level definition, it can be stated precisely and technically.

**Bivariate data.** Pairs of linked numerical observations. *Example: a list of heights and weights for each player on a football team.* (MA)

**Box plot.** A graphic method that shows the distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data. (DPI)

**Calculus.** The mathematics of change and motion. The main concepts of calculus are limits, instantaneous rates of change, and areas enclosed by curves. (MA)

**Capacity.** The maximum amount or number that can be contained or accommodated, e.g., a jug with a one-gallon *capacity*; the auditorium was filled to *capacity*. (MA)

**Cardinal number.** A number (such as 1, 5, 15) that is used in simple counting and that indicates how many elements there are in a set. (MA)

**Cardinality.** ~~Cardinal numbers, known as the "counting numbers," indicate quantity. The cardinality of a finite collection of objects is the number of objects in the set. (For example, in PK-Grade 1 students are still learning that "5" represents the number of objects in any group of "five" objects.)~~

**Comment [CY4]:** Was this supposed to be bold, as in a new term?

**Cartesian plane.** A coordinate plane with perpendicular coordinate axes. (MA)

A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak/akstandards/standards/akstandards\\_elaandmath\\_080812.pdf](http://www.eed.state.ak/akstandards/standards/akstandards_elaandmath_080812.pdf). Accessed 12/1/2015.



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**Cavalieri's Principle.** A method, with formula given below, of finding the volume of any solid for which cross-sections by parallel planes have equal areas. This includes, but is not limited to, cylinders and prisms. Formula: Volume =  $Bh$ , where  $B$  is the area of a cross-section and  $h$  is the height of the solid. (MW)

**Combinations.** a selection of objects without regard to order. (PASS)

**Coefficient.** Any of the factors of a product considered in relation to a specific factor. Often, this will be a numerical factor in a product of numbers and variables, e.g.  $3x^2$  has coefficient 3. (W)

**Commutative property.** See Table 3 in this Glossary.

**Complementary angles.** Two angles whose measures have a sum of 90 degrees. (PASS)

**Complex fraction.** A fraction  $A/B$  where  $A$  and/or  $B$  are fractions ( $B$  nonzero). (MA)

**Complex number.** Numbers of the form  $a + bi$ , where  $a$  and  $b$  are real numbers and  $i$  equals the square root of  $-1$ . (PASS) (Might add: the number  $a$  is called the real part and  $b$  the imaginary part of the complex number  $a + bi$ .)

**Complex plane.** ~~The coordinate plane used to graph complex numbers.~~ (MW) A Cartesian plane in which the point  $(a, b)$  is used to represent  $a + bi$ .

**Compose numbers.** a) Given pairs, triples, etc. of numbers, identify sums or products that can be computed; b) Each place in the base ten place value is composed of ten units of the place to the left, i.e., one hundred is composed of ten bundles of ten, one ten is composed of ten ones, etc. (MA)

The definition above ↑ is wordy and unclear. Perhaps something simpler: To compose numbers is to create new numbers using any of the four operations with other numbers. For example, students compose 10 in many ways ( $9+1, 8+2, \dots, 5+5, \dots$ ). Also, each place in the base ten place value is composed of ten units of the place to the left, i.e., one hundred is composed of ten bundles of ten, one ten is composed of ten ones, etc.

**Compose shapes.** Join geometric shapes without overlaps to form new shapes. (MA)

**Composite number.** Any positive integer exactly divisible by one or more positive integers other than itself and 1. (PASS)

**Computation algorithm.** A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: **algorithm**; **computation strategy**. (MA)

**Computation strategy.** Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: **computation algorithm**. (MA)

**Comment [CY5]:** If "divisible" is defined in a precise way, then "exactly" is unnecessary here.

**Comment [CY6]:** I haven't gotten to "prime number" yet. The technical definition of prime number is a number  $p > 1$  such that if  $k$  is a factor (divisor) of  $p$ , then  $k=1$  or  $k=p$ . Then, one defines a composite number as a number greater than 1 that is *not* prime.

A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak/akstandards/standards/akstandards\\_elaandmath\\_080812.pdf](http://www.eed.state.ak/akstandards/standards/akstandards_elaandmath_080812.pdf). Accessed 12/1/2015.



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**Congruent.** Geometric figures having exactly the same size and shape. (PASS)

Two geometric objects are congruent if one can be mapped onto the other using a sequence of rigid motions (rigid motions are geometric transformations that preserve lengths and angles).

**Comment [CY7]:** No! A 1×4 rectangle and a 2×2 square are (1) both rectangles, and (2) have the same area. So they are “same size and shape.” As we discussed in the Geometry standards, this needs a precise definition.

**Conjugate.** The result of writing a sum of two terms as a difference, or vice versa. *For example, the conjugate of  $x - 2$  is  $x + 2$ .* (MW)

**Conic sections.** Circles, parabolas, ellipses, and hyperbolas, which can all be represented by intersecting a plane with a hollow double cone. (PASS)

**Conjecture.** A statement believed to be true but not yet proved. (PASS)

**Coordinate plane.** A plane in which two coordinate axes are specified, i.e., two intersecting directed straight lines, usually perpendicular to each other, and usually called the x-axis and y-axis. Every point in a coordinate plane can be described uniquely by an ordered pair of numbers, the coordinates of the point with respect to the coordinate axes. (MA)

A plane in which a point is represented using two coordinates that determine the precise location of the point. In the Cartesian plane, two perpendicular number lines are used to determine the locations of points. In the polar coordinate plane, points are determined by their distance along a ray through that point and the origin, and the angle that ray makes with a pre-determined horizontal axis.

**Comment [CY8]:** I don't think this is correct. For example, a plane represented with polar coordinates is still a “coordinate plane”. Alternate definition idea follows.

**Cosine (of an acute angle).** In a right triangle, the cosine of an acute angle is the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse. (PASS)

**Counting number.** A number used in counting objects, i.e., a number from the set

$$\{1, 2, 3, 4, 5, \dots\}.$$

See also “Natural number.”

~~1, 2, 3, 4, 5, 1/4.~~ See Illustration 1 in this Glossary. (MA)

**Comment [CY9]:** My hope is this wasn't intentional, but for some reason “1/4” was included on the list!

**Counting on.** A strategy for finding the number of objects in a group without having to count every member of the group. *For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again; one can find the total by counting on—pointing to the top book and saying “eight,” following this with, “nine, ten, eleven. There are eleven books now.”* (MA)

**Decimal expansion.** Writing a rational number as a decimal. (MA)

**Comment [CY10]:** Strange definition since the term is a noun but the definition is a verb.

**Decimal fraction.** A fraction (as  $0.25 = 25/100$  or  $0.025 = 25/1000$ ) or mixed number (as  $3.025 = 3 25/1000$ ) in which the denominator is a power of ten, usually expressed by the use of the decimal point. (M)

**Decimal number.** Any real number expressed in base 10 notation, such as 2.673. (MA)

A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak.akstandards/standards/akstandards\\_eaandmath\\_080812.pdf](http://www.eed.state.ak.akstandards/standards/akstandards_eaandmath_080812.pdf). Accessed 12/1/2015.



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**Decompose numbers.** Given a number, identify pairs, triples, etc. of numbers that combine to form the given number using subtraction and division. (MA)

**Decompose shapes.** Given a geometric shape, identify geometric shapes that meet without overlap to form the given shape. (MA)

**Dependent events.** Events that influence each other. If one of the events occurs, it changes the probability of the other event. (PASS)

**Digit.** a) Any of the Arabic numerals 1 to 9 and usually the symbol 0; b) One of the elements that combine to form numbers in a system other than the decimal system. (MA)

**Digital.** Having to do with data that is represented in the form of numerical digits; providing a readout in numerical digits, e.g., a digital watch. (MA)

**Dilation.** A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. (MA)

**Directrix.** A fixed curve with which a generatrix maintains a given relationship in generating a geometric figure; specifically: a straight line the distance to which from any point in a conic section is in fixed ratio to the distance from the same point to a focus. (M)

**Discrete mathematics.** The branch of mathematics that includes combinatorics, recursion, Boolean algebra, set theory, and graph theory. (MA)

**Domain of a relation.** The set of all the first elements or x-coordinates of a relation. (PASS)

**Dot plot.** See: *line plot*.

**Expanded form.** A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example,  $643 = 600 + 40 + 3$ . (MA)

**Expected value.** For a random variable, the weighted average of its possible values, with weights given by their respective probabilities. (MA)

**Exponent (Whole Number).** The number that indicates how many times the base is used as a factor, e.g., in  $4^3 = 4 \times 4 \times 4 = 64$ , the exponent is 3, indicating that 4 is repeated as a factor three times. (MA)

**Exponential function.** an exponential function with base  $b$  is defined by  $y = b^x$  where  $b > 0$  and  $b$  is not equal to 1. (PASS)

**Expression.** A mathematical phrase that combines operations, numbers, and/or variables (e.g.,  $3^2 \div a$ ). (H)

**Fibonacci sequence.** The sequence of numbers beginning with 1, 1, in which each number that follows is the sum of the previous two numbers, i.e., 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144.... (PASS)

**Comment [CY11]:** I thought writing " $14+16 = 10+4+10+6 = 10+10+10$ " was an example of decomposing (the 14 and 16) to make the problem easier? If that's the case then this definition is inaccurate.

**Comment [CY12]:** Only valid for whole number exponents.

**Comment [CY13]:** I'm not the biggest fan of this term here, but defining "expression" in a simple yet precise way can be challenging.

A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak/akstandards/standards/akstandards\\_eaandmath\\_080812.pdf](http://www.eed.state.ak/akstandards/standards/akstandards_eaandmath_080812.pdf). Accessed 12/1/2015.



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**First quartile.** For a data set with median  $M$ , the first quartile is the median of the data values less than  $M$ . Example: For the data set  $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$ , the first quartile is 6.<sup>[1]</sup> See also: **median, third quartile, interquartile range.** (MA)

**Fraction.** A number expressible in the form  $a/b$  where  $a$  is a whole number and  $b$  is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) See also: **rational number.** (MA)

**Function.** A mathematical relation for which each element of the domain corresponds to exactly one element of the range. (MW)

A rule that assigns to every element of one set (the domain) exactly one element of another set (the range). A function is often thought of as an “input/output” rule, as in every input determines an output (usually according to mathematical operations performed on the input).

**Function machine.** An input/output model (often made with milk cartons, boxes, or drawn on the board) to show one number entering and a different number exiting. Students guess the rule that produced the second number (e.g., enter 3, exit 5, rule: add 2). (PASS)

**Function notation.** A notation that describes a function. For a function  $f$ , when  $x$  is a member of the domain, the symbol  $f(x)$  denotes the corresponding member of the range (e.g.,  $f(x) = x + 3$ ).

**Fundamental Theorem of Algebra.** The theorem that establishes that, using complex numbers, all polynomials can be factored into a product of linear terms. A generalization of the theorem asserts that any polynomial of degree  $n$  has exactly  $n$  zeros, counting multiplicity. (MW)

**Geometric sequence (progression).** An ordered list of numbers that has a common ratio between consecutive terms, e.g., 2, 6, 18, 54~~4~~. (H)

**Histogram.** A type of bar graph used to display the distribution of measurement data across a continuous range. (MA)

**Identity property of 0.** See Table 3 in this Glossary.

**Imaginary number.** A complex number of the form  $bi$

**Independent events.** events that do not influence one another. Each event occurs without changing the probability of the other event. Specifically, two events  $A$  and  $B$  are independent if  $P(A \text{ AND } B) = P(A) \cdot P(B)$ . (PASS)

**Independently combined probability models.** Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair. (MA)

<sup>1</sup> Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., “Quartiles in Elementary Statistics,” *Journal of Statistics Education* Volume 14, Number 3 (2006).

A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak/akstandards/standards/akstandards\\_elaandmath\\_080812.pdf](http://www.eed.state.ak/akstandards/standards/akstandards_elaandmath_080812.pdf). Accessed 12/12/2015.

**Comment [CY14]:** I personally do not like defining “function” in terms of “relation”, since for example I believe in the OK standards functions are studied before relations! (Check that.)

**Comment [CY15]:** “real part” of a complex number had not been defined before.



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**Integer.** ~~All positive and negative whole numbers, including zero. (MW)~~ The set of numbers that contains the whole numbers and their additive inverses (opposites). I.e.,  $\{\dots, -2, -1, 0, 1, 2, 3, \dots\}$ .

**Interquartile range.** A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set  $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$ , the interquartile range is  $15 - 6 = 9$ . See also: **first quartile, third quartile.** (MA)

**Inverse function.** ~~A function  $g$  that satisfies  $g(f(x)) = x$  and  $f(g(x)) = x$  is said to be an inverse function for  $f$ . The inverse of  $f$  is often denoted by  $f^{-1}$ . A function obtained by expressing the dependent variable of one function as the independent variable of another; that is the inverse of  $y = f(x)$  is  $x = f^{-1}(y)$ .~~ (NCTM)

**Inverse operations.** Operations that undo each other (e.g., addition and subtraction are inverse operations; multiplication and division are inverse operations). (PASS)

**Irrational number.** Numbers that are not rational. Irrational numbers have nonterminating, nonrepeating decimal expansions (e.g., square root of 2, pi). (MA)

**Law of Cosines.** An equation relating the cosine of an interior angle and the lengths of the sides of a triangle. (MW)

**Law of Sines.** Equations relating the sines of the interior angles of a triangle and the corresponding opposite sides. (MW)

**Line plot.** A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot. (DPI)

**Linear association.** A set of bivariate data exhibits a linear association if a scatter plot of the data can be well-approximated by a line. (MA)

**Linear equation.** Any equation that can be written in the form  $Ax + By + C = 0$  where  $A$  and  $B$  cannot both be 0. The graph of such an equation is a line. (MA)

**Linear function.** ~~A mathematical function in which the variables appear only in the first degree, are multiplied by constants, and are combined only by addition and subtraction. For example:  $f(s) = Ax + By + C$ .~~ (M) A function  $f$  is linear if it can be written in the form  $f(x) = mx + b$ .

**Logarithm.** The exponent that indicates the power to which a base number is raised to produce a given number. *For example, the logarithm of 100 to the base 10 is 2.* (M)

**Logarithmic function.** Any function in which an independent variable appears in the form of a logarithm; they are the inverse functions of exponential functions. (MA)

**Manipulatives.** Concrete materials (e.g., buttons, beans, egg and milk cartons, counters, attribute and pattern blocks, interlocking cubes, base-10 blocks, geometric models, geoboards, fractions pieces, rulers, balances, spinners, dot paper) to use in mathematical calculations used to represent mathematical concepts, operations, and relationships. (PASS)

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**Comment [CY16]:** Are these too non-specific as to be rendered basically not helpful? Someone would need to do further research to find what these are precisely.

Do these even appear in the K-12 standards?

**Comment [CY17]:** Bivariate data has been defined above, so use it here.



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**Matrix (pl. matrices).** A rectangular array of numbers or variables. (MA)

**Mean.** A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.<sup>2</sup> *Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.* (MA)

**Mean absolute deviation.** A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.* (MA)

**Measure of variability.** A determination of how much the [performance of a group data in a set](#) deviates from [a measure of center \(usually the mean or median\)](#). The most frequently used measure is standard deviation. (MA)

**Median.** A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list; or the mean of the two central values, if the list contains an even number of values. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.* (MA)

**Midline.** In the graph of a [trigonometric sine or cosine](#) function, the horizontal line halfway between its maximum and minimum values. (MA)

**Model.** A mathematical representation (e.g., number, graph, matrix, equation(s), geometric figure) for real-world or mathematical objects, properties, actions, or relationships. (DPI)

**Modulus of a complex number.** The distance between a complex number and the origin on the complex plane. The [absolute value modulus](#) of  $a + bi$  is written  $|a + bi|$ , and the formula for  $|a + bi|$  is  $\sqrt{a^2 + b^2}$ . For a complex number in polar form,  $r(\cos \theta + i \sin \theta)$ , the modulus is  $|r|$ . (MW)

**Multiplication and division within 100.** Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0–100. *Example:  $72 \div 8 = 9$ .* (MA)

**Multiplicative inverses.** Two numbers whose product is 1 are multiplicative inverses of one another. *Example:  $\frac{3}{4}$  and  $\frac{4}{3}$  are multiplicative inverses of one another because  $\frac{3}{4} \cdot \frac{4}{3} = \frac{4}{3} \cdot \frac{3}{4} = 1$ .* (MA)

**Natural numbers.** [A number used in counting objects, i.e., a number from the set](#)

$\{1, 2, 3, 4, 5, \dots\}$ .

[See also "Counting number."](#)

[See Illustration 1 in this Glossary](#)

<sup>2</sup> To be more precise, this defines the *arithmetic mean*.

A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak.gov/akstandards/standards/akstandards\\_elaandmath\\_080812.pdf](http://www.eed.state.ak.gov/akstandards/standards/akstandards_elaandmath_080812.pdf). Accessed 12/1/2015.

**Comment [CY18]:** Are students graphing sine and cosine functions in K-12 ?



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(counting numbers) 1,2,3,4,... (PASS)

**Network.** a) A figure consisting of vertices and edges that shows how objects are connected, b) A collection of points (vertices), with certain connections (edges) between them. (MA)

**Comment [CY19]:** Necessary in K-12?

**Non-linear association.** The relationship between two variables is nonlinear if a change in one is associated with a change in the other and depends on the value of the first; that is, if the change in the second is not simply proportional to the change in the first, independent of the value of the first variable. (MA)

**Nonstandard measurement.** A measurement determined by the use of nonstandard units such as hands, paper clips, beans, cotton balls, etc. (PASS)

**Number line diagram.** A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity. (MA)

**Number sense.** involves the understanding of number size (relative magnitude), number representations, number operations, referents for quantities and measurement used in everyday situations, etc. (PASS)

**Comment [CY20]:** I'd be surprised if NCTM doesn't have a definition for "Number sense".

**Numeral.** A symbol or mark used to represent a number. (MA)

**Operation.** General term for any one of addition, subtraction, multiplication, and division. (PASS)

**Order of Operations.** Convention adopted to perform mathematical operations in a consistent order. 1. Perform all operations inside parentheses, brackets, and/or above and below a fraction bar in the order specified in steps 3 and 4; 2. Find the value of any powers or roots; 3. Multiply and divide from left to right; 4. Add and subtract from left to right. (NCTM)

**Ordinal number.** A number designating the place (as first, second, or third) occupied by an item in an ordered sequence. (M)

**Partition.** A process of dividing an object into parts or a set into (smaller) subsets. (MA)

**Pascal's triangle.** A triangular arrangement of numbers in which each row starts and ends with 1, and each other number is the sum of the two numbers above it. (H)

**Comment [CY21]:** A very specific object and thus perhaps an illustration would be useful.

**Percent rate of change.** A rate of change expressed as a percent. *Example: if a population grows from 50 to 55 in a year, it grows by  $5/50 = 10\%$  per year.* (MA)

**Periodic phenomena.** Events that recur over regular intervalsoccurring events, for example, ocean tides, machine cycles. (MA)

**Picture graph.** A graph that uses pictures to show and compare information. (MA)

**Comment [CY22]:** As it stands, this is a very vague definition. Is this referring to data representations using pictures to represent 1 unit of a given quantity, as used in K-2?

**Polar form.** The polar coordinates of a complex number on the complex plane. The polar form of a complex number is written in any of the following forms:  $r \cos \theta + r i \sin \theta$ ,  $r(\cos \theta + i \sin \theta)$ ,

A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak/akstandards/standards/akstandards\\_elaandmath\\_080812.pdf](http://www.eed.state.ak/akstandards/standards/akstandards_elaandmath_080812.pdf). Accessed 12/1/2015.



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or  $r \operatorname{cis} \theta$ . In any of these forms,  $r$  is called the modulus or absolute value.  $\theta$  is called the argument. (MW) The polar form of the complex number  $a + bi$  is any of the following forms:

$$r \cos \theta + ir \sin \theta, \quad r(\cos \theta + i \sin \theta), \quad r \operatorname{cis} \theta$$

where  $(r, \theta)$  are the polar coordinates of  $a + bi$ , that is  $r$  is the modulus and  $\theta$  the argument of  $a + bi$ .

**Polynomial.** The sum or difference of terms which have variables raised to positive integer powers and which have coefficients that may be real or complex. The following are all polynomials:  $5x^3 - 2x^2 + x - 13$ ,  $x^2 y^3 + xy$ , and  $(1 + i)a^2 + ib^2$ . (MW)

**Polynomial function.** Any function whose value is the solution output is given by a polynomial expression of the input.

**Postulate.** A statement accepted as true without proof. (MA)

**Prime factorization.** A number written as the product of all its prime factors. (H)

**Prime number.** A whole number greater than 1 whose only factors are 1 and itself. (MA)

**Probability distribution.** The set of possible values of a random variable with a probability assigned to each. (MA)

**Properties of equality.** See Table 4 in this Glossary.

**Properties of inequality.** See Table 5 in this Glossary.

**Properties of operations.** See Table 3 in this Glossary.

**Probability.** The study and measure of the likelihood of an event happening. (PASS)

**Probability model.** A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also: **uniform probability model.*** (MA)

**Proof.** A method of constructing a valid argument using deductive reasoning. (MA)

**Proportion.** An equation that states that two ratios are equivalent, e.g.,  $\frac{4}{8} = \frac{1}{2}$  or  $4 : 8 = 1 : 2$ . (MA)

**Pythagorean theorem.** For any right triangle, the sum of the squares of the lengths of the legs equals the square of the lengths of the hypotenuse. (MA)

**Quadratic equation.** An equation that is equivalent to  $ax^2 + bx + c = 0$ , where  $a \neq 0$ . An equation that includes only second degree polynomials. Some examples are  $y = 3x^2 - 5x^2 + 1$ ,  $x^2 + 5xy + y^2 = 1$ , and  $1.6a^2 + 5.9a - 3.14 = 0$ . (MW)

Comment [CY23]: For purposes of K-12...

A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak/akstandards/standards/akstandards\\_elaandmath\\_080812.pdf](http://www.eed.state.ak/akstandards/standards/akstandards_elaandmath_080812.pdf). Accessed 12/1/2015.



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**Quadratic expression.** ~~An expression that contains the square of the variable, but no higher power of it. (MA)~~ An expression that contains variables raised to whole number exponents no higher than 2.

**Quadratic function.** A function that can be represented by an equation of the form  $y = ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  are arbitrary, but fixed, numbers and  $a \neq 0$ . The graph of this function is a parabola. (DPI)

**Quadratic polynomial.** A polynomial where the highest degree of any of its terms is 2. (MA)

**Radical.** The  $\sqrt{\quad}$  symbol, which is used to indicate square roots or  $n$ th roots. (MW)

**Random sampling.** A smaller group of people or objects chosen from a larger group or population by a process giving equal chance of selection to all possible people or objects. (H)

**Random variable.** An assignment of a numerical value to each outcome in a sample space. (M)

**Range of a relation.** The set of all the second elements or y-coordinates of a relation is called the range. (PASS)

**Ratio.** A multiplicative comparison of two numbers or quantities, e.g., 4 to 7 or 4 : 7 or 4/7. (MA)

OR: A relationship between quantities such that for every  $a$  units of one quantity there are  $b$  units of the other. A ratio is often denoted by  $a : b$ , and read “ $a$  to  $b$ .”

**Rational expression.** A quotient of two polynomials with a non-zero denominator. (MA)

**Rational number.** A number expressible in the form  $a/b$  or  $-a/b$  for some fraction  $a/b$ . The rational numbers include the integers. See Illustration 1 in this Glossary. (MA)

**Real number.** An element of the set of numbers consisting of all rational and all irrational numbers. See Illustration 1 in this Glossary. (MA)

**Rectangular array.** An arrangement of mathematical elements into rows and columns. (MA)

**Rectilinear figure.** A polygon, all angles of which are right angles. (MA)

**Recursive pattern (or sequence).** Patterns in which each number is found from the previous number by repeating a process (e.g. Fibonacci numbers). (PASS)

**Reflection.** A type of transformation that flips points about a line, called the *line of reflection*. Taken together, the image and the pre-image have the line of reflection as a line of symmetry. (MA)

**Relation (Real numbers (set of)).** The set of of all rational and irrational numbers (PASS)

**Relation.** A collection of ordered pairs of real numbers.

**Relative frequency.** The empirical counterpart of probability. If an event occurs  $N'$  times in  $N$  trials, its relative frequency is  $N'/N$ . (M)

**Comment [CY24]:** Note that “degree” is not defined.

**Comment [CY25]:** As per our conversation about defining ratio more precisely...

**Comment [CY26]:** Not sure what happened here, but this looks like the definition of the “set of real numbers.”

A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak/akstandards/standards/akstandards\\_eaandmath\\_080812.pdf](http://www.eed.state.ak/akstandards/standards/akstandards_eaandmath_080812.pdf). Accessed 12/1/2015.



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**Remainder Theorem.** If  $f(x)$  is a polynomial in  $x$  then the remainder on dividing  $f(x)$  by  $x - a$  is  $f(a)$ . (M)

**Repeating decimal.** A decimal in which, after a certain point, a particular digit or sequence of digits repeats itself indefinitely; ~~the decimal form of a rational number~~. (M) See also: **terminating decimal**. (MA)

**Rigid motion.** A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures. (MA)

**Rotation.** A type of transformation that turns a figure about a fixed point, called the *center of rotation*. (MA)

**SAS congruence.** (Side-angle-side congruence.) If in two triangles two corresponding sides and the angles formed by those sides are congruent, then the triangles are congruent. (MW)

**SSS congruence.** (Side-side-side congruence.) If two triangles have corresponding sides that are congruent, then the triangles are congruent. (MW)

**Sample space.** In a probability model for a random process, a list of the individual outcomes that are to be considered. (MA)

**Scatter plot.** A graph in the coordinate plane representing a set of **bivariate data**. *For example, the heights and weights of a group of people could be displayed on a scatter plot.* (DPI)

**Comment [CY27]:** Notice how this definition used the term bivariate but “linear association” above didn’t.

**Scientific notation.** A widely used floating-point system in which numbers are expressed as products consisting of a number between 1 and 10 multiplied by an appropriate power of 10, e.g.,  $562 = 5.62 \times 10^2$ . (MW)

**Sequence, progression.** A set of elements ordered so that they can be labeled with consecutive positive integers starting with 1, e.g., 1, 3, 9, 27, 81. In this sequence, 1 is the *first term*, 3 is the *second term*, 9 is the *third term*, and so on. (MA)

**Significant figures.** (digits) Digits included in a measurement that purposely indicate the precision of the measurement. For example, writing a measurement as 3.50 seconds instead of 3.5 seconds indicates that the measurement is accurate to the hundredths place.

**Comment [CY28]:** I took a stab at something a little clearer.

**Similar (Shapes).** Two geometric shapes are said to be similar (to each other) if one can be mapped onto the other by a sequence of similarity transformations.

**Comment [CY29]:** Unless I missed it, I didn’t see SIMILAR anywhere.

**Similarity transformation.** A rigid motion followed by a dilation. (MA)

**Simultaneous equations.** Two or more equations containing common variables. (MW)

**Sine (of an acute angle).** The trigonometric function that for an acute angle is the ratio between the leg opposite the angle when the angle is considered part of a right triangle and the hypotenuse. (M)

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**Spatial sense.** [The ability to build and manipulate](#) mental representations of 2- and 3-dimensional objects and ideas. (PASS)

**Standard deviation.** [A measurement of](#) how much each value in the data differs from the mean of the data. (PASS)

**Comment [CY30]:** Include specific formula for SD?

**Statistics -** [The](#) study of data (PASS)

**Stem-and-leaf plot.** a frequency distribution made by arranging data in the following way (e.g., student scores on a test were 96, 87, 77, 93, 85, 85, and 75 would be displayed as:

9] 6,3

8] 7,5,5

7] 7,5

**Subitize.** Instantly knowing “how many.” Recognizing a number without using other mathematical processes. (Clements)

**Comment [CY31]:** Include an example here, for the uninitiated?

**Supplementary angles.** Two angles whose measures have a sum of 180 degrees. (PASS)

**Supposition.** (act of supposing) [Making](#) a statement or assumption without proof. (PASS)

**Tangent.** a) Meeting a curve or surface in a single point if a sufficiently small interval is considered. b) [\(of an acute angle\)](#) The trigonometric function that, for an acute angle, is the ratio between the leg opposite the angle and the leg adjacent to the angle when the angle is considered part of a right triangle. (MW)

**Tape diagram.** A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model. (MA)

**Terminating decimal.** A decimal is called terminating if its repeating digit is 0. [Every](#) terminating decimal is the decimal form of [some](#) rational number. *See also: **repeating decimal**.* (MA)

**Third quartile.** For a data set with median  $M$ , the third quartile is the median of the data values greater than  $M$ . *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also: **median, first quartile, interquartile range**.* (MA)

**Transformation.** A prescription, or rule, that sets up a one-to-one correspondence between the points in a geometric object (the *pre-image*) and the points in another geometric object (the *image*). Reflections, rotations, translations, and dilations are particular examples of transformations. (MA)

**Transitivity principle for indirect measurement.** If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the

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length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well. (MA)

**Translation.** A type of transformation that moves every point in a graph or geometric figure by the same distance in the same direction without a change in orientation or size. (MW)

**Trigonometric function.** A function (as the sine, cosine, tangent, cotangent, secant, or cosecant) of an arc or angle most simply expressed in terms of the ratios of pairs of sides of a right-angled triangle. (M)

**Trigonometry.** ~~The study of trigonometric functions.~~ ~~The study of triangles, with emphasis on calculations involving the lengths of sides and the measure of angles.~~ (MW)

**Uniform probability model.** A probability model which assigns equal probability to all outcomes. See also: **probability model**.

**Unit fraction.** A fraction with a numerator of 1, such as  $\frac{1}{3}$  or  $\frac{1}{5}$ . (MA)

**Valid.** a) Well-grounded or justifiable; being at once relevant and meaningful, e.g., a *valid* theory; b) Logically correct. (MW)

**Variable.** (a) A quantity that can change or that may take on different values. (b) A symbol (often a letter of the alphabet) that represents a number in a mathematical expression.

**Vector.** A quantity with magnitude and direction, usually represented by an arrow that emanates from one point and has arrowhead ending at another point. defined by an ordered pair or triple of real numbers. (MA)

**Visual fraction model.** A diagram or representation to show the relative size of a fraction, for example, A-a tape diagram, number line diagram, or area model. (MA)

**Whole numbers.** The numbers 0, 1, 2, 3, ... . See Illustration 1 in this Glossary.

Some possible missing terms:

**Range (of a Data Set).** The difference between the maximum and minimum values of a data set, a measure of the spread of the data.

**One-to-one Correspondence (PK.N.2.2, others).** A matching of the elements of two sets such that each element from the first set is matched with one and only one element of the second set, and such that each element of the second set is matched with some element of the first. Early grades students use this to establish the concept of cardinal use of numbers (as in “5” can represent any collection of five objects; if I can match the fingers on one hand to all the elements of a given set then that set has “5” objects.)

- A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak/akstandards/standards/akstandards\\_elaandmath\\_080812.pdf](http://www.eed.state.ak/akstandards/standards/akstandards_elaandmath_080812.pdf). Accessed 12/1/2015.

**Comment [CY32]:** Please note that this definition really only applies to acute angles. I can't cite anything specific but apparently there is some research suggesting that defining trig functions *first* as functions of acute (only) angles in right triangles is limiting, as opposed to defining them more generally in terms of the unit circle (for all real numbers thought of as angles). Regardless, to be precise some mention of this definition being accurate for acute angles should be made.

**Comment [CY33]:** It is surprisingly hard to give an accurate definition of “variable.”

**Comment [CY34]:** This is only true if the vector is emanating from the origin.

Also, vectors can be used on the number line, as in the number line model for representing operations with integers.



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**Categorical Data (PK.D.1.2, others).** Data that measures the number of occurrences of a discrete set of outcomes. E.g., noticing the different colors of shoes in the class and then recording the number of each color.

**Length (of a Segment) (1.GM.2.1 and others).** The length of a (straight) line segment is a measurement of the distance from one endpoint of the object to the other. Once a unit of length is specified, the length of a segment is found by placing such units end-to-end without gaps or overlaps and counting how many such units are used.

**Place Value (2.N.1.3, others).** The concept that the order in which digits are written in the base-10 number system determines the value of that digit. Thus, in the number 245, the digit 2 is in the “hundreds place”, indicating that the value of that particular 2 is actually 2 hundreds or 200.

**Area (2.N.3.1 and others).** A measurement of the amount of space within a closed two-dimensional shape. Area is usually measured in terms of “square units”, in which 1 square unit is the amount of space within a square that measures 1 unit by 1 unit (for a given unit of length). For example, area may be measured in “square centimeters”, 1 square centimeter being the amount of space within a 1cm by 1cm square.

**Right Angle (2.GM.1.4).** Informally, an angle whose measure is 90 degrees. Formally, if two congruent copies of a given angle are supplementary (that is, they form a straight line when one matches an edge of one copy with one edge of the other), then the given angle is said to be a right angle. (We can then define the measure of this angle to be 90 degrees and measure other angles in terms of a right angle.)

**Unit of Measurement (2.GM.2.1 others).** When measuring a given attribute of an object, a “unit” is defined in terms of which all other measurements are determined. That a given unit is fixed is a concept to be learned by young students (e.g. we wouldn’t measure the length of a room in hands because your hand is different from mine, and we wouldn’t measure the length of a room using cm and inches at the same time).

**Bar Graph (2.D.1.1 and others).** A display of categorical data in which vertical or horizontal bars represent the count of a category. The relative lengths of the various bars in the graph are commensurate with the relative sizes of the counts of the data.

**Set Model (for Fractions) (3.N.3.2).** The use of a discrete set of objects to represent the whole and a subset of those objects to represent a fraction. For example, since 3 of the 15 students in class are wearing blue shirts,  $3/15$  of the students are wearing blue shirts.

**Equivalent Fractions (3.N.3.3).** Two fractions  $a/b$  and  $c/d$  are said to be equivalent if there exists a non-zero number  $n$  such that  $na/nb = c/d$ . Equivalent fractions represent the same amount by changing both the size and the number of parts of a given fraction.

**Input/Output Table (3.A.1.2).** Usually a two-column table (or two-row table) with one column (row) listing the inputs of a rule and the other column (row) listing the corresponding outputs for each input.

- A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak/akstandards/standards/akstandards\\_elaandmath\\_080812.pdf](http://www.eed.state.ak/akstandards/standards/akstandards_elaandmath_080812.pdf). Accessed 12/1/2015.



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**Perimeter (of a Polygon) (3.GM.2.3).** The total length of all the edges of a polygon. Often, perimeter is thought of as the distance around an object, traversed once along the edges starting from one vertex and ending at the same vertex.

**Frequency Table (3.D.1.1).** A representation of data in which categories are listed in one column (row) of a table and the number of occurrences (frequency) of each category is indicated in another column (row).

**Line Plot (3.D.1.1).** A representation of data in which categories are listed underneath points on a number line, and in which the number of occurrences (frequency) of each category is represented by a corresponding number of marks (X's, dots) above each category's point.

**Quadrilateral (4.GM.1.2).** A polygon with 4 sides. Important classes of quadrilaterals:

**Trapezoids:** A quadrilateral in which at least two sides are parallel.

**Parallelogram:** A quadrilateral in which opposite sides are parallel.

**Rhombus:** A parallelogram in which opposite sides are congruent (have the same length).

**Rectangle:** A parallelogram that has at least one right interior angle.

**Square:** A rectangle that has all sides congruent.

**Kite:** A quadrilateral that has two pairs of congruent adjacent sides.

**Polygon (4.GM.2.2):** A closed, two-dimensional figure comprised of line segments connected end-to-end, and such that no two segments cross each other. The segments are typically called sides or edges, and the common endpoints of adjacent segments are called vertices (sing. vertex). The space within the polygon is called its *interior*. The angles formed by adjacent sides that lie in the interior of a polygon are called its *interior* angles.

**Venn Diagram (4.D.1.2).** A data display in which (typically) circles are used to represent categories and in which the overlapping of two (or more) circles indicates data that lies in each category in the overlap.

**Quotient (5.N.1.3).** The result of a division problem. Also, given whole numbers  $n$  and  $m$  with  $n > m$ , if we write

$$n = mq + r$$

with  $0 \leq r < m$ , then we say  $q$  is the quotient and  $r$  is the remainder.

**Mixed Number (5.N.2.3).** A number written in the form  $A\frac{b}{c}$ , which is a shorthand way to represent the quantity  $A + \frac{b}{c}$ . A mixed number may be written as a fraction greater than 1 by writing

$$A\frac{b}{c} = A + \frac{b}{c} = \frac{Ac}{c} + \frac{b}{c} = \frac{Ac + b}{c}.$$

**Equivalent Expressions (5.A.2.1).** Two expressions (numerical or otherwise) are said to be equivalent if one can be obtained from the other using the properties of operations, such as the commutative, associative and distributive properties, as well as by representing numbers in the expressions in different but equivalent forms.

**Comment [CY35]:** Am I thinking of the right thing here?

- A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>, and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak.gov/akstandards/standards/akstandards\\_eaandmath\\_080812.pdf](http://www.eed.state.ak.gov/akstandards/standards/akstandards_eaandmath_080812.pdf), Accessed 12/1/2015.



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**Triangle (5.GM.1.1).** A polygon with three sides. Important classes of triangles:

**Equilateral Triangle:** A triangle with all sides congruent.

**Right Triangle:** Contains an interior angle that is a right angle.

**Scalene Triangle:** A triangle with no side congruent to another.

**Isosceles Triangle:** A triangle with two congruent sides.

**Rectangular Prism (5.GM.1.2).** A three-dimensional object constructed from three pairs of parallel rectangles (called *faces* in this context) that share common edges so as to form an enclosed space and such that opposite rectangles are congruent. The vertices of the rectangles are the vertices of the prism, and the sides of the rectangles are called edges. A **cube** is a rectangular prism in which each face is a square of the same size as the other faces.

**Pyramid (5.GM.1.2).** A three-dimensional shape constructed from a polygon (called the *base*) and triangles that have one edge matching the edges of the base and such that the triangles share a common vertex. (DIAGRAM.)

**Net (5.GM.1.3).** A two-dimensional representation of a three-dimensional figure constructed of polygons, such that if folds were made on certain edges of the net and appropriate sides were “glued” together, the resulting figure would be the original three-dimensional figure.

**Percent (6.N.1.3).** A percent represents a certain number of parts out of 100. Equivalently, a percent can be considered as a ratio:  $n\%$  represents a ratio of  $n:100$ .

**Greatest Common Factor (6.N.1.6).** The GCF of a finite set of whole numbers is a whole number that is simultaneously a factor of all of the numbers in the set, and such that any other common factor of all of the numbers in the set is itself a factor of the GCF.

**Least Common Multiple (6.N.1.6).** The LCM of a finite collection of whole numbers is a whole number that is simultaneously a multiple of all of the numbers in the set, and such that any other multiple of all of the numbers in the set is itself a multiple of the LCM.

**Unit Rate (6.N.3.2):** A ratio  $a:b$  is said to have unit rate  $a/b$  units of the first quantity per 1 unit of the second quantity.

**Equivalent Ratios (6.N.3.3).** Two ratios  $a:b$  and  $c:d$  are equivalent if there is a non-zero number  $k$  such that  $ka = c$  and  $kb = d$ . Equivalent ratios can be shown to have the same unit rate.

**Angle (6.GM.2.1 and earlier).** Two rays with a shared vertex determine an angle. The shared vertex is called the vertex of the angle, and the two rays the sides of the angle. We often think of the angle as the “space within” the two rays; note there are technically two different angles created when two rays share a vertex. (DIAGRAM?)

Some special angle pairs:

**Supplementary Angles:** Angles that share one side but such that their unshared sides form a straight line.

**Vertical Angles:** The non-adjacent angles formed when two distinct lines intersect. Vertical angles can be shown to always be congruent.

- A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak/akstandards/standards/akstandards\\_elaandmath\\_080812.pdf](http://www.eed.state.ak/akstandards/standards/akstandards_elaandmath_080812.pdf). Accessed 12/1/2015.

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**Complementary Angles:** Angles that share a side but such that their unshared sides form a right angle.

**Line of Symmetry (2D) (6.GM.4.3).** A line that can be drawn that intersects the interior of a shape and such that the resulting shapes on either side of this line are reflections of each other is called a line of symmetry for that shape.

**Proportional Relationship (7.A.1.1).** A collection of pairs of quantities that exhibit equivalent ratios. The collection  $(x, y)$  can be shown to be proportional if and only if there is a non-zero number  $k$  with  $y = kx$ .

**Constant of Proportionality (7.A.1.2).** Given a proportional relationship expressed as  $y = kx$ , the number  $k$  is often called the constant of proportionality.

**Slope (of a Line) (7.A.2.1).** A measure of the steepness of a line in a Cartesian plane, found by determining the constant change in the  $y$ -coordinate per 1-unit change in the  $x$ -coordinate.

**Surface Area (of a Rectangular Prism) (7.GM.1.1).** The total measure of the area of the faces of a rectangular prism. Equivalently, the total area of a net for the prism.

**Volume (of a 3D Object) (7.GM.1.2).** A measurement of the amount of space within a closed three-dimensional shape. Volume is often measured in terms of “cubic units”, in which 1 cubic unit is the amount of space within a cube that measures 1 unit by 1 unit by 1 unit (for a given unit of length). For example, volume may be measured in “cubic centimeters”, 1 cubic centimeter being the amount of space within a 1cm by 1cm by 1cm cube. Note that since one can measure the volume of a liquid by placing said liquid into a 3D shape, volume has historically been measured in various units such as cups, fluid ounces, and liters. Note that 1 cubic centimeter is equal to 1 milliliter, one way to connect such fluid units to cubic units.

**Circle (7.GM.3.1 and others).** The set of all points that are equidistant from a given point, called the center of the circle. The set of points that lie inside the circle is called the *interior* of the circle. Important terms:

**Radius of a Circle:** Both a segment with one endpoint the center of the circle and the other endpoint on the circle, and the length of this segment (which is necessarily the same for any point on the circle).

**Diameter of a Circle:** Both a segment with endpoints on the circle that contains the center, and the length of this segment.

**Circumference of a Circle:** The length of the circle if cut and opened up to make a straight line segment, which can be found with  $C = 2\pi r$  where  $r$  is the radius and  $\pi$  is the irrational number “pi”. (Can be thought of as the perimeter of the circle.)

**Area of a Circle:** The area of the interior of the circle, which can be found with  $A = \pi r^2$  where  $r$  is the radius and  $\pi$  the irrational number “pi”.

**Pi ( $\pi$ ) (7.GM.3.1).** The irrational number that is derived by finding the ratio of the circumference to the diameter of circles. That this ratio is constant and an irrational number are important concepts and challenging to prove, so they are often arrived at empirically by students.

- A. Adapted from Massachusetts Department of Elementary & Secondary Education. <http://www.doe.mass.edu/frameworks/math/0311.pdf>. and from Alaska English/Language Arts and Mathematics Standards, June 2012. [www.eed.state.ak/akstandards/standards/akstandards\\_elaandmath\\_080812.pdf](http://www.eed.state.ak/akstandards/standards/akstandards_elaandmath_080812.pdf). Accessed 12/1/2015.



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**Scale Factor (7.GM.4.1).** For similar shapes, the common ratio of corresponding side lengths is called the scale factor. Informally, it is the multiplicative amount by which the lengths of one shape are “blown up” or “shrunk down” to obtain the other shape to which it is similar.

**Exponents (Integer) (PA.N.1.1).** A negative integer exponent denotes the reciprocal of the base raised to the corresponding opposite integer. Thus  $x^{-2} = \frac{1}{x^2}$ .

**Perfect Square (PA.N.1.5).** A number that is a whole number squared, that is, a number that can be expressed as  $n^2$  for  $n$  a whole number.

**Intercepts (of a Graph) (PA.A.2.3).** Geometrically, where a graph intersects an axis in a Cartesian plane.

**Substitution (PA.A.3.1).** The substitution of one expression for an equivalent expression, used when rewriting expressions as equivalent ones or solving equations. It is based on the *transitive property of equality*, which states, “If  $A = B$ , and  $B = C$ , then  $A = C$ .”

**Outlier (PA.D.1.2).** A data point that is far outside a representative range of the data set. For example, once the inter-quartile range (IQR) is computed, one might calculate the interval of  $1.5 \times$  IQR above the median and  $1.5 \times$  IQR below the median and decide that any data point that lies outside this range is considered an outlier.

**Literal Equation (A1.A.3.1).** An equation involving multiple variables and numbers, often that cannot be solved for an explicit numerical value of any of the individual variables. In such a case one may solve for one variable as an expression of the others.

**Arithmetic Sequence (Progression) (A1.A.3.5).** A sequence in which successive terms exhibit a common difference.

**Parallel Lines (A1.A.4.2).** Lines that do not intersect. Distinct lines can be shown to be parallel if and only if they have equal slopes.

**Perpendicular Lines (A1.A.4.2).** Lines that intersect such that all four angles that are created are congruent. Two lines can be shown to be perpendicular if and only if the product of their slopes is  $-1$ .

**Independent Variable (A1.F.1.2).** The input of a function. The quantity whose value is changed to affect the output.

**Dependent Variable (A1.F.1.2).** The output of a function. The quantity that is affected when the input is changed.

**Piecewise Function (A1.F.1.4).** A function that is defined differently on different intervals.

**Summary Statistics (A1.D.1.1).** A collection of statistics (measurements based on data) that describe the data set. For example, the range, mean, and standard deviation of a given data set indicate certain features of the data set and hence are summary statistics.

**Discrete Graph (of Data) (A1.D.1.3).** A graph is discrete if it consists of separated data points and contains no intervals of data.

**Continuous Graph (of Data) (A1.D.1.3).** A graph is continuous if it contains intervals of data points.

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**Multiplication Counting Principle (A1.D.2.1).** If  $k$  actions can be taken in  $N_1, N_2, \dots, N_k$  different ways, then there are a total of  $N_1 \cdot N_2 \cdot \dots \cdot N_k$  different ways to perform those actions in sequence.

**Intersection (of Sets) (A1.D.2.2).** For two sets  $A$  and  $B$ , the intersection  $A \cap B$  is the set of all elements that are members of both sets simultaneously.

**Union (of Sets) (A1.D.2.2).** For two sets  $A$  and  $B$ , the union  $A \cup B$  is the set of all elements that are members of one or both of the sets.

**Complement (of a Set) (A1.D.2.2).** A set  $A$  is typically considered to be a subset of an understood “universal set”. The complement of  $A$ , denoted by  $A^C$  is the set of all elements of the universal set that are not members of  $A$ .

**Experimental Probability (A1.D.2.3).** When trials of a probability experiment are run and data is collected, the experimental probability of a desired outcome is the relative frequency of that outcome as a ratio of the number of such outcomes to the total number of outcomes. For example, if a coin is flipped 100 times, and heads comes up 45 times, then the experimental probability of heads is  $45/100$  or  $0.45$ . (The *theoretical probability* is  $0.50$ , and if the number of trials is increased the experimental probability will get closer and closer to  $0.50$ .)

**Inductive Reasoning (G.RL.1.2).** Informally, the process of examining patterns and making conclusions based on observed patterns.

**Deductive Reasoning (G.RL.1.2).** Informally, the process of using known facts and relationships to derive new facts and relationships.

**Conditional Statement (G.RL.1.2).** A statement of the form, “If  $P$ , then  $Q$ ,” where each of  $P$  and  $Q$  are themselves statements. For example, “If it rains, then the streets get wet,” is a conditional statement.

If the conditional statement “If  $P$ , then  $Q$ ,” is *true*, then this means that it is never the case that the statement  $P$  is true while the statement  $Q$  is false. For example, it will never be the case that “it rained” but “the streets are not wet”.

Related statements are:

**Converse:** “If  $Q$ , then  $P$ .” This may or may not be true if the original statement is true.

**Inverse:** “If NOT  $P$ , then NOT  $Q$ .” This may or may not be true if the original statement is true.

**Contrapositive:** “If NOT  $Q$ , then NOT  $P$ .” This is always true if the original statement is true, and vice versa. For an example, notice that, “If the streets are NOT wet, then it did NOT rain,” is logically equivalent to the example statement above.

**Counterexample (G.RL.1.3).** An example to show that a given statement is false. For example, to disprove the statement “All right triangles are isosceles,” all one needs to do is produce a right triangle that is scalene.

**Transversal Line (G.2D.1.1).** A line that crosses two or more other lines is called a transversal.

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**Exterior Angles (of a Polygon) (G.2D.1.1).** The supplement of an interior angle of a polygon that is formed by extending one of the line segments determining the interior angle at a given vertex.

**HL Congruence (G.2D.1.8).** If two right triangles have hypotenuse and one corresponding leg congruent, then the triangles are congruent.

**Hypotenuse (G.2D.1.8).** The longest side of a right triangle, necessarily opposite to the right angle. The other sides are called the *legs* of the right triangle (*longer* and *shorter* if applicable).

**G.C.1.2:** A whole bunch of circle terms: arcs, secants, chords, tangents.

**Extraneous Solution (A2.A.1.3).** A solution introduced to an equation by an irreversible algebraic step. (EXAMPLE.)

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**Tables and Illustrations  
of Key Mathematical Properties, Rules, and Number Sets**

TABLE 1. Common addition and subtraction situations.<sup>[3]</sup>

	Result Unknown	Change Unknown	Start Unknown
<b>Add to</b>	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
<b>Take from</b>	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	<b>Total Unknown</b>	<b>Addend Unknown</b>	<b>Both Addends Unknown<sup>[4]</sup></b>
<b>Put Together/</b>	Three red apples and	Five apples are on	Grandma has five

<sup>3</sup> Adapted from Box 2-4 of the Mathematics Learning in Early Childhood. National Research Council (2009,op.cit.pp.32,33)

<sup>4</sup> These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

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<b>Take Apart</b> <sup>5</sup>	two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$
	<b>Difference Unknown</b>	<b>Bigger Unknown</b>	<b>Smaller Unknown</b>
<b>Compare</b> <sup>6]</sup>	<p>("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?</p> <p>("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? <math>2 + ? = 5, 5 - 2 = ?</math></p>	<p>(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? <math>2 + 3 = ?, 3 + 2 = ?</math></p> <p>(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?</p>	<p>(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? <math>5 - 3 = ?, ? + 3 = 5</math></p> <p>(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?</p>

<sup>5</sup> Either addend can be unknown so there are three variations of these problem situations. Both addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

<sup>6</sup> For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

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TABLE 2. Common multiplication and division situations.<sup>[7]</sup>

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
<b>Equal Groups</b>	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
<b>Arrays,<sup>[8]</sup> Area<sup>[9]</sup></b>	There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area

<sup>7</sup> The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

<sup>8</sup> The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

<sup>9</sup> Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

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	by 6 cm rectangle?	18 square centimeters. If one side is 3 cm long, how long is a side next to it?	18 square centimeters. If one side is 6 cm long, how long is a side next to it?
<b>Compare</b>	<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p><i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
<b>General</b>	$a \cdot b = ?$	$a \cdot ? = p$ and $p, a = ?$	$? \cdot b = p$ and $p, b = ?$

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**TABLE 1. The properties of operations.**

Here  $a$ ,  $b$  and  $c$  stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

<p><i>Associative property of addition</i></p> <p><i>Commutative property of addition</i></p> <p><i>Additive identity property of 0</i></p> <p><i>Existence of additive inverses</i></p>	$(a + b) + c = a + (b + c)$ $a + b = b + a$ $a + 0 = 0 + a = a$ <p>For every <math>a</math> there exists <math>-a</math> so that <math>a + (-a) = (-a) + a = 0</math>.</p>
<p><i>Associative property of multiplication</i></p> <p><i>Commutative property of multiplication</i></p> <p><i>Multiplicative identity property of 1</i></p> <p><i>Existence of multiplicative inverses</i></p>	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ $a \cdot b = b \cdot a$ $a \cdot 1 = 1 \cdot a = a$ <p>For every <math>a \neq 0</math> there exists <math>1/a</math> so that <math>a \cdot 1/a = 1/a \cdot a = 1</math>.</p>
<p><i>Distributive property of multiplication over addition</i></p>	$a \cdot (b + c) = a \cdot b + a \cdot c$

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**TABLE 2. The properties of equality.**

Here  $a$ ,  $b$ , and  $c$  stand for arbitrary numbers in the rational, real, or complex number systems.

<i>Reflexive property of equality</i>	$a = a$
<i>Symmetric property of equality</i>	If $a = b$ , then $b = a$ .
<i>Transitive property of equality</i>	If $a = b$ and $b = c$ , then $a = c$ .
<i>Addition property of equality</i>	If $a = b$ , then $a + c = b + c$ .
<i>Subtraction property of equality</i>	If $a = b$ , then $a - c = b - c$ .
<i>Multiplication property of equality</i>	If $a = b$ , then $a \cdot c = b \cdot c$ .
<i>Division property of equality</i>	If $a = b$ and $c \neq 0$ , then $\frac{a}{c} = \frac{b}{c}$ .
<i>Substitution property of equality</i>	If $a = b$ , then $b$ may be substituted for $a$ in any expression containing $a$ .

**TABLE 3. The properties of inequality.**

Here  $a$ ,  $b$ , and  $c$  stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$ , $a = b$ , $a > b$ . If $a > b$ and $b > c$ then $a > c$ .  If $a > b$ , then $b < a$ .  If $a > b$ , then $-a < -b$ .  If $a > b$ , then $a \pm c > b \pm c$ .  If $a > b$ and $c > 0$ , then $a \cdot c > b \cdot c$ .
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If  $a > b$  and  $c < 0$ , then  $a \cdot c < b \cdot c$ .

If  $a > b$  and  $c > 0$ , then  $a \cdot c > b \cdot c$ .

If  $a > b$  and  $c < 0$ , then  $a \cdot c < b \cdot c$ .

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## SAMPLE OF Works Consulted

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[1] Adapted from Boxes 2–4 of *Mathematics Learning in Early Childhood*, National Research Council (2009, pp. 32–33).

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